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INVESTIGATION OF THE DYNAMIC PROPERTIES OF THE LIQUID-FILLED PIPEWORK SYSTEMS

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1. INTRODUCTION

In fluid filled pipework systems there is a need to know the vibration transmission to the supports or end termination, arising from both vibration inputs and fluid pressure inputs. This is necessary as a machine may transmit vibrational energy via many different paths simultaneously, for example vibration isolators or airborne sound; the pipework system represents one of these transmission paths. If the vibration transmission was found to be excessive, steps could be taken to reduce it. The overall objective is to vary the stiffness and damping parameters to minimise the total power transmission from the pipe coupling points and the pipe outlet.

Pipework systems are quite complex and a few authors have attempted to model them using the finite element model or the mobility method[1 - 5]. Frid has used modal superposition based on a mobility matrix method which is analogous to the standard stiffness method used in solid body structural mechanics for liquid elements[5]. The main difficulty is the complexity or overelaboration of these methods. The general approach used here is to analyze pipework systems using the impedance matrix and transfer matrix method[6,7] within the framework of the power flow philosophy[8,9]. The pipework system is simplified to mass, and spring elements for the pipe wall structure, with a longitudinal wave solution for the fluid elements within. Allowance is made for the fluid loading to both the mass in the structural elements and the wavespeed in the fluid elements. In the approach here masses and springs are used to model low frequency behaviour when only longitudinal or flexural vibration is occurring in the fluid-filled pipe walls. This is particularly appropriate when there are many short pipe sections between discontinuities. The impedance or mobility of the pipework system is by using the transfer matrix method which is very efficacious for pipe systems. The solution is then converted to an impedance matrix form, giving the transmitted force or power to each point if necessary.

This paper describes the theoretical model of a three-dimensional pipework system and the experimental results for a simple pipework system. The prediction results agree well with the measurement results.

2. APPROACH METHOD AND TRANSFER MATRICES

2.1 Approach Method

The pipework system, which is geometrically three dimensional, is first divided into subsystems. Each subsystem may consist of one or several typical elements. In this study, a pipe element is represented as an equivalent massless beam and concentrated mass in series. The transfer matrices of the typical elements between the output and input parameters vector, which include the generalized force and velocity, can be established from the nature parameters of the structure, i.e. mass, stiffness, damping, etc. An external support or excitation can been considered using an additional connective element. For a pipe wall element, the internal fluid influence is included using a parallel fluid element with the structural elements. Combination of the elements of the component systems gives the assembled transfer matrix.

2.2 Impedance and Transfer Matrices

The generalized force F_1 and generalized velocity V_1 at one end of a single element can be written in terms of the force and velocity F_2, V_2 at the other end using the impedance matrix of the structure:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} \qquad (1)$$

where Z_{11} and Z_{21} are the point and transfer impedances found by setting V_2=0. Z_{22} and Z_{12} are found by setting V_1=0. Unfortunately, to build the assembled impedance matrix directly for several combined elements is too inconvenient. It is numerically more efficient to rewrite equation (1) in the transfer matrix form:

$$\begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} \qquad (2)$$

When n elements are combined to give the total transmission [T], only multiplication is necessary of the individual elements. The relationship between the transfer and impedance matrices can be obtained as follows,

$$\begin{aligned} Z_{11} &= -T_{12}^{-1}\, T_{11}\,, \quad & Z_{12} &= T_{12}^{-1} \\ Z_{21} &= T_{21} - T_{12}^{-1}\, T_{11}\,, \quad & Z_{22} &= T_{22}\, T_{12}^{-1} \end{aligned} \qquad (3)$$

2.3 Transfer Matrix for Fluid Vibration in the Pipe

Let P, V be the pressure and velocity of liquid respectively, ϱ_f is fluid density, c_f is wave speed of fluid.

For a straight pipe element, the pressure and velocity at end *i* can be written in terms of the parameters at end *i-1*.

$$\begin{Bmatrix} V \\ \\ P \end{Bmatrix}_i = \begin{bmatrix} \cos\frac{\omega}{c_f}L & -\frac{i}{\rho_f c_f}\sin\frac{\omega}{c_f}L \\ i\rho_f c_f \sin\frac{\omega}{c_f}L & \cos\frac{\omega}{c_f}L \end{bmatrix} \begin{Bmatrix} V \\ \\ P \end{Bmatrix}_{i-1} \qquad (4)$$

For a sudden change cross-section area

$$\begin{Bmatrix} V \\ P \end{Bmatrix}_i = \begin{bmatrix} \frac{A_{i-1}}{A_i} & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} V \\ P \end{Bmatrix}_{i-1} \qquad (5)$$

2.4 Transfer Matrix for a Liquid-filled Straight Pipe Element and the Joint Element as an Equivalent Concentrated Mass of Three Dimensional Pipework System

The pipe wall and fluid can be combined to approach as spring element and mass element in series. Each element has fourteen input parameters for three dimensional system at each end {**P**},

$$\{P\} = \{\dot{x}\ \dot{y}\ \dot{z}\ \dot{\theta}_x\ \dot{\theta}_y\ \dot{\theta}_z\ \dot{u}\ F_x\ F_y\ F_z\ M_x\ M_y\ M_z\ p\}^T \qquad (6)$$

The transmission matrices are represented as [**T**]. i.e.

$$\{P\}_k = [T]_k \{P\}_{k-1} \qquad (7)$$

where k --- denotes the output end of *k*th element
k-1 --- denotes the input end of *k*th element

The $\dot{x}$, $\dot{y}$, $\dot{z}$, and $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$ are translational velocities and rotational velocities respectively of pipe wall, $\dot{u}$ is velocity of fluid. The forces, moments and pressure are indicated in Figure 1. The vector directions at the input end for all the parameters including generalized fores and velocities are the same, i.e. (X-Y-Z) Cartesian coordinate direction at the input end. At the output end, the generalized velocities are the same but the forces are in the opposite direction to that of the coordinate direction.

The example for the straight pipe sections including fluid of length L are represented as massless springs with mass terminations from X direction to anti-Y direction, their transfer matrices are as Equations (8),(9).

Where E is the Young's modulus. A_i, A_{i-1} are the cross-sectional area at the end *i* and *i-1* respectively, I is the second moment of area of the pipe wall. J_x, J_y, and J_z are inertial moments in X-axle, Y-axle and Z-axle respectively. K is the longitudinal stiffness EA/L. K_t is torsional stiffness of pipe wall.

3. NUMERICAL EXAMPLE OF THEORETICAL MODEL

In order to predict the dynamic properties of complex pipework system, a computer program called DAPST (Dynamic Analysis of Pipework Systems using Transfer matrix method) has been developed. The parameters of a pipework system and the analysis commands (impedance or mobility, frequency range, results filename, etc.) are inputted by the control file. A pipe configuration with three elements (two copper pipe elements and one Nylon pipe element) in

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$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -i\omega\frac{1}{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & L & 0 & 0 & i\omega\frac{L^3}{6EI} & 0 & 0 & 0 & -i\omega\frac{L^2}{2EI} & 0 \\
0 & 0 & 1 & 0 & -L & 0 & 0 & 0 & 0 & i\omega\frac{L^3}{6EI} & 0 & i\omega\frac{L^2}{2EI} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -i\omega\frac{1}{K_t} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -i\omega\frac{L^2}{2EI} & 0 & -i\omega\frac{L}{EI} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & i\omega\frac{L^2}{2EI} & 0 & 0 & 0 & -i\omega\frac{L}{EI} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cos\frac{\omega}{c_f}L & 0 & 0 & 0 & 0 & 0 & 0 & -i\frac{\sin\frac{\omega}{c_f}L}{\rho_f c_f} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -L & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -i\rho_f c_f \sin\frac{\omega}{c_f}L & 0 & 0 & 0 & 0 & 0 & 0 & \cos\frac{\omega}{c_f}L
\end{bmatrix}
\qquad (8)
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{A_{i-1}}{A_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-i\omega m & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & A_{i-1} \\
0 & -i\omega m & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & A_i \\
0 & 0 & -i\omega m & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -i\omega J_x & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -i\omega J_y & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i\omega J_z & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\qquad (9)
$$

Figure 2 was chosen as representative of the complex systems amenable to the present analysis for example for a three-dimensional pipework system. Figure 3 is shown the flexural point mobility of separated one copper pipe element without fluid, and Figure 4 is for with water. Figure 5 and Figure 6 are the flexural point mobilities for the three-element pipework system without water and with water respectively. The Nylon pipe section acts an isolator with the input mass moving freely upon the copper pipe.

4. COMPARISON OF THE THEORETICAL AND EXPERIMENTAL RESULTS

As the section 3 for the flexural point mobilities prediction, the measurement results for that parameters pipe element and the pipework system without and with water respectively are drawn on the same graphics in Figures 3 - 6. They are shown that the theoretical model agrees well with the experimental results.

5. CONCLUSIONS

It has been demonstrated that the point or transfer mobilities of liquid-filled pipework system can be obtained by the transfer matrix method. The use of the approach method could then be used to predict the properties of pipework system which agree well with the experimental results, and to optimize the parameters such as damping, support position, etc.

6. REFERENCES

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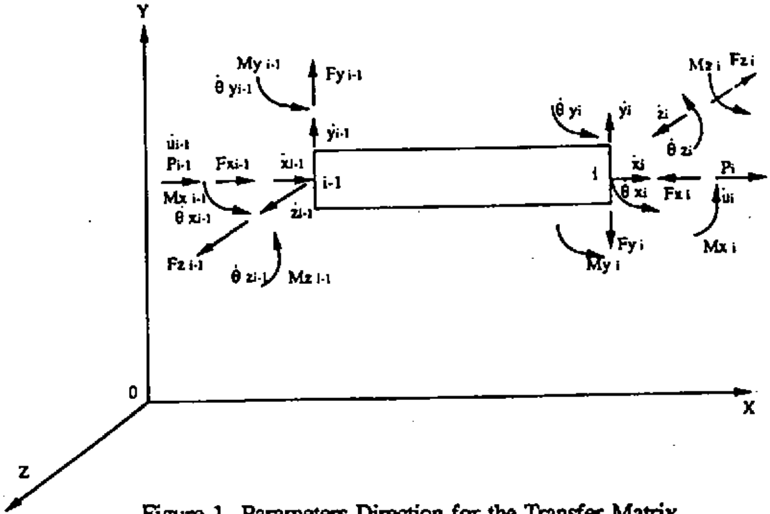

Figure 1 Parameters Direction for the Transfer Matrix

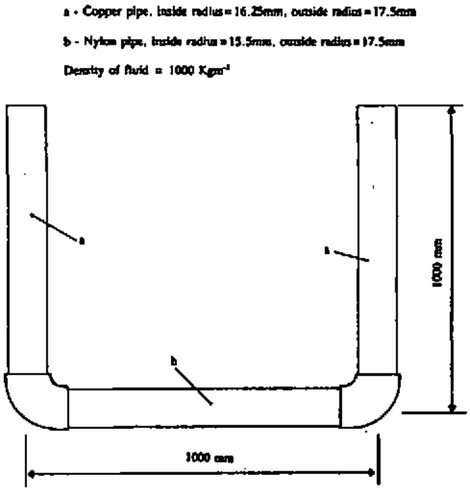

Figure 2 Copper + Nylon + Copper Pipework System

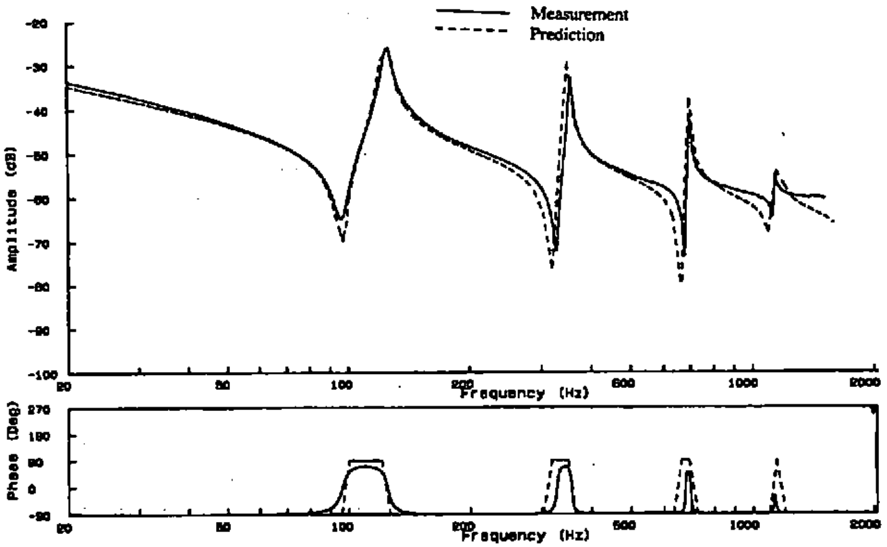

Figure 3 Modulus of Flexural Point Mobility of a Copper Pipe Element

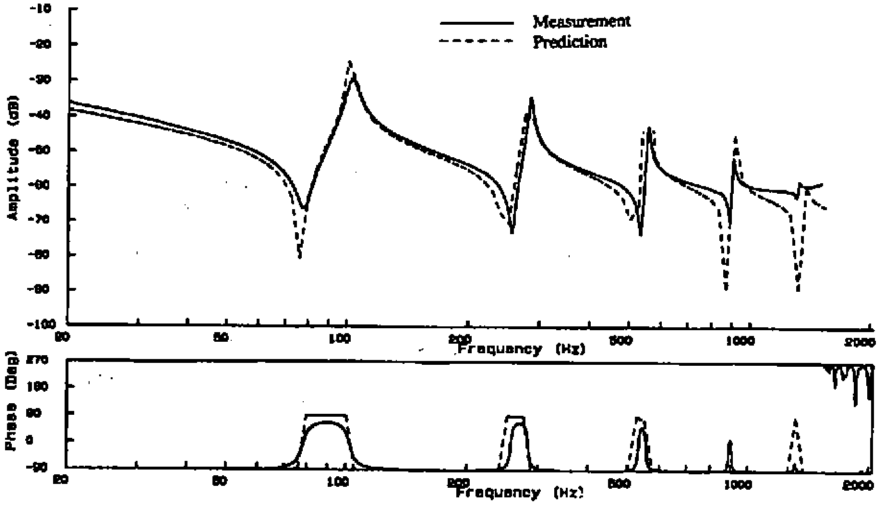

Figure 4 Modulus of Flexural Point Mobility of a Copper Pipe Element with Water

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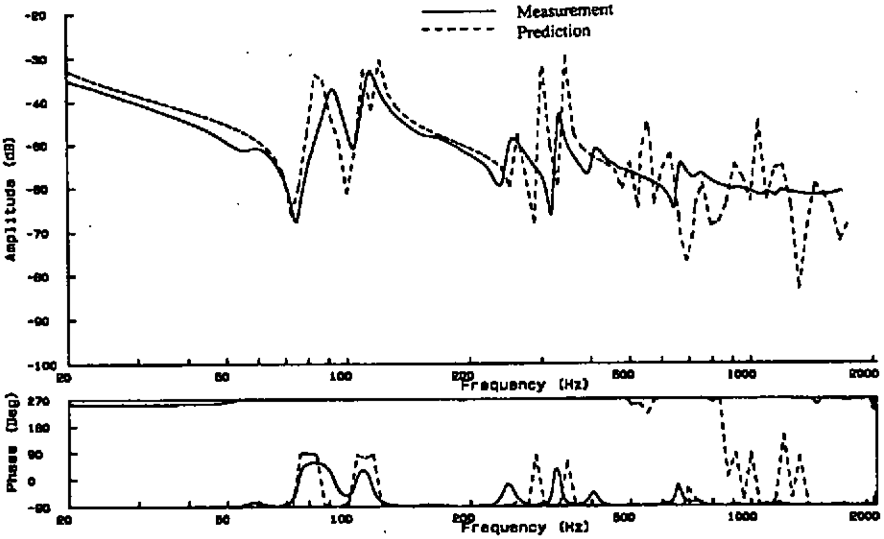

Figure 5 Modulus of Flexural Point Mobility of a Pipework System without Water

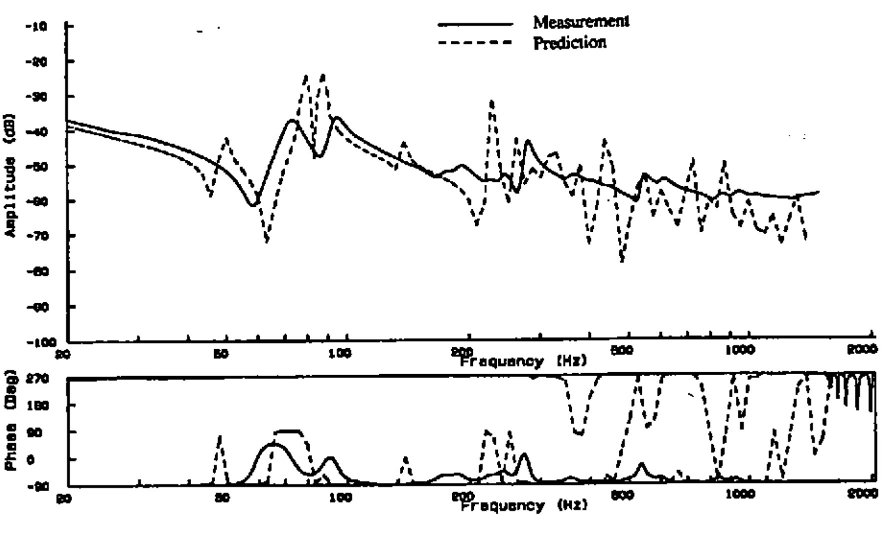

Figure 6 Modulus of Flexural Point Mobility of a Pipework System with Water