DETECTION OF UNBALANCE IN A ROTOR-BEARING SYSTEM USING A DETERMINISTIC-STOCHASTIC APPROACH

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Disk unbalance in rotating shaft of machines is a common problem. Direct measurement of unbalance forces due to disk unbalance is not possible, therefore, usually the unbalance forces are predicted via response measurements. In the present work, a model-based approach with joint input-state (JIS) estimation technique to detect the amount of unbalance present in a disc is proposed. The JIS technique is of a deterministic-stochastic type, therefore, capable of estimating the unbalance forces in noisy measurements. A numerical case study is presented for a single disk rotor system. The state-space form of a modally reduced order model for the rotor-bearing system and a limited set of response measurements are used. Mode shapes are obtained from a finite element analysis of the rotor-bearing system. Results are presented for different levels of measurement errors, and the effect of different vibration measurements (displacements and accelerations) are shown. The efficacy of the proposed method is validated for different unbalance conditions and rotor speeds.

Keywords: Disk unbalance; rotor-bearing system; joint input-state estimation.

1. Introduction

Rotating machines typically consist of a rotor-disk-bearing and often subjected to periodic forces e.g. force due to unbalance [1, 2]. Fault identification in rotating machines via signal processing techniques are widely used in practice for qualitative analysis. For quantitative analysis, model-based approaches are an active area of research. Meanwhile, these approaches are combined with mathematical models for optimal estimates of system states from noisy measurements.

Optimal state estimation is one of the state-of-the-art methods for fault identification under model-based approach. Kalman filter and its variants for linear and nonlinear systems are applied to rotordynamics for various purposes e.g. crack detection [3], parameter identification [4], fault identification [5], etc. These techniques are capable of dealing with the presence of noise in the measured signal as well as the presence of exogenous disturbances.

Finite element methods (FEM) are widely used for modeling and analyzing structural systems. Dynamic FE models of complex systems have high dimensionality and may not be efficient and accurate, therefore, reduced order models are needed for efficient performance. Qu [6] has discussed different model order reduction techniques used in finite element analysis. In the proposed technique the finite element method is used to model the rotor system.

Forward analysis in structural dynamics is well developed and easy to perform, where system models and forcing functions are known. However, when the forcing functions are not known, then input estimation techniques need to be adopted using the responses. A joint-input estimation technique is developed for simultaneous estimation of input and state of the system [7], it uses a state-space representation of the system. Later, this technique is applied in structural dynamics for force and response estimation [8]. The JIS technique has been successfully applied for damage prediction in a metallic body via limited response measurements [9]. Using reduced number of measured responses in JIS technique modal displacements are estimated to calculate strain time history.

In the present work, a modally reduced order model is used in the algorithm. A joint input-estimation technique is used to estimate unbalance forces using a reduced order model and noisy response measurements. Different set of measurements are used that consists of accelerations, displacements, and strains. The responses are measured from a full order finite element model. Section 2 consist of the formulations for the system model and JIS technique. A numerical case study is presented in section 3 for a single disk rotor system to demonstrate the efficacy of the proposed technique. Effects of the different noise levels are also presented with normalized mean square error (NMSE) between exact and estimated forces.

2. Formulation

2.1 State-space description

In structural dynamics, finite element models are widely used. A vibrating structure modeled as a linear dynamic system is described by:

$$[M]\{\ddot{x}\} + [C_d]\{\dot{x}\} + [K]\{x\} = S_p p(t) \tag{1}$$

where $[M]$, $[C_d]$, and $[K]$ are mass, damping, and stiffness matrices, respectively. $p(t)$ is the force vector, S_p is a force selection matrix relating the forces with the corresponding degrees of freedom of the structure, $x \in R^n$ is a vector of nodal displacement and the dot over the vector represents differentiation with respect to time.

The proposed algorithm requires a modal domain representation of the system. Using the coordinate transformation $x(t) = \Phi\eta(t)$, where the matrix Φ contains the eigenvectors in each column, η represent the modal coordinates, and premultiplying the equation of motion (Eq. (1)) by Φ^T (transpose of Φ), we obtain the following set of equations:

$$\Phi^T[M]\Phi\{\ddot{\eta}\} + \Phi^T[C_d]\Phi\{\dot{\eta}\} + \Phi^T[K]\Phi\{\eta\} = \Phi^T S_p p(t) \tag{2}$$

Using mass-normalized eigenvectors Φ and assuming proportional damping, the mass, stiffness and damping matrices can be diagonalized. In turn, the following expression holds,

$\Phi^T[M]\Phi = I$, $\Phi^T[K]\Phi = \Omega^2$, and $\Phi^T C_d \Phi = \Gamma = \text{diag}\{2\xi_j\omega_j\}$.

where I is a $n \times n$ unit matrix, $\Omega = \text{diag}\{\omega_j\}$, ξ_j and ω_j represents j^{th} modal damping ratio and natural frequency, respectively. In modal coordinates, the equation of motion becomes

$$\{\ddot{\eta}\} + \Gamma\{\dot{\eta}\} + \Omega^2\{\eta\} = \Phi^T S_p p(t) \tag{3}$$

The equation of motion in continuous time state-space form is represented by,

$$\dot{x}(t) = A_c x(t) + B_c p(t) \tag{4}$$

$$y(t) = G_c x(t) + J_c p(t) \tag{5}$$

By using the modal reduction technique, higher order systems can be reduced to lower order systems, which decreases the computational efforts. The model order reduction of large structures is necessary because the number of response measurements is less compared to its size. For this case, the number of columns of the modal matrix will be reduced to the selected number of modes.

$$x(t)=\begin{Bmatrix}\eta\\ \dot{\eta}\end{Bmatrix},\quad A_c=\begin{bmatrix}0 & I\\ -\Omega^2 & -\Gamma\end{bmatrix},\quad B_c=\begin{bmatrix}0\\ \Phi_r^T S_p\end{bmatrix},G_c=\begin{bmatrix}S_d\Phi_r-S_a\Phi_r\Omega^2 & S_v\Phi_r-S_a\Phi_r\Gamma\end{bmatrix}$$

$$J_c=\begin{cases}[S_a\Phi_r\Phi_r^T S_p], & \text{if the output is acceleration}\\ 0, & \text{otherwise}\end{cases}$$

Vector $x(t)$ contains modal states of the reduced order model and matrix Φ_r contains modal vectors corresponding to the selected modes of vibration.

The measurements are observed at discrete time steps, hence, the continuous time state-space model need to be converted into a discrete model. For a sampling time of Δt, a discrete time state-space model is expressed by

$$x_{k+1}=Ax_k+Bp_k \tag{6}$$

$$y_k=Gx_k+Jp_k \tag{7}$$

2.2 Joint input-state estimation

The purpose of the joint input-state estimation is to identify the unknown inputs along with the state estimate. The joint estimator used in the present research is based on the linear minimum-variance unbiased estimation, a recursive filter, which is derived by Gillijns and De Moor [7]. The unknown input is estimated from the predicted states and the current measurement. In this algorithm, it is assumed that the unknown input can be a signal of any type without any prior information.

For a linear discrete-time system represented by following set of equations,

$$x_{k+1}=Ax_k+Bp_k+w_k \tag{8}$$

$$y_k=Gx_k+Jp_k+v_k \tag{9}$$

An initial unbiased estimate and its covariance are assumed known. Three step filter equations are summarized as:

Initialization

$$\hat{x}_0=E[x_0] \tag{10}$$

$$P_0^x=E[(x_0-\hat{x}_0)^T] \tag{11}$$

Estimation of unknown input

$$\tilde{R}_k=GP_{k/k-1}^xG^T+R \tag{12}$$

$$M_k=(J^T\tilde{R}_k^{-1}J)^{-1}J^T\tilde{R}_k^{-1} \tag{13}$$

$$\hat{p}_k=M_k(y_k-G\hat{x}_{k/k-1}) \tag{14}$$

$$P_k^p=(J^T\tilde{R}_k^{-1}J)^{-1} \tag{15}$$

Measurement Update

$$K_k=P_{k/k-1}^xG^T\tilde{R}_k^{-1} \tag{16}$$

$$\hat{x}_{k/k}=\hat{x}_{k/k-1}+K_k(y_k-G\hat{x}_{k/k-1}-J\hat{p}_k) \tag{17}$$

$$P_{k/k}^x=P_{k/k-1}^x-K_k(\tilde{R}_k-JP_k^pJ^T)K_k^T \tag{18}$$

$$P_k^{xp}=(P_k^{px})^T=-K_kJP_k^p \tag{19}$$

Time Update

$$\hat{x}_{k+1/k}=A\hat{x}_{k/k}+B\hat{p}_k \tag{20}$$

$$P_{k+1/k}^x=\begin{bmatrix}A & B\end{bmatrix}\begin{bmatrix}P_{k/k}^x & P_k^{xp}\\ P_k^{px} & P_k^p\end{bmatrix}\begin{bmatrix}A^T\\ B\end{bmatrix}+Q \tag{21}$$

where $\hat{x}_{k/k}$ is the state estimate, $\hat{p}_k$ is the force estimate, P_k^p is the error covariance matrix for input estimate, $P_{k/k}^x$ is the error covariance matrix for state estimate, and the matrix P_k^{xp} is the error cross-covariance (between input and state estimates) matrix. K_k is the gain matrix, R and Q are the measurement and process noise covariance matrices, respectively.

For the case when number of measurements (n_d) and number of applied forces (n_p) exceeds the number of modes (n_m), the matrices $\tilde{R}_k$, $J^T\tilde{R}_k^{-1}J$ and $JP_k^pJ^T$ becomes rank deficient which leads to instability. Therefore, the bases of these matrices need to be reduced [8].

3. Numerical Example

In this section, a numerical case study is presented for the verification of the proposed strain estimation technique. The rotor-disk-bearing assembly[1] is shown in the Fig. 1 whose geometrical dimensions and mechanical properties are presented in Table 1. A cartesian coordinate is attached to the rotor at left bearing, y and z axes are shown and the direction of positive x axis (not shown) is normal and outward to y-z plane. In finite element modeling the rotor system is discretized into six beam elements. The ratio of shaft diameter/length is ≈ 22.8 hence modeled with Euler-Bernoulli beam elements with uniform cross-section, dimensions and same elastic properties.

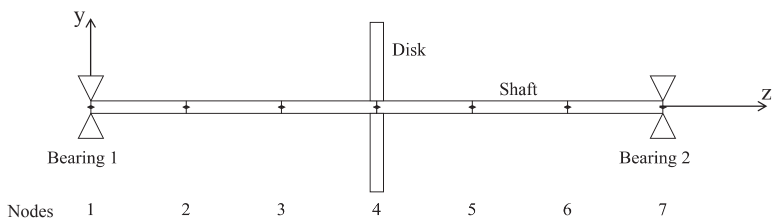

Figure 1: Schematic diagram of single disc rotor system.

Table 1: Rotor-disk-bearing data.

Shaft :	Length – 362 mm, Diameter – 15.875 mm Modulus of elasticity – 2.08×10^{11} N/m^2
Disk :	Mass – 782 g, Thickness – 15.875 mm, Diameter – 152.4 mm
Bearing :	Stiffness – 2.57×10^7 N/m, Damping – 250 Ns/m

In the present work, the disk unbalance is considered as an excitation to the shaft rotating with a constant speed (ω_{shaft}) as represented in Fig. 2. The rotor bow can be neglected because of the thin disk. The unbalance force can be given as,

$$F_{unb} = me\omega_{shaft}^2 \tag{22}$$

The unbalance mass m is located at the radial distance e, and ω_{shaft} is the rotating speed of the shaft.

3.1 Unbalance detection

First, the responses are generated from the full order model of the system and contaminated with white noise. To simulate the measurement error, we have added random noise which can be expressed as, $noise = \gamma\sigma r_k$, where γ is the noise level (%), σ is the standard deviation of the response signal and

[1]The rotor system layout is chosen to match the machinery fault simulator (MFS) facility available at Acoustics and Condition Monitoring Laboratory, Indian Institute of Technology Kharagpur, India.

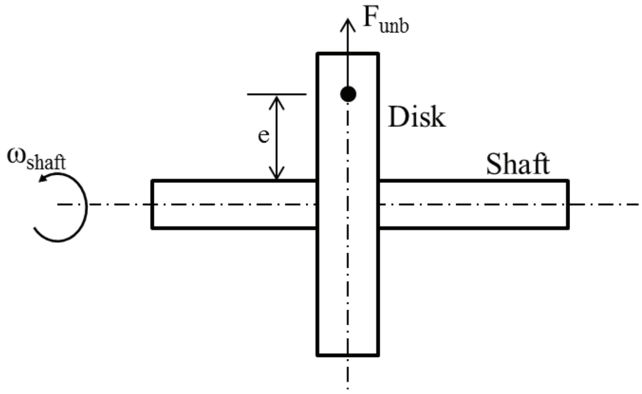

Figure 2: Static disk unbalance.

r_k is the random sequences. Then, these noisy responses are used to estimate forces with a reduced order model. The first natural frequency of the system is ≈ 124Hz, and unbalance force excites the bending mode of vibration. Hence, first two orthogonal bending modes are selected in the reduced order model. The natural frequencies, mode shapes, and damping ratios are calculated from the same model used in the response generation. Hence, the modeling error is not considered in the present analysis. Only the effect of measurement noise is observed.

Three different measurement sets are used in the present study. In set 1 the accelerations in x and y direction at disk location are used, set 2 consists of displacements along-with accelerations. In set 3, the accelerations at node 4 and normal strains on shaft at node 3 in x-z and y-z plane are considered. It is noted here that, two components of the unbalance force are to be estimated by JIS technique, therefore, two accelerations are necessary [10].

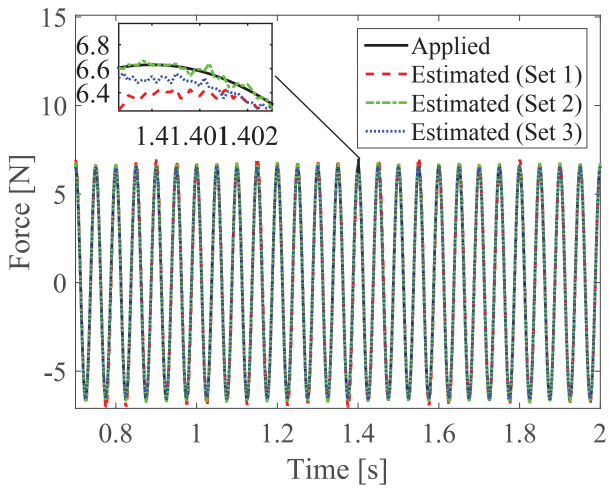

Figure 3: Exact and estimated unbalance force for rotor speed 1200 rpm.

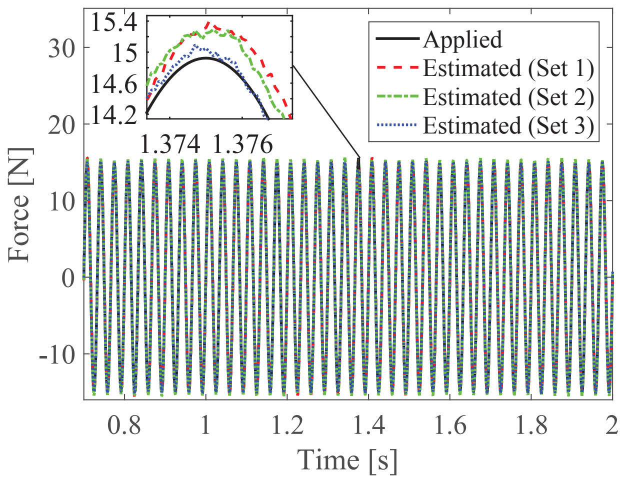

Figure 4: Exact and estimated unbalance force for rotor speed 1800 rpm.

The detection of amount of unbalance (me) depends on the estimated forces. The exact and estimated forces (obtained from different set of measurements) are shown in Figs. 3 and 4. The modal states obtained using the algorithm are affected with low frequency drift, therefore, a Butterworth high-pass filter with 5 Hz cut-off frequency is used to remove unwanted trends.

The spectrum of the exact and estimated force for 1200 rpm and 1800 rpm are shown in Figs. 5 and 6, respectively. The amount of unbalance (me) can be obtained by dividing the amplitude of estimated force by ω_{shaft}^2. The calculated amount of unbalances for both cases are 4.256×10^{-4}kg-m and 4.23×10^{-4}kg-m with 1.3% and 0.7% error when compared with exact amount of unbalance

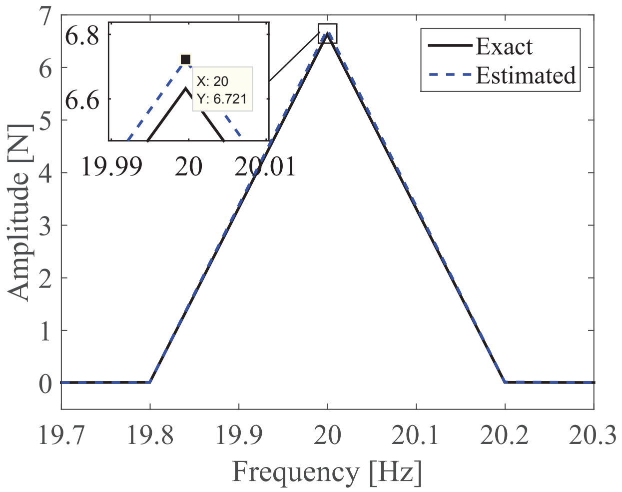

Figure 5: Frequency domain representation of exact and estimated unbalance force for rotor speed 1200 rpm.

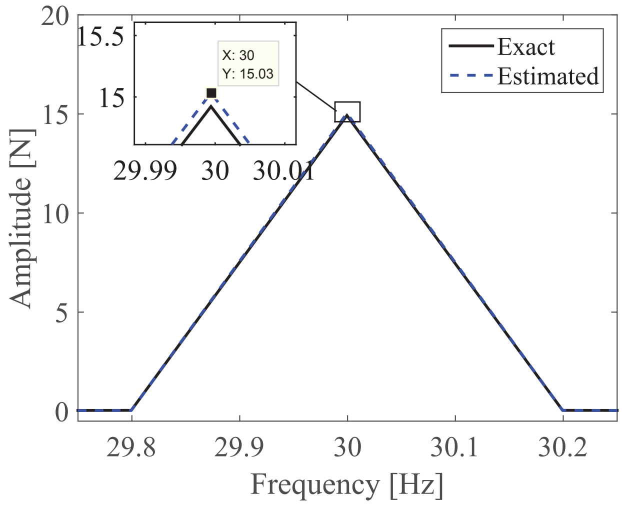

Figure 6: Frequency domain representation of exact and estimated unbalance force for rotor speed 1800 rpm.

(4.2×10^{-4}kg-m).

To observe the effects of measurement noise on estimated forces, three levels of noise (5%, 10%, and 20%) are considered. The error estimates are presented in Table 2 for two different running speed. The normalized mean square errors between exact and estimated unbalance forces are evaluated, which is given by,

$$\text{NMSE}(\%) = \frac{\sqrt{\sum_{i=1}^{N} [F_{exact}(i) - F_{estimated}(i)]^2}}{\sqrt{\sum_{i=1}^{N} [F_{exact}(i)]^2}} \times 100 \tag{23}$$

where F_{exact} is the exact force, $F_{estimated}$ is the estimated force, and N is the number of time-steps.

Table 2: Normalised mean square error (NMSE).

Measurement set	Rotor speed (RPM)	Measurement noise (%)	NMSE (%)
Set 1	1200	5	6.48
		10	7.0
		20	8.13
	1800	5	4.91
		10	7.26
		20	12.46
Set 3	1200	5	3.4
		10	6.81
		20	8.78
	1800	5	2.77
		10	6.55
		20	11.83

4. Conclusions

In this paper, a technique to quantify the amount of unbalance present in the rotor system is presented. The joint-input state estimation technique is used to estimate unbalance forces using a reduced order model and with noisy measurements. The numerical example verifies the efficacy of the present technique which is tested for different shaft speed and levels of measurement noise. The force estimates are followed by calculation of the amount of unbalance for known running speed. The algorithm provides better estimates when displacements/strains are included in the measurement set.

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