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AN IMPEDANCE APPROACH TO PIPEWORK SYSTEM ANALYSIS USING THE TRANSMISSION MATRIX METHOD

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1. INTRODUCTION

Machines in industrial applications are frequently isolated with compliant mounts as a vibration control measure. Unfortunately, the performance of these isolators is offset by other vibrational paths such as airborne noise or pipework. The pipework transmission can occur due to mechanical excitation of the machine or on account of acoustic pressure pulsations generated by pumps. The significance of each of these vibrational paths can be assessed before construction from an estimate of the vibrational power that they will transmit individually. For the case of the pipework this requires the force and velocity at the various mounting points and the pressure and volume velocity at the termination.

The transmission matrix method is used to couple straight pipe subsystems into a two-dimensional pipe system. The pipe subsystems are chosen to be sufficiently short so as to be represented as a lumped mass and stiffness element. Flexural vibration, involving rotation and lateral vibration and axial vibration are considered. A much more rigorous approach is given by El-Raheh [1].

The overall objective is to vary the stiffness and damping parameters to minimise the total power transmission from the pipe coupling points and the pipe outlet. Reported here is the initial stage where the effect of the internal fluid is not included.

2. IMPEDENCE AND TRANSMISSION MATRIX

For a simple empty straight pipe segment moving only in longitudinal motion (Figure 1) the force and velocity at one end F_1, V_1 can be written in terms of the force and velocity at the other end F_2, V_2 using the impedance matrix of the pipe:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} \qquad (1)$$

where Z_{11} and Z_{21} are the point and transfer impedances found by setting $V_2 = 0$. It is possible to obtain expressions for these impedances for each straight pipe segment, and then combine these matrices to get the overall response for a coupled system of segments. However, this procedure is rather ungainly as some matrix inversion is required. It is numerically more efficient to rewrite equation (1) in the transmission matrix form:

$$\{P\}_i = [T]_i \{P\}_{i-1} \quad , \quad [T] = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \tag{2}$$

where $\{P\}_i = \begin{Bmatrix} F_i \\ V_i \end{Bmatrix}$

is the force and velocity at the input of the i^{th} element. $[T]_i$ is the transmission matrix of the i^{th} element, $\{P\}_{i-1}$ are the force and velocity of the other end of the i^{th} element. When n elements are combined to give the total transmission [T], only multiplication is necessary according to the scheme below:

$$[T] = [T]_n [T]_{n-1} \cdots [T]_i \cdots [T]_1 \tag{3}$$

The coefficients in the transmission matrix are found by expanding equation (2) and making a comparison with the expansion of the impedance matrix 1. The transfer matrix [T] can be used to give all relevant information such as transfer functions, resonance frequencies and mode shapes.

3. APPLICATION OF THE TRANSMISSION MATRIX TO A PRACTICAL EXAMPLE

Figure 2 shows a possible pipe configuration with three elements, two outer steel pipes and one central rubber pipe to act as an isolator. Each element has six input parameters at each end {P}.

$$\{P\} = (x, y, \theta, F_x, F_y, M_z)^T \tag{4}$$

where x,y,θ are displacements and rotations, forces and moments as indicated in Figure 2.

The straight pipe sections of length L are represented as massless springs with mass terminations. The symmetric impedance matrix, allowing for translation in two directions and rotation, for the spring is

$$[Z] = \frac{EA}{i\omega L} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ & \frac{12\beta}{L^2} & \frac{6\beta}{L} & 0 & -\frac{12\beta}{L^2} & \frac{6\beta}{L} \\ & & 4\beta & 0 & -\frac{6\beta}{L} & 2\beta \\ & & & 1 & 0 & 0 \\ & & & & \frac{12\beta}{L^2} & -\frac{6\beta}{L} \\ & & & & & 4\beta \end{bmatrix} \tag{5}$$

where E is the Young's modulus, A the cross-sectional area, β is the radius of gyration squared for the cross-section. The transmission matrices for the massless spring, derived from equation (5), and for the end mass are:

$$[T]_i = \begin{bmatrix} 1 & 0 & 0 & \frac{L}{EA} & 0 & 0 \\ 0 & 1 & L & 0 & \frac{L^3}{6EI} & \frac{L^2}{2EI} \\ 0 & 0 & 1 & 0 & \frac{L^2}{2EI} & \frac{L}{EI} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & L & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -m\omega^2 & 0 & 0 & 1 & 0 & 0 \\ 0 & m\omega^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -I_i\omega^2 & 0 & 0 & 1 \end{bmatrix}$$

The transmission matrices for the pipe sections are now combined to give results for the system in Figure 2.

4. CALCULATED INPUT AND TRANSFER IMPEDANCES

Figures 3 to 7 give the input impedances in translation and rotation. All graphs give a stiffness characteristic at low frequencies until about 10 Hz above which the rubber pipe acts as an isolator with the input mass moving freely upon it.

For motion in the y direction there are resonances at 10 Hz and 150 Hz associated with the bending and stretching of the rubber pipe.

It is interesting to see a comparison of the high frequency values of point impedance at ends 1 and 2. End 1 gives a stiffness characteristic at high frequencies, while end 2 gives a mass characteristic. This is because the lumped mass for this example has been placed at the right hand end of the element. This is seen in the impedance at end 2, but not at end 1. It is obviously better to split the mass into one half at each end.

The transfer impedances, which are of greater interest, are not affected much by the position of the mass. The compliant element is immediately effective as a spring until at least 50 Hz. Various resonances occur until 1000 Hz. Above this frequency the limitation of the lumped parameter model is seen in the rather spectacular attenuation rate.

5. CONCLUSIONS

The transmission matrix method seems appropriate for pipework and so, in future, is to be extended to include fluid and a third dimension.

6. REFERENCES

[1] M. EL RAHEH, 'Vibration of three dimensional pipe systems with acoustic coupling', *Journal of Sound and Vibration*, 78(1), 39-67 (1981).

Figure 1

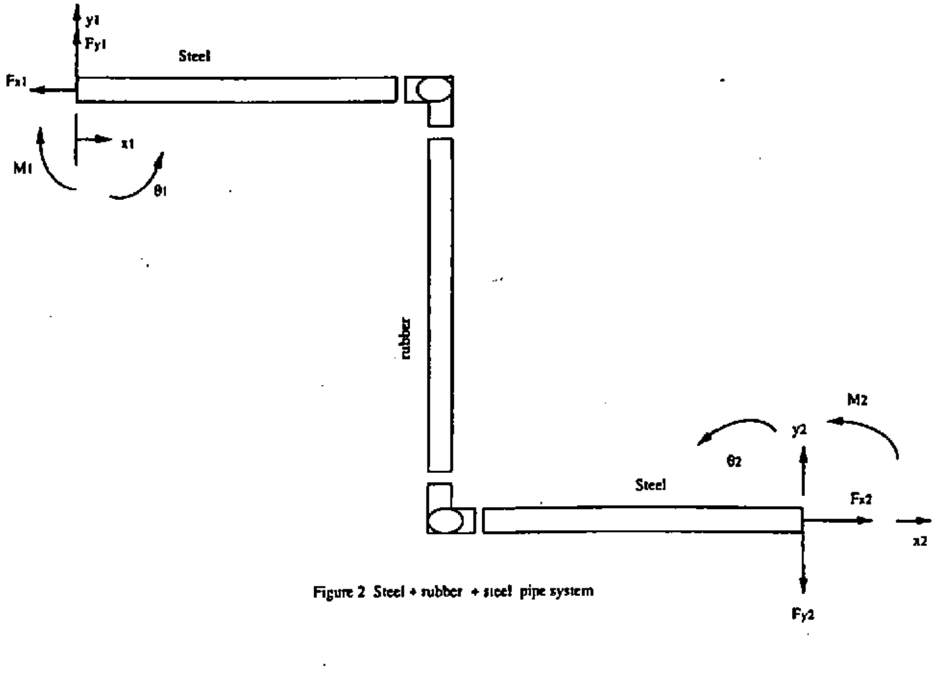

Figure 2 Steel + rubber + steel pipe system

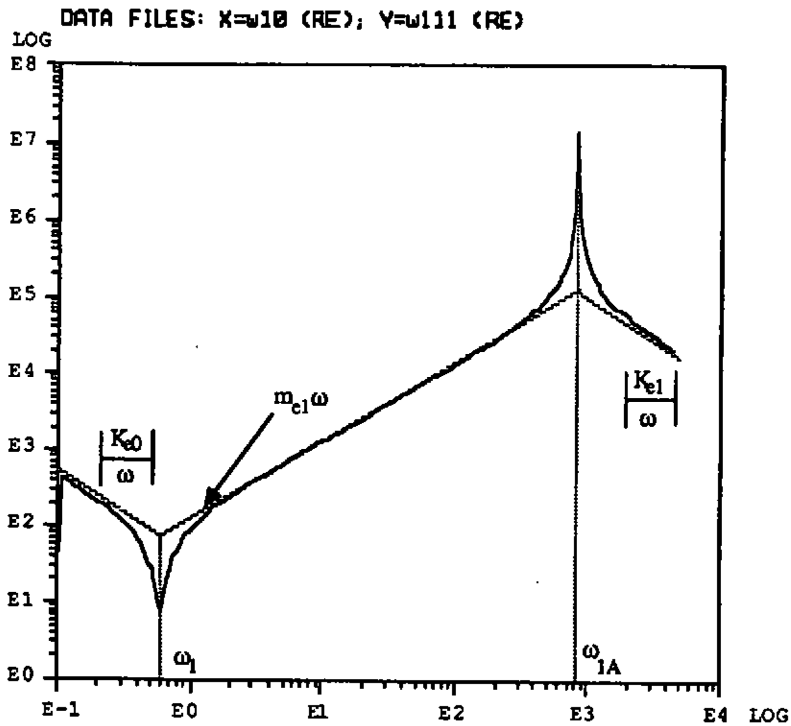

Figure 3 Modulus of impedance $F_{x1}/\dot{X}_1$

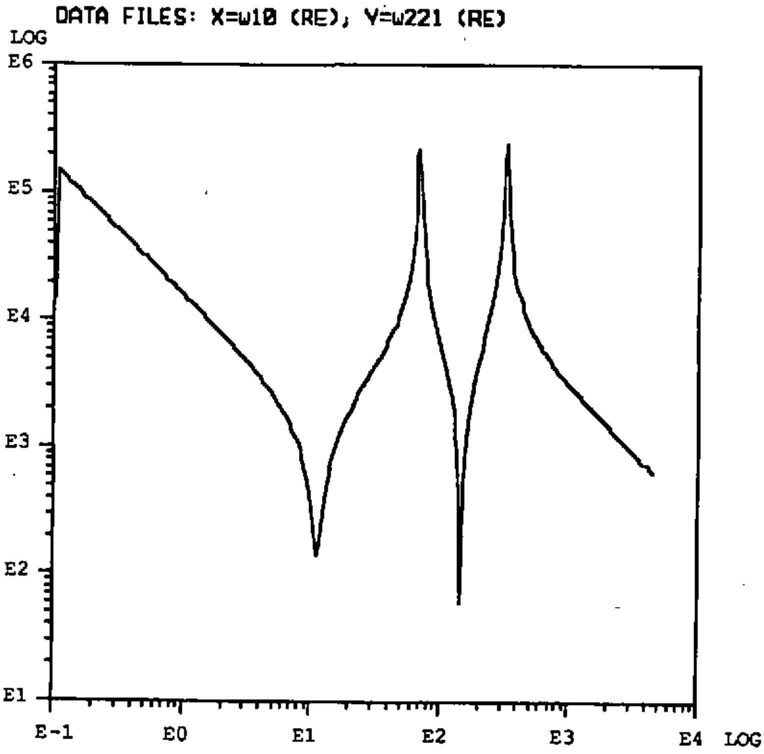

Figure 4 Modulus of impedance Fy1/Ẏ1

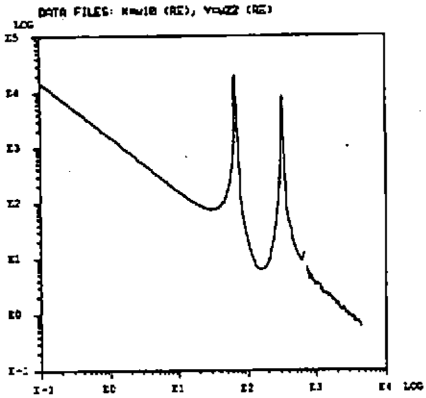

Figure 4.1 Real part of impedance Fy1/Y1

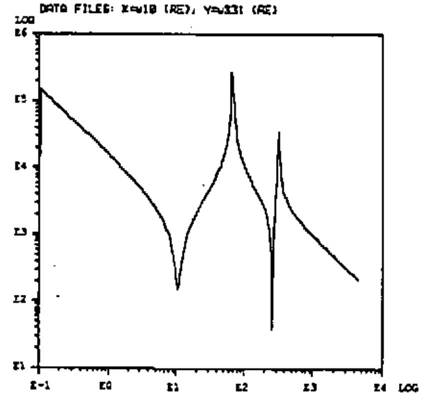

Figure 5 Modulus of impedance M1 / $\dot{\theta}_1$

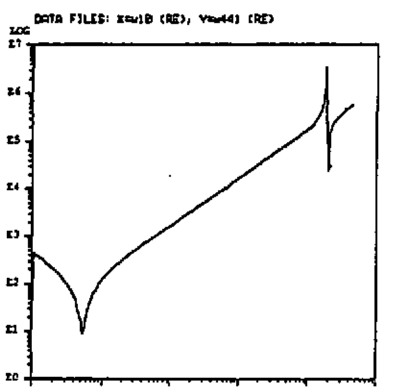

Figure 6 Modulus of impedance Fx2 / X2

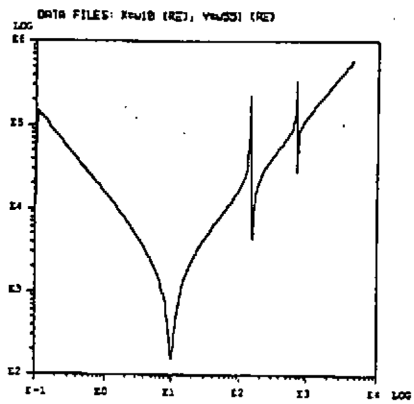

Figure 7 Modulus of impedance Fy2/ Y2

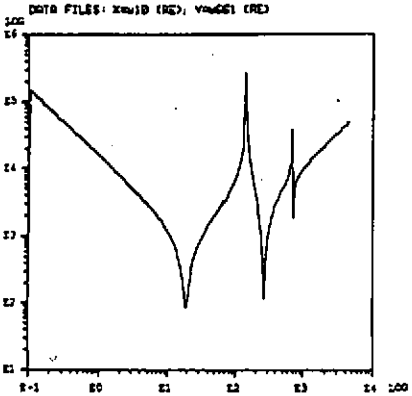

Figure 8   Modulus of impedance M2 / θ 2

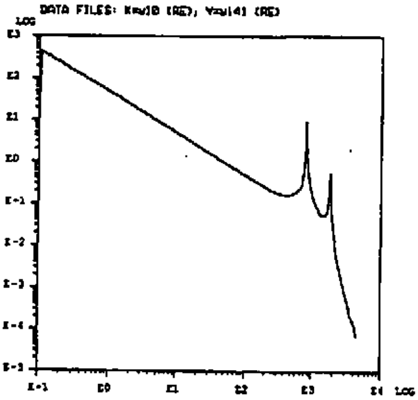

Figure 9   Modulus of impedance Fx1/ X2

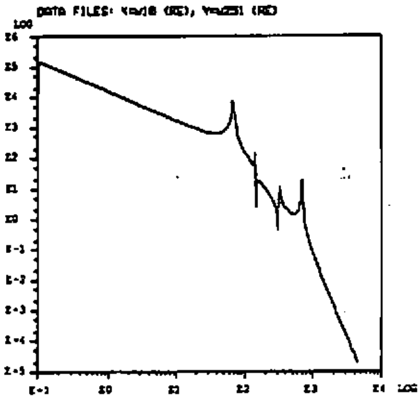

Figure 10 Modulus of impedance Fy1 / Y2

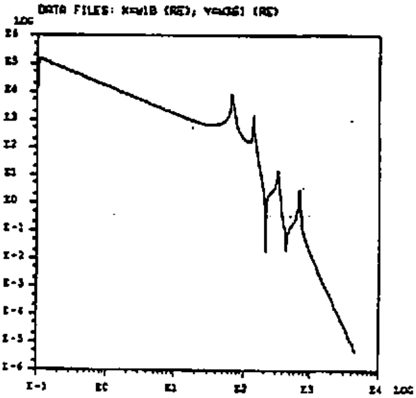

Figure 11   Modulus of impedance M1 / θ 1