

# VIBRATION OF FUNCTIONALLY GRADED CYLINDRICAL SHELLS

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Functionally gradient materials (FGMs) have attracted a growing interest as advanced structural materials because of their heat-resistance properties. In this paper, an experimental study on the vibration of cylindrical shells made of a functionally gradient material (FGM) composed of Polyethylene terephthalate (PET) is presented: to obtain functional gradient proprieties the PET shell had been exposed at a thermal temperature gradient in the range of its glass transition temperature of 79°C.

The setting up of the experiment is explained and deeply described along with the thermal characterisation of the specimen. The linear and the nonlinear dynamic behaviour have been investigated. The shell behaviour is also investigated by means of a finite element model, in order to enhance the comprehension of experimental results.

Keywords: FGM, CYLINDRICAL SHELL, EXPERIMENTAL, DYNAMIC BEHAVIOUR

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## 1. Introduction

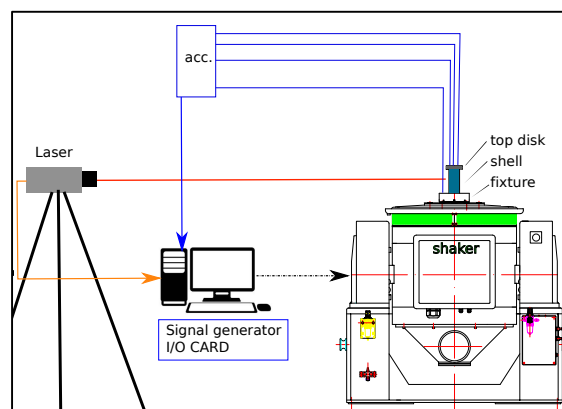
The characterization of the dynamic properties of a circular cylindrical shell made of Polyethylene terephthalate (PET) is the object of the present work, this is a first part of a more complex work regarding Functionally gradient materials (FGMs) models applied to shell dynamics. The dynamic of shell under base excitation is part of a long research activity covering theoretical and experimental aspects of the shell behaviour. More specifically, here a campaign of experiments is presented in order to reveal the extreme complexity of simple structures such as circular cylindrical shells made with PET. A complex setup has been specifically designed and built to characterize with dynamic test the structural properties of the specimen on temperature change from -10°C up to about 90°C. Predicting the mechanical properties of shells, panels and plates is one of the main concern of structural engineers; since shell elements present complicated stability behaviours, rich linear vibration spectra (high modal density), high sensitivity to perturbations and strong interactions with surrounding elements, the problem of shells connected with a top mass was analysed in detail both from a theoretical and experimental point of view. An accurate experimental modal analysis was used for setting up and validate a FEM (Nastran based) modelling capable of handling complex boundary conditions, new experimental data were presented and compared with a FEM model.

## 2. Experiment Setup

This section provides information about the experimental setup developed to test thin shell structures with a top mass subjected to a harmonic load. In Figure 1 a schematic representation of the setup is shown. A shaker is rigidly connected to a shell by mean of a fixture, designed to have the first resonance at more than 5000Hz.

The system under investigation consists of a circular cylindrical shell, made of PET plastic, see Figure 2a and Figure 2b, clamped at the base to rigid supports by means of steel shaft collars (see Figure 2b) and at the end top glued at the top disk with epoxide glue. The geometric and physical parameters of the circular cylindrical shell are reported in Figure 2a and Table 1. The bottom support is an aluminium alloy thick circular disk rigidly bolted to the shaker with a triaxial accelerometer. The top disk is connected to the shell by means of an epoxy glue resistant to high temperature; three triaxial accelerometers are located and equispaced on the top disk. In Figure 2b is clearly visible a cartridge heater that was used to adjust the inner temperature of the shell. The cartridge heater is a cylindrical shaped, industrial Joule heating element and will be used in next step of the research to create the thermal gradient between inside and outside the shell wall.

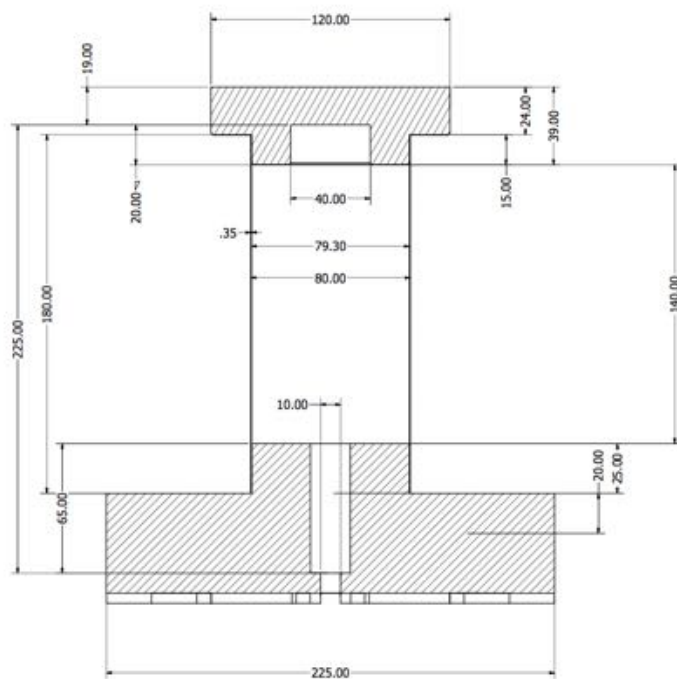
Moreover, a laser vibrometer is used to measure the lateral vibration of the shell and two thermocouples are used to measure the temperature, one in the inner side and one outside the shell. All the test has been performed coupling the system with a climate chamber that allows to control the temperature outside the shell from -70°C up to 250°C.



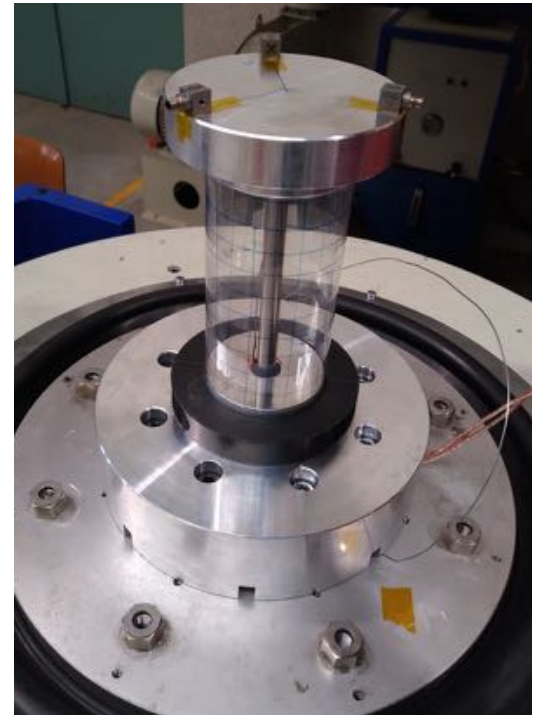
**Figure 1.** Experimental setup: acquisition and control chain

**Table 1. physical parameters of the circular cylindrical shell and fixture and top disk**

Fixture and Top disk		
Material	Alluminium Alloy	
General	Mass Density	2.7 g/cm <sup>3</sup>
Stress	Young's Modulus	68.9 GPa
	Poisson's Ratio	0.33 ul
Circular Cylindrical Shell		
Material	PET Plastic	
<b>General</b>	<b>Mass Density</b>	<b>1.366 g/cm<sup>3</sup></b>
<b>Stress</b>	<b>Young's Modulus</b>	<b>4 GPa</b>
	<b>Poisson's Ratio</b>	<b>0.4</b>



(a)



(b)

**Figure 2. Scheme of the shell-top disk system (dimensions mm)**

### 3. Modal analysis

A modal analysis has been carried out for understand the shell behaviour and for identifying the natural frequencies, damping ratios and modal shapes, this step it's crucial for the choice of the mode that will be used for identify the dynamic property of the PET shell. The micro-hammer used for the test is a PCB Modally Tuned, ICP instrumented impact hammer. Triaxial accelerometers are used on the top disk in conjunction with the laser vibrometer hammer to perform a measurement of the structural response, moreover more tests were executed using the shaker to better identify the beam-like and axisymmetric modes.

In Table 2 the measured natural frequencies are reported together with the description of the corresponding modal shapes. In this table  $m$  represents the number of axial half waves, while  $n$  is the number of nodal diameters. In Table 3 the first nine mode shapes are presented; these modes are obtained with comparison with experimental tests. The first four modes are strongly influenced by the presence of the top disk; the first mode at 52.73Hz is quite similar to the one of a cantilever beam, it is extremely influenced by the lateral displacement of the disk; the second mode (144.92Hz) involve the rotation of the top disk along the shell axis; the third mode (237.5Hz) is axisymmetric, it is virtually the first longitudinal bar-like mode, the disk motion is a pure translation along the shell axis; the fourth mode (356.3Hz) is the last mode involving the disk that mainly rotates and marginally moves horizontally and vertically, so it appears as a combination of the previous modes, this is quite interesting as it can justify modal interactions in nonlinear field; the fifth mode (608 Hz with five nodal diameters and one longitudinal wave), the sixth mode (664 Hz with four nodal diameters and one longitudinal wave), the seventh mode (764Hz with six nodal diameters and one longitudinal wave), the eighth mode (850 Hz with three nodal diameters and one longitudinal wave) and the ninth mode (963 Hz with seven nodal diameters and one longitudinal wave) are shell like modes. In Figure 3 the FRF sum between 10 responses ( 3 triaxial accelerometer on the top disk and one monoaxial accelerometer on the base) and 131 references (points on shell).

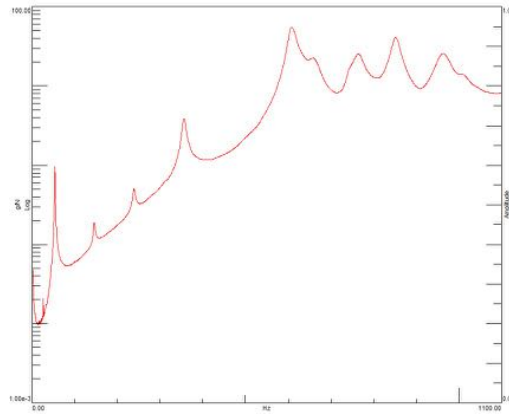


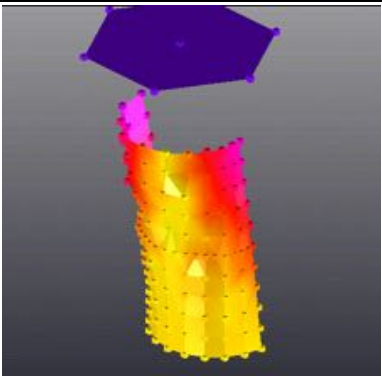
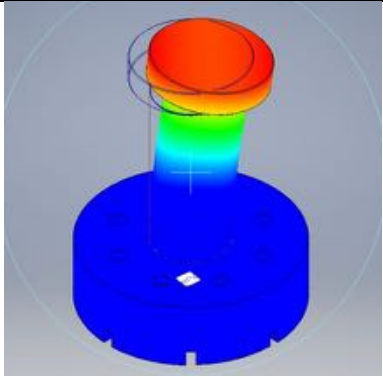
Figure 3. FRF sum between responses (10) and references (131)

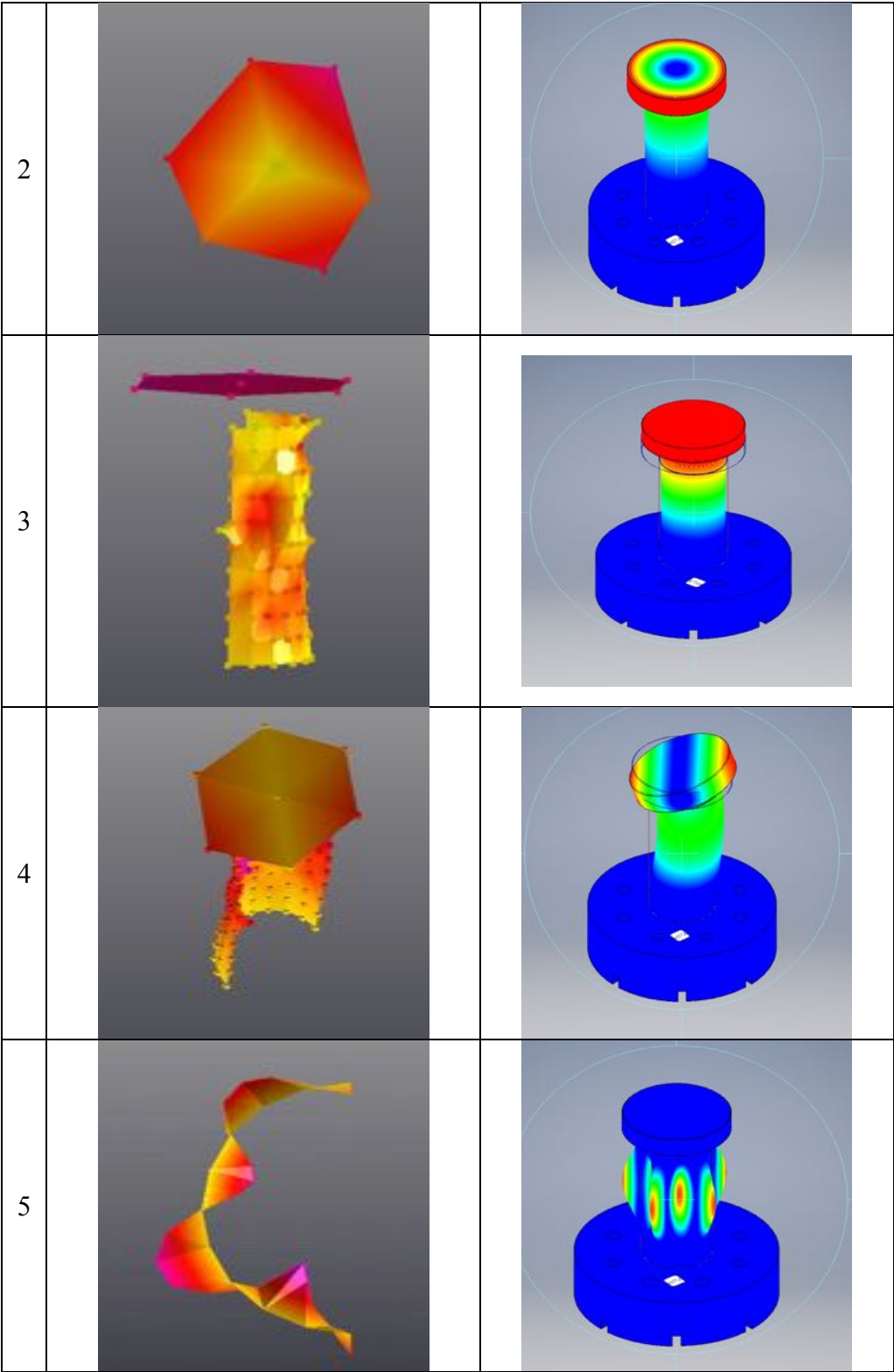
Table 2. Experimental and FEM frequencies

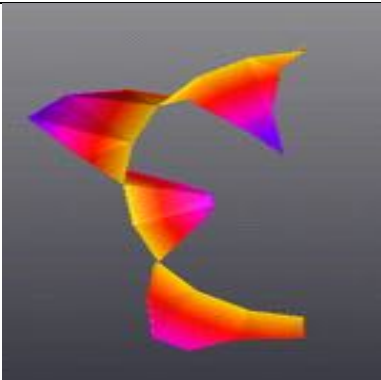
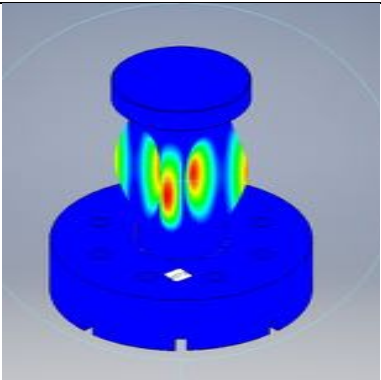
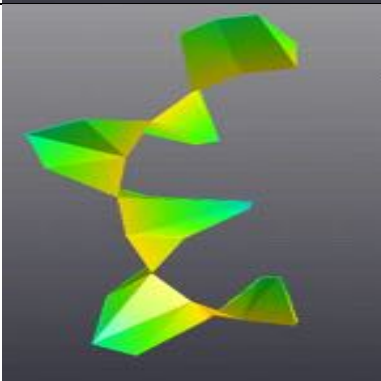
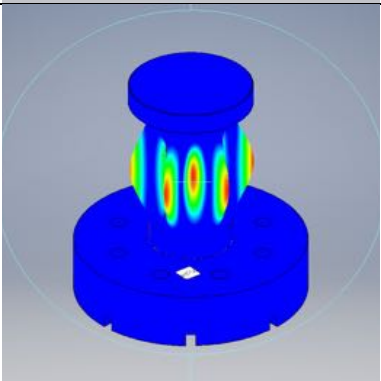

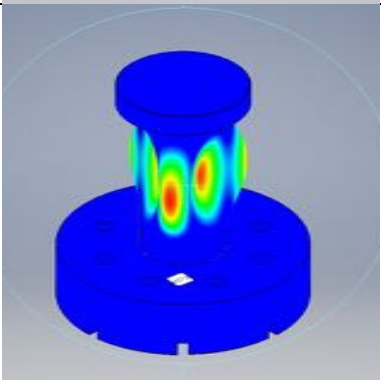

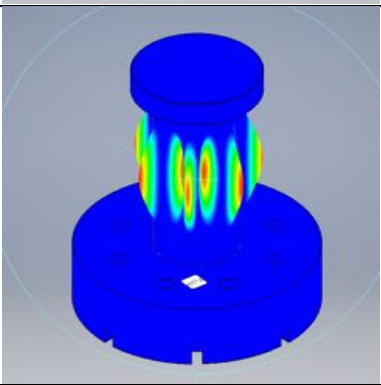
n	Mode	m	n	Experimental [Hz]	FEM [Hz]	% error
1	Cantilever beam like	--	--	52.73	59.52	12.9%
2	Axisymmetric Rotational	--	--	144.92	150.95	4.2%
3	Axisymmetric compressive	--	--	237.5	262.33	10.5%
4	Top disk rotation	--	--	356.3	376.44	5.7%
5	1-5	1	5	608	607.78	0.0%
6	1-4	1	4	664	634.0	4.5%
7	1-6	1	6	764	729.02	4.6%
8	1-3	1	3	850	865.52	1.8%
9	1-7	1	7	963	941.14	2.3%

In Table 3 a comparison between experimental and FEM modal analysis.

Table 3. Experimental and FEM modal shapes

N	Experimental	FEM
1		



6		
7		
8		
9		



#### 4. Temperature Change

After the modal analysis was chosen the third mode, the axisymmetric compressive mode, to perform a series of tests changing the temperature of the specimen. The first shaker test was performed at -10°C and several test was carry out up to 92°C; In figure 4 is shown the FRFs at temperature changes, between each the test, the time required for the temperature to stabilize was expected. In Table 3 the results are shown, the quality factor was calculated with the Half-Band method. In Figure 5 and 6 the results are plotted.

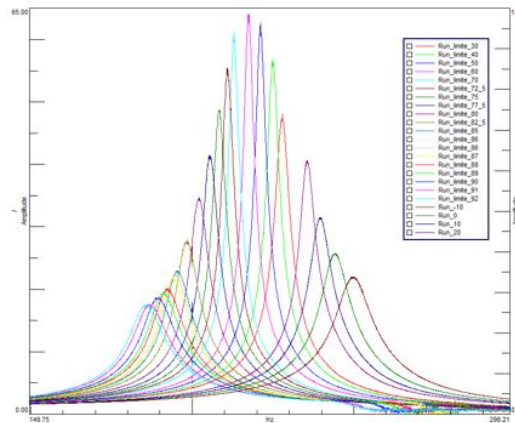
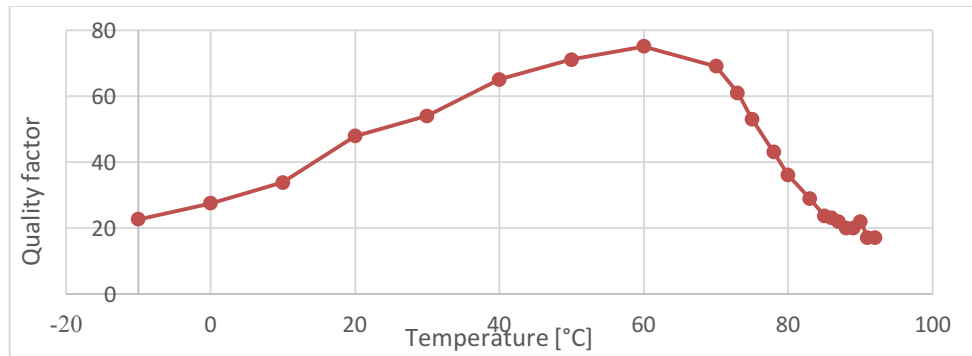


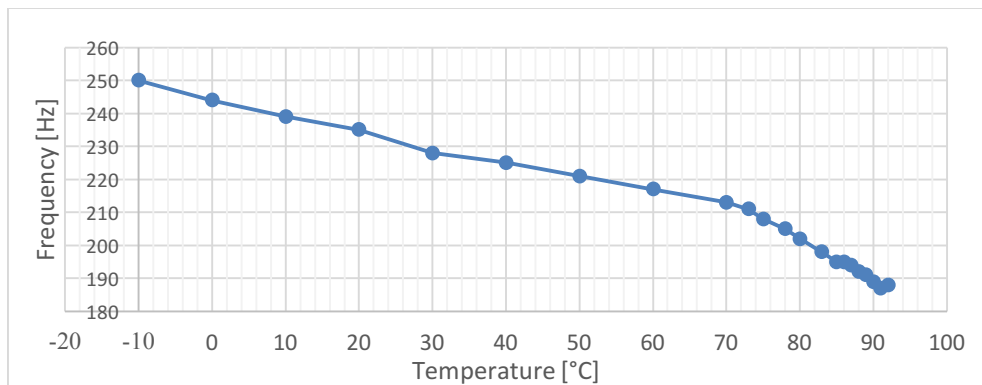
Figure 4. FRF of third mode on temperature change

Table 4. frequency and quality factor on temperature change

T [° C]	f [Hz]	Quality factor
-10	250	23
0	244	28
10	239	34
20	235	48
30	228	54
40	225	65
50	221	71
60	217	75
70	213	69
73	211	61
75	208	53
78	205	43
80	202	36
83	198	29
85	195	24
86	195	23
87	194	22
88	192	20
89	191	20
90	189	22
91	187	17
92	188	17



**Figure 5. Quality factor on temperature change**



**Figure 6. Frequency of third mode on temperature change**

## 5. Conclusion

In the present work the experimental setup to identify the dynamic property of a PET circular cylindrical shell was deeply explained and the results of experimental and FEM modal analysis were shown. Moreover a characterization of the properties was carry out in a wide temperature range with several tests.

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