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FIRST EXPERIENCES USING A COMMERCIAL FINITE ELEMENT PACKAGE - A CASE HISTORY

Amanda B. Gallaher

Admiralty Research Establishment, Portland, Dorset DT5 2JS.

1. INTRODUCTION

Background

Over a number of years computer programs have been written to model various characteristics and, more specifically, to determine the resonance frequency of the transducers used by the Transducer Design Section at A.R.E. Portland. Each program is specific to one type of transducer and cannot be used to model any transducers which vary significantly from the standard design. It was therefore hoped that some form of analysis could be found which would be general enough to be applied to many different types of transducers and which could handle possible modifications to standard designs. To this end it was decided to investigate the use of the PAFEC Finite Element Program (Level 6.1)[1][3].

In an attempt to assess the usefulness of the models, the first step was to model the standard types of transducers and compare the results with those obtained from the existing programs and from experiment. This study is confined to assessing how accurately the resonance frequency in air of a given transducer can be determined. The standard resonance frequencies, against which the accuracy of the computer program results are measured, are obtained experimentally as described in APPENDIX A.

The Objective

This report documents the early attempts to use the PAFEC Finite Element Program to model piezoelectric transducers. It is intended to highlight some of the advantages, disadvantages and problems encountered and to compare the results with those obtained using the conventional transducer analysis programs.

In the early studies a number of standard types of transducer were considered. The scope of this discussion will be confined to two basic types of transducer:

1. Ring Transducers

2. Piston Stack Transducers

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The Ring Transducer is one of the simplest designs of transducer. Its simplicity of design and symmetrical shape make it probably ideal for a novice to begin a first attempt at modelling a transducer.

The Piston Stack Transducer is perhaps the type which is most widely used and therefore of most general interest. It forms a useful study in that it highlights most of the points relevant to modelling all types of piezoelectric transducer.

2. RING TRANSDUCERS

Description
These consist of a ring of piezoelectric ceramic polarized radially and driven so as to produce radial motion which is omnidirectional in a plane perpendicular to its axis of symmetry [5](Figure 2.1). In many cases a metal ring is thermally shrunk on to the outside of the ceramic. The purpose of this is to keep the ceramic in compression up to the maximum vibrational amplitude it is likely to experience. This is necessary because of the relatively low tensile strength of the ceramic.

The existing, conventional program for analysing ring type transducers is called RINGPAN and is described in APPENDIX C.

PAFEC Model
It is possible to create a 3-dimensional model for this structure, however, as we are only interested in uniform radial motion, the problem is greatly simplified by using an axisymmetric model. This allows the structure to be represented by a 2-dimensional generator and the structure is assumed to be a solid of revolution of this generator about the horizontal axis. An axisymmetric model, however, rules out the use of PAFEC's special piezoelectric brick elements to represent the ceramic because, in Level 6.1, these are only available in 3-dimensions (see APPENDIX B). The ceramic is simply treated as a normal material with the Y-33 value quoted for the Young's Modulus.

The generator for an axisymmetric model of this type of transducer simply consists of a rectangle, ie. a radial section through the ring, as shown in Figure 2.1. and Figure 2.2. This is made up of simple 2-dimensional 8 noded rectangular elements. At this stage PAFEC allows us a choice of two possible element types [3].

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If we require to analyse mode shapes with harmonic numbers higher than zero, we must use the element 36610 which has been specially designed for axisymmetric Fourier applications. However, if as in this case, we are only interested in harmonic number zero we can simplify the analysis by using the more simple eight noded quadrilateral element 36210 and invoking the AXISYMMETRIC COMMAND in the CONTROL module.

If we now run the natural frequencies analysis for such a mesh the mode shapes obtained are as shown in Figure 2.3 [2]. Mode shape 1 represents a rigid body mode. Mode shape 3 represents uniform radial motion so clearly this is the mode of operation of the transducer and the frequency at which this occurs is its resonance frequency.

It is worth noting that if, as in this case, we are only interested in displaying the first five mode shapes, the processing time is significantly reduced by specifying in the MODES AND FREQUENCIES module that full back substitution is only required for the first five modes. Back substitution is the process whereby displacements at all the nodes are found after the displacements at the masters are calculated. By default PAFEC will perform back substitution for the first twenty modes which is often unnecessary and may be very time consuming.

<u>EXAMPLE</u> - For a ring with the following dimensions

Inner radius	=	50.8	mm
Outer radius	=	57.15	mm
Height	=	28.0	mm

the experimentally determined resonance frequency is 9.83 kHz. The conventional ring analysis program RINGPAN (as described in APPENDIX C)gives a value of 9.68 kHz and PAFEC gives a value of 9.64 kHz. We can see that the PAFEC value and the RINGPAN value are very close. The discrepancy between the two theoretical values and the measured value is likely to be due to the tolerance in the fabrication of the ceramic ring as mentioned in APPENDIX A. To reduce this effect an average value for frequency should be taken over a large number of rings. However, the experimental result quoted here is simply the resonant frequency of a single ring found by experiment.

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If we now wish to include the effect of a metal stress ring around the outside of the ceramic it is a simple job to add an extra rectangular cross-section to our axisymmetric model (Figure 2.4). However, including the effect of the stress that has been introduced is not so easy. We must use the TEMPERATURE module to decrease the temperature of the metal ring by the required amount to cause it to contract and apply the stress. This involves the use of the PAFEC non-linear package called SNAKES to modify the stiffness matrix of the structure before the natural frequency calculation is performed.

In theory the effect on the resonance frequency of prestressing the ceramic should be negligible so it is worth investigating the effects on the model of removing the temperature shrinkage. In this case the only significant effect on the resonance frequency, when the metal ring is introduced, is due to the additional mass loading of the metal. If we perform the natural frequencies analysis on a mesh (such as shown in Figure 2.4) twice, once with the prestress and once without, the results are found to differ by less than 0.0001%. Even a very severe prestress, much greater than should ever occur in practice, makes no significant difference so it is clear that the resonance frequency of a ring transducer is virtually independent of prestress. It is therefore preferable to perform the analysis without any prestress as this makes the data file much easier to set up and the program much quicker to run.

<u>EXAMPLE</u> - Three standard ceramic rings of the dimensions given in the previous example were prestressed using three aluminium rings of different thicknesses as follows

1. SMALL   Aluminium thickness = 1/16" = 1.5875 mm
2. MEDIUM   Aluminium thickness = 1/8" = 3.175 mm
3. LARGE   Aluminium thickness = 3/16" = 4.7625 mm

These were modelled without prestress so that the aluminium part was considered to be a passive component loading the ring. The mesh used in each case was the same as that shown in Figure 2.4. The results are shown in Table 2.1 along with those from a version of RINGPAN adapted to include the theory for a composite ring. From TABLE 2.1 we can see that the results from PAFEC are generally better than those from RINGPAN. In fact, the results from PAFEC are all within 3% of the experimental resonance frequency.

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Conclusion.
We have seen that with a Finite Element Program such as PAFEC, it is relatively straightforward to produce a model of a ceramic ring transducer. In general, using a very simple mesh, PAFEC will produce results for an unloaded ceramic ring which are at least as accurate as those from the conventional ring transducer analysis program RINGPAN. However in the case of composite rings the PAFEC results are consistently better than those from RINGPAN. PAFEC has the added advantage that, if we wish to model composite rings with a slightly different structure (eg. more layers of different materials) this can be done simply by altering the data file, whereas RINGPAN, at least in its present form, would be unable to tackle this problem.

3. PISTON STACK TRANSDUCERS

Description
A transducer of this type [6] (such as in Figure 3.1) consists of a "piston" mass, designed to transmit energy into the water, which is driven by a "stack" of piezoelectric rings, acting as a spring. The other end of the stack is attached to a counter mass known as the tail mass. In most modern transducers, the piezoelectric materials used for the rings are types of ceramic. These are weak in tension but strong in compression. To maintain the ceramic always in compression, a mechanical bias on the ceramic is imposed by a bolt down through the centre of the stack. A tensioning nut screws on to the tail mass providing the compressive force required. The transducer is cemented together, with a form of epoxy resin, to provide acoustic coupling across the joints.

There are two conventional programs used at ARE for analysing piston stack transducers. The two programs called PETPAN and FJEUXB use two distinctly different techniques as described in APPENDIX D.

PAFEC Model
As with the ring type of transducer, it is possible to create a 3-dimensional model for this structure, however, as the transducer is symmetrical about its central axis and we are only interested in longitudinal motion which is symmetrical about the axis, the problem is greatly simplified by the use of an axisymmetric model. Again this rules out the use of PAFEC's special piezoelectric brick elements to represent the ceramic stack because, in Level 6.1, these are only available in 3-dimensions (as mentioned in APPENDIX

B). The ceramic is simply treated as a normal material with the Y-33 value quoted for the Young's Modulus.

It is often valuable to be able to model the piston independent of the rest of the transducer to determine its flapping frequency. To do this we must either produce a 3-dimensional mesh of the piston or, more simply, we can produce an axisymmetric model using the element 36610 which is specially designed for axisymmetric Fourier applications. This will allow us to look at mode shapes with harmonic numbers higher than zero.

In this case we only wish to study the longitudinal motion of the whole transducer so we are only interested in mode shapes of harmonic number zero. We can therefore simplify the analysis by using the simple 2-dimensional eight noded quadrilateral element 36210 and invoking the AXISYMMETRIC command.

A series of simple transducers were designed and built at ARE specifically for the purpose of testing the conventional analysis programs PETPAN and FJEUXB (see APPENDIX D). These transducers were designed to work at a range of frequencies from 5 kHz to 32 kHz, three at each frequency. It was decided to pick four of these different types, well spaced in frequency, and attempt to model them using the PAFEC Finite Element Program. The transducers were as shown in Figures 3.3 to 3.6. The analysis was done in a number of stages, starting with a simple mesh which was then systematically refined in an attempt to improve the results.

<u>STAGE I</u> - With, at this early stage, very little knowledge of how best to create a mesh, a simple mesh was drawn up of each of the four transducers, as shown in Figures 3.7 to 3.10. In each case the joints in the stack were ignored, the centre bolt was considered to be bonded to the tail mass at the tail nut and there was no prestress on the bolt. The results from this model are shown in Table 3.1 along with the results from the two conventional transducer analysis programs PETPAN and FJEUXB for comparison. It can be seen from the table that, using this simple model, the PAFEC results obtained for the resonance frequencies are very poor.

A number of test files were run to investigate the effect of varying the coarseness of the mesh. If, as in this case, PAFBLOCKS have been used to generate the mesh, changing the mesh density is simply a matter of altering the values given in the SPACING LIST of the

MESH module [3]. It was found that this had little effect on the frequency. At best, an improvement of about 1% was all that could be achieved on the two results with the worst error, L109A and L109S, still leaving them around 15% too high. The best results from this simple mesh are plotted in Graph 3.1 along with the full range of results from PETPAN and FJEUXB. We can clearly see that this very simple PAFEC model is not nearly as good at predicting the resonance frequency as FJEUXB and is only better than PETPAN at high frequencies where the Lump Mass Model is not valid (see APPENDIX D).

STAGE II - So far we have not considered the joints in the ceramic stack. These epoxy joints have a significant effect on the compliance of the stack and cannot be ignored. PETPAN and FJUEXB both include the joints in their calculations. The major problem in modelling these joints, and the reason they were omitted from the simplest models, is that they are very thin, around 0.083 mm. There is a limitation on PAFEC elements that the ratio of the longest to the shortest side should not be more than 5:1. It would therefore take a large number of tiny elements to model each thin sliver of epoxy. A further constraint is that the corner nodes of adjacent elements must connect as shown in Figure 3.11. This means that the high density of elements in the epoxy joints will propagate into the rest of the structure, causing the mesh to be extremely fine so the job will take an unreasonably long time to run.

In an attempt to solve this problem it was decided to try to model the joints by adding them all together to form a thicker layer of epoxy in the middle of the stack. This thicker layer is, however, still very thin and the problem of requiring a high density of elements still occurs, although to a lesser extent. It is possible to confine this high density of elements to the stack by careful use of triangular elements as shown in Figure 3.12.

Application of this technique to transducers L109A and L109S gave the results shown on Graph 3.2. We can see that these are a substantial improvement on the results from the simple mesh. However, for two main reasons, it is not feasible to use this method for modelling all such transducers.

1. This technique is very time consuming, both in terms of the man hours required to build up such a mesh and the C.P.U. time to run the job.

2. More importantly, it is not possible to apply this technique to transducers, such as L108, which have only a short stack consisting of very few rings. Such transducers have so few joints that, even when all the epoxy is lumped together, the layer is so thin that a very large number of tiny elements is required. The problem is further enhanced by the fact that, as the stack is so short, the high density of elements cannot be prevented from propagating beyond the stack into the rest of the structure.

STAGE III - At this stage it was thought worth investigating the effect of introducing the prestress on the centre bolt. This is done in much the same way as for the stress ring on the ring transducers. We use the TEMPERATURE module to decrease the temperature of the centre bolt by the required amount to apply the same stress as that due to the torque on the tail nut. SNAKES, the non-linear package, is then invoked to modify the stiffness matrix of the structure before the natural frequency calculations are performed. This technique also has its limitations as, in Level 6.1, SNAKES cannot be used in conjunction with triangular elements, however this problem has been overcome in subsequent versions.

As with composite ring transducers, the introduction of prestress makes a negligible difference to the resonance frequency, so for simplicity, we model the transducers unstressed.

STAGE IV - The main problem still to be solved is how to successfully model the joints in the stack without introducing hundreds of extra elements. An attempt was therefore made to represent the joints by a number of 1-dimensional spring elements. These elements, type 30100, have elasticity but no mass. This would seem to be a reasonable representation of the joints since they are very thin and contribute very little in terms of mass to the overall structure of the transducer but their contribution to the compliance of the stack is significant. For simplicity we can make the springs of zero length by using coincident points for the two nodes which define the ends of the spring and use three springs spaced radially across each joint, as shown in Figure 3.13.

The stiffness of each of the three springs was simply taken as a third of the stiffness of a joint. We need not vary the stiffness with radius because we are only looking for axial modes. In an attempt to create the effect of a continuous epoxy layer rather than distinct springs, the REPEATED.FREEDOMS module was used to link the three axial degrees of freedom down each side of the joint.

The results from our four test transducers, when modelled in this way, are shown in Table 3.2 and Graph 3.3. We can see that these results represent a substantial improvement on the original analysis although they are not as good as those obtained for L109A and L109S by modeling the joints as a single epoxy layer. However, representing the joints by springs seems to be a method which is universally applicable, whereas the single epoxy layer method is limited to transducers with long stacks.

It is unlikely that we will be able to improve the modelling of the epoxy joints until Level 7 of PAFEC is available. Level 7 has a GENERALIZED CONSTRAINTS option, removing the need for corner nodes of adjacent elements to connect. This would allow consistent modelling of very thin layers, such as epoxy joints, to be carried out without the rest of the mesh becoming too fine.

Further Developments

French engineers have been modelling transducers using their own specially written Finite Element code [15][16]. They are able to obtain good results using relatively coarse meshes and do not appear to model the joints in the ceramic stack explicitly. Also they use a number of simple 1-dimensional rod elements to produce a simplified representation of the centre bolt [15]. This not only reduces the processing time, but also appears to reduce the stiffness of the central section of the transducer, thereby making some allowance for the effect of the joints. The French engineers have perfected their simplified meshing technique, over a number of years, by exhaustive comparison with more elaborate models and with experiment [16].

Producing PAFEC meshes similar to those use by the French for their Finite Element program (see Figure 3.14) resulted in, at best, an error of 8%. No attempt has been made in these PAFEC models to include the epoxy joints. To achieve any further improvement in the results requires that some account of their effect must be taken.

4. CONCLUSION

We have seen that, in most cases, the resonance frequency of a transducer can be determined by the PAFEC Finite Element Program to within 10% using a fairly simple mesh. However, the major limitation on the accuracy of the results is the difficulty in modelling the epoxy joints. The most accurate results obtained from PAFEC were for transducers, such as the ceramic ring type, which contain no epoxy. Unfortunately most types of transducer include epoxy joints in their construction and any further improvements to the results for these transducers requires a major increase in the complexity of the mesh. In a number of cases it seems unlikely that any improvement to the simple model will be possible without the use of some of the new features available in Level 7 of PAFEC.

A method of modelling the joints has been suggested whereby the length or Young's modulus of some of the components in the model are adjusted to include the additional compliance due to the joints. This was considered to be unsatisfactory as it is largely an empirical technique and an ab initio method is required if new designs of transducer are to be modelled.

In principle the Finite Element method should be able to calculate the resonance frequency of a structure precisely, providing each component can be represented accurately. In terms of modelling transducers, accurate resonance frequency predictions can be expected when the piezoelectric components and epoxy joints are precisely defined. Thus it is hoped that, with the introduction of Level 7 of PAFEC which includes some new features, more accurate calculation of the resonance frequencies of transducers may be possible.

APPENDIX A

EXPERIMENTAL METHOD

The standard resonance frequencies for all the different types of transducer are found using the same experimental arrangement. This consists of a digital impedance analyser controlled by a desk top computer and linked to a plotter. This is used to apply a small A.C. voltage across the terminals of the transducer at a range of frequencies, chosen by the operator, to obtain the admittance response over that range. The output from the system is a plot of the admittance loop of the transducer [9], such as that shown in Figure A.1. The resonance frequency is the frequency at which the maximum conductance is obtained. This is unlikely to be accurate to more than three significant figures although this may be further limited by the size of the steps chosen between sampling frequencies. Possibly a more important limitation on the accuracy is the reproducibility of the result over a number of transducers. Due to the nature of some of the materials and construction methods employed, there are marked variations in response characteristics between different transducers of the same design. The piezoelectric ceramic, for example, which forms the active part of all the transducers in this study, is such that there is some variation in its parameters, particularly between batches but also within a batch. Greater accuracy in results may be obtained by taking an average value of the resonance frequency over a large number of transducers.

APPENDIX B

USE OF PIEZOELECTRIC ELEMENTS

In Level 6.1 of PAFEC piezoelectric materials can be modelled in three dimensions using the piezoelectric elements type 35115 [3]. These elements can be mixed with the ordinary 3-dimensional elements. As these materials are orthotropic their mechanical properties are defined using the ORTHOTROPIC.MATERIAL module, while their dielectric and piezoelectric properties are defined using the PIEZOELECTRIC module. The material constants required for the PIEZOELECTRIC module are:

1. The clamped dielectric constants in Farads/metre.

2. The piezoelectric constant stress/electric field at constant strain or charge density/strain at constant electric field in Coulombs/metre^2 .

The material constants are published by manufacturers of piezoelectric ceramics and often referred to as e_{33}, e_{11}, e_{31} and e_{15}. The properties are highly sensitive to thoroughness of poling and it may be more appropriate to use degraded values [13].

Piezoelectric ceramics are isotropic in the plane perpendicular to the axis of polarization [14]. Hence PAFEC assumes that $e_{22} = e_{11}$, $e_{32} = e_{31}$, $e_{24} = e_{15}$, etc.

Although, in Level 6.1, piezoelectric elements are only available in three dimensions, a form of 2-dimensional piezoelectric element, for use in axisymmetric models, is available in subsequent versions.

APPENDIX C

RING TRANSDUCER ANALYSIS PROGRAM

The existing, conventional ring transducer analysis program used at ARE is called RINGPAN [4]. For rings where the ratio of the height to the mean radius is much less than π, it uses the standard formula for the natural frequency of uniform radial motion of a ring.

$$\text{FREQUENCY} = \frac{1}{2\pi R\,[\rho / E]^{1/2}}$$

where E is the Young's Modulus
ρ is the Density
R is the Mean Radius

The program requires that the ratio of the outside diameter to the thickness is much greater than 8, otherwise modes of vibration, which cannot be calculated from the simple formula used in this program, become significant and affect the performance of the ring.

APPENDIX D

PISTON STACK TRANSDUCER ANALYSIS PROGRAM

There are conventionally two distinct ways in which we can analyse a piston stack transducer [7],[8].

1. In the case where the length of the transducer is shorter or of the same order as the wavelength of sound within it, it can be considered to behave like two masses connected by a spring [9]. This simple model is shown in Figure 3.2. This is known as the "Lumped Mass" approach and characterizes the method used by the program PETPAN [10] to calculate the resonance frequency of a transducer. The spring representing the stack is considered to have no mass, but the compliance is that of the stack. The masses representing the head and tail are adjusted to include a contribution from the effective mass of the stack. The equations for the normal modes of this system are then solved to give the natural frequency of the transducer.

2. At higher frequencies, when the transducer is long relative to the wavelength of sound within it, a distributed parameter approach is required [11]. The program FJEUXB [12] uses this form of analysis where each component of the transducer is represented by an acoustic transmission line. The solution of the wave equation for plane waves for each part is written as a complex matrix. These are combined according to the boundary conditions to give a matrix equation representing the complete transducer. This is then solved to give the natural frequency of the transducer.

REFERENCES

[1] PAFEC "Theory" - PAFEC Limited 1984.

[2] PAFEC "PIGS User Manual Level 6.1" - PAFEC Limited.

[3] PAFEC "Data Preparation User Manual Level 6.1" - PAFEC Limited.

[4] ELLIOTT, A G "Ceramic Ring Transducer Analysis" - ARE Unpublished Report.

[5] CAMP, L "Underwater Acoustics" - Wiley-Interscience 1970.

[6] GALE, R L and SNELL, D J "Notes on Sonar Transducer Construction" - ARE Report No. 61793, 1983.

[7] ELLIOTT, A G and GALLAHER, A B "A Comparison of Piston Stack Transducers using a Lumped Mass Model, a Distributed Parameter Model and a Finite Element Model " - ARE Unpublished Report.

[8] ELLIOTT, A G and BUTTERWORTH, W "A Computer System for the Design and Analysis of Sonar Transducers at ARE" - ARE Unpublished Report.

[9] AUWE TRANSDUCER SECTION "Notes on Sonar Transducer Design" - ARE Report No. 20730, Revised 1980.

[10] SNELL, D "Computer Programs used in the Design and Analysis of Sonar Transducers at AUWE" - ARE Report No. 61792, 1980.

[11] EARWICKER, M J "Mathematical Modelling of Piezoelectric Transducers" - ARE Report No.482/73, 1973.

[12] EARWICKER, M J "Piezoelectric Transducer Analysis" - ARE Report No. 681.3.06, 1974.

[13] ALLIK, H and WEBMAN, K M "Vibrational Response of Sonar Transducers using Piezoelectric Finite Elements' - J Acoust Soc Am Vol 56 No 6 pp1782-1791, 1974.

[14] ALLIK, H and HUGHES, T J R "Finite Element Method for Piezoelectric Vibration" - Int Jou Mech Eng Vol 2 pp 151-157, 1970.

[15] DECARPIGNY, J N, DEBUS, J C, TOCQUET, B and BOUCHER, D "In-Air Analysis of Piezoelectric Tonpilz Transducers in a Wide Frequency Band" - J Acoust Soc Am Vol 78 pp 1499-1507, 1985.

[16] DECARPIGNY, J N, DEBUS, J C, HARMONIC, B, BOSSUT, R, TIERCE, P, MOREL, D, BOUCHER, D and TOCQUET, B "Finite Element Analysis of Low Frequency Sonar Transducers'. Proc IoA Vol 9 Part 2, 1987.

Figure 2.1 : Ceramic Ring Transducer.

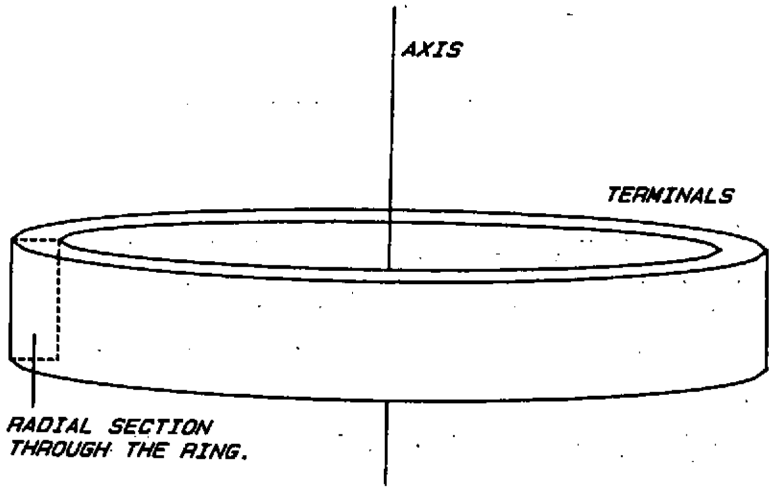
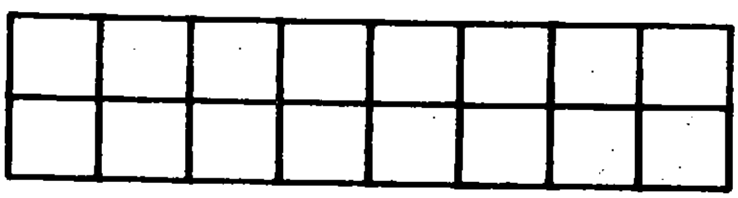


Figure 2.2 : Mesh for a Ceramic Ring.

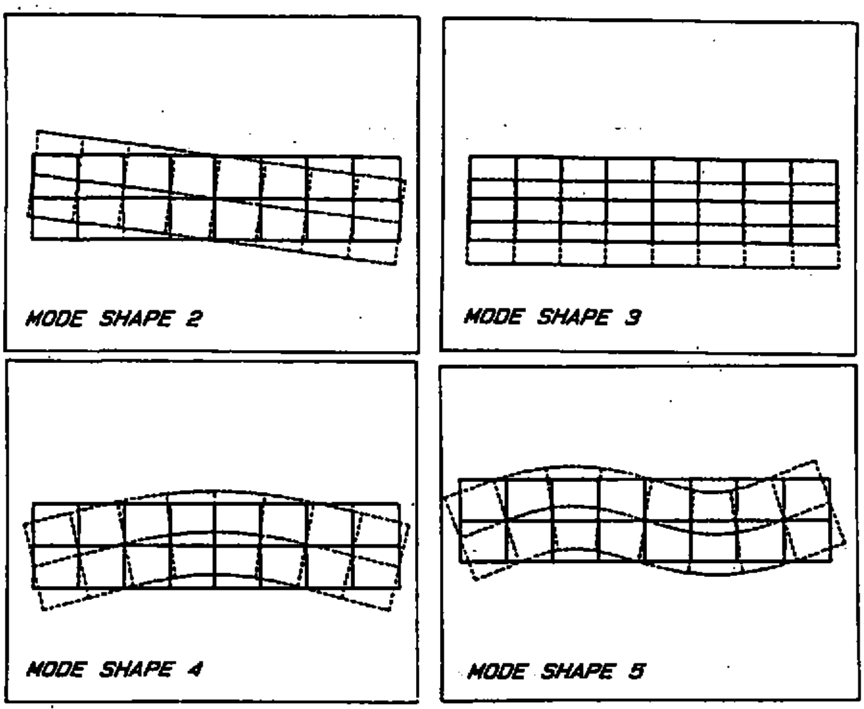


Axisymmetric mesh consisting of a radial section through the ring.

(90° rotation relative to Figure 2.1)

Figure 2.3

Mode Shapes of a Ceramic Ring Transducer.



The dashed lines represent the deformed shape
and the solid lines the undeformed shape.

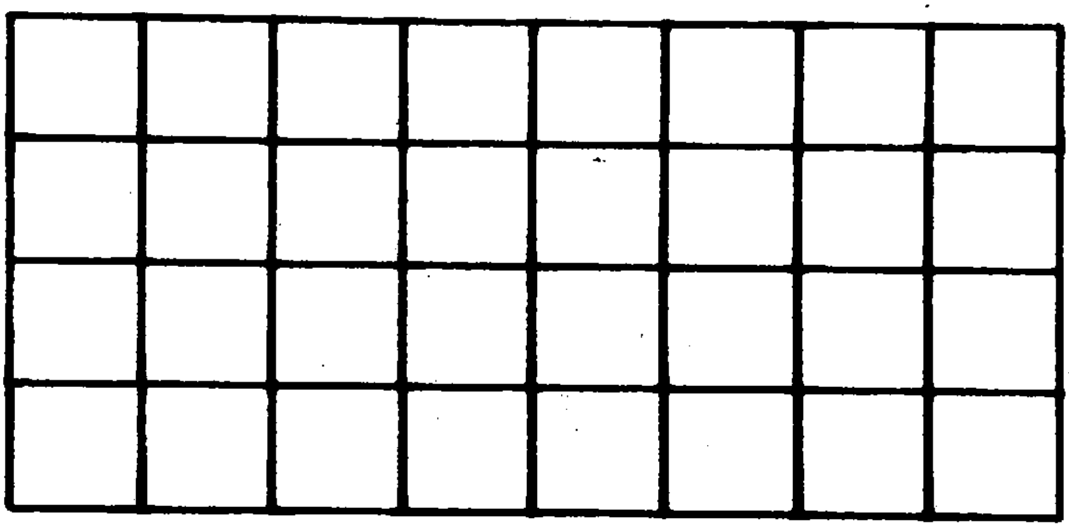


Figure 2.4

Mesh for a Ceramic Ring with a Metal Stress Ring around the outside.

TABLE 2.1 : Results for the three Ring Transducers

	Resonance Frequency from Experiment	Results from RINGPAN	ERROR*	Results from PAFEC	ERROR*
SMALL	10.2 kHz	10.9 kHz	6.9%	10.1 kHz	1.0%
MEDIUM	10.7 kHz	11.0 kHz	2.8%	10.4 kHz	2.8%
LARGE	10.8 kHz	11.2 kHz	3.7%	10.8 kHz	0.0%

* ERROR : This is defined as the difference between the experimental value for the resonance frequency and that calculated by the program as a percentage of the experimental value.

Figure 3.1 : Piston Stack Transducer.

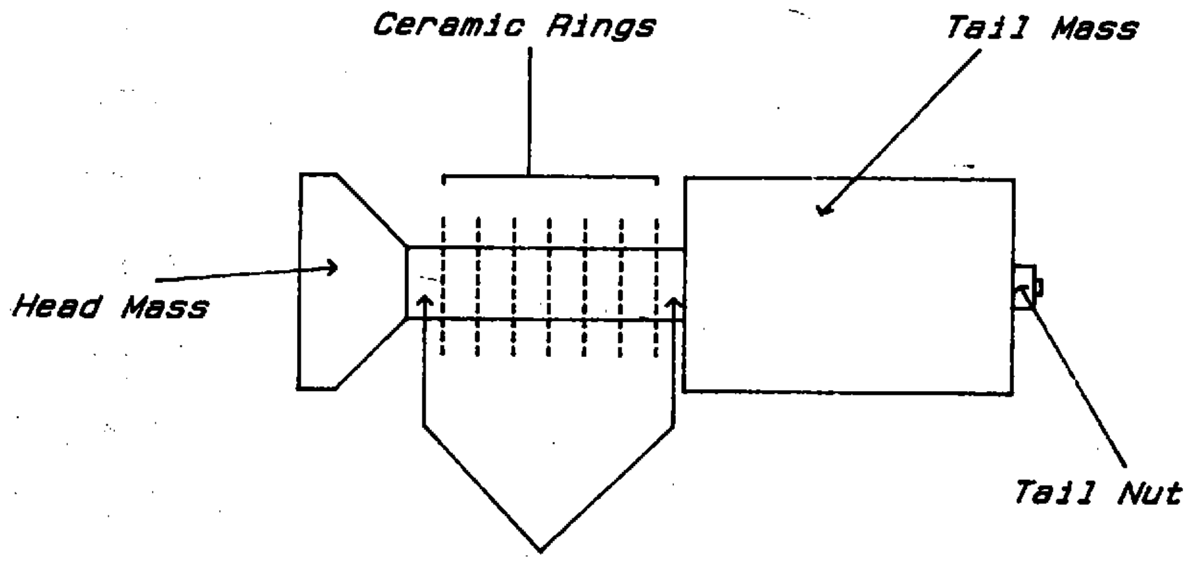


Figure 3.2 : Lumped Mass Model.

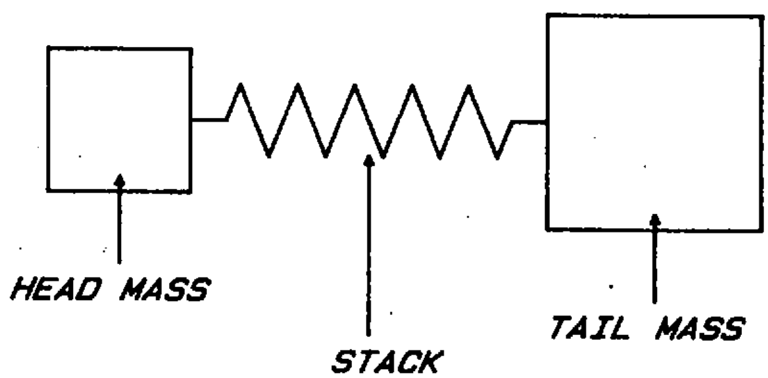


FIGURE 3.3 : Transducer L104

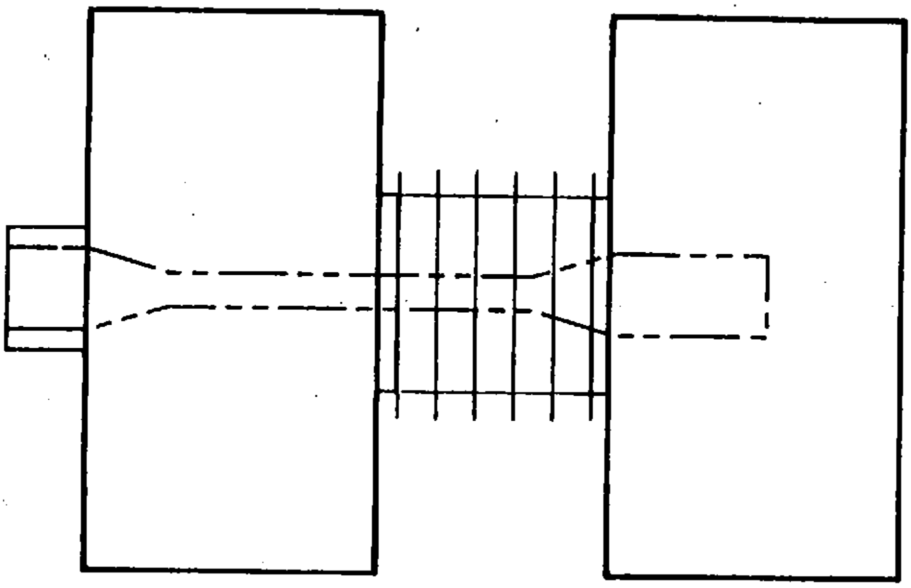


FIGURE 3.4 : Transducer L109S

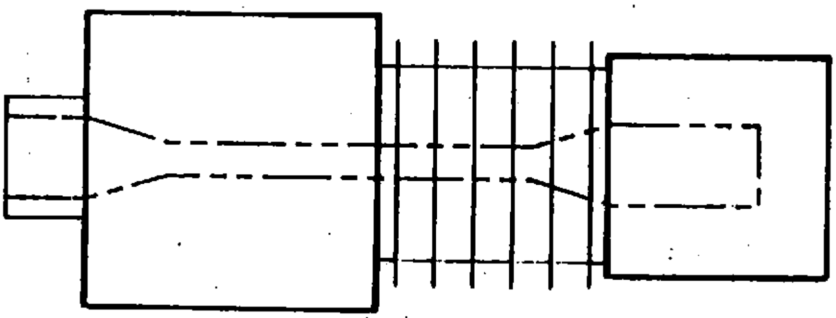


FIGURE 3.5 : Transducer L190A

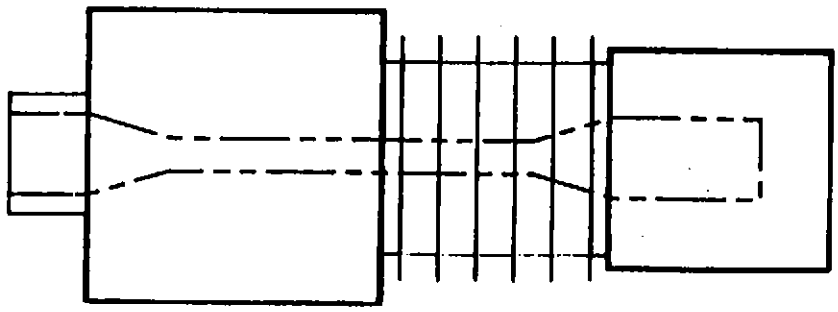


FIGURE 3.6 : Transducer L108

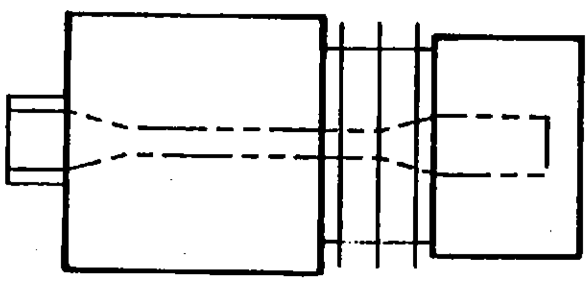


FIGURE 3.7 : Mesh for Transducer L104

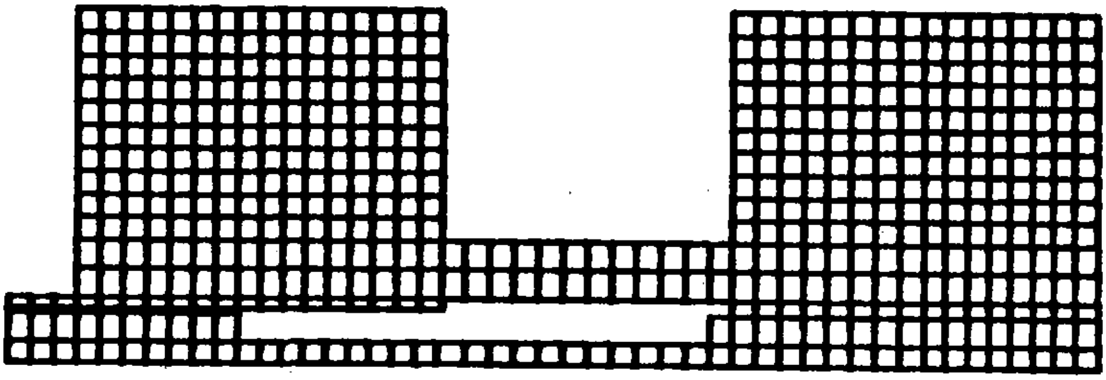


FIGURE 3.8 : Mesh for Transducer L109S

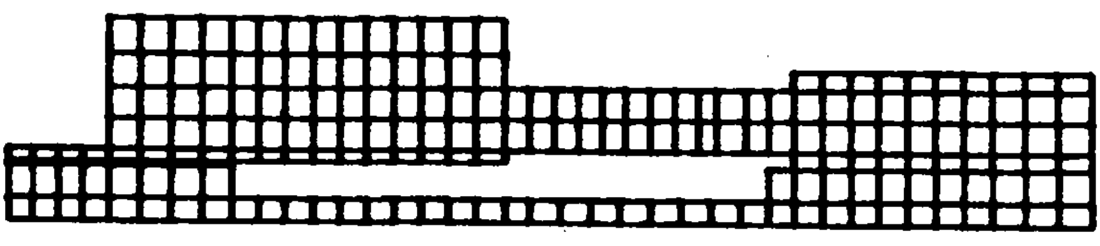


FIGURE 3.9 : Mesh for Transducer L190A

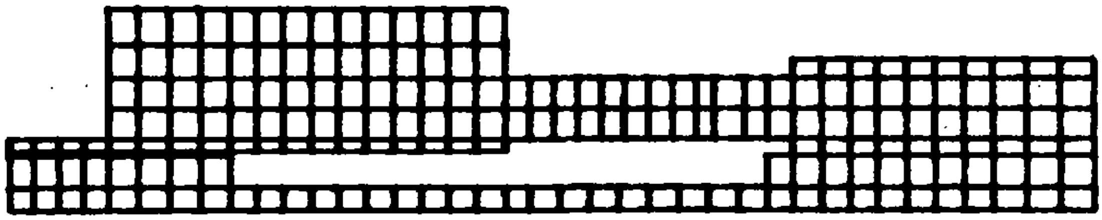


FIGURE 3.10 : Mesh for Transducer L108

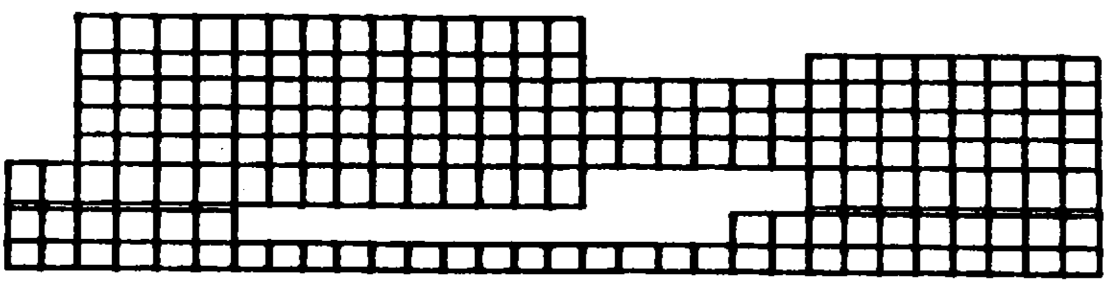


FIGURE 3.11

Corner nodes of adjacent elements must connect.

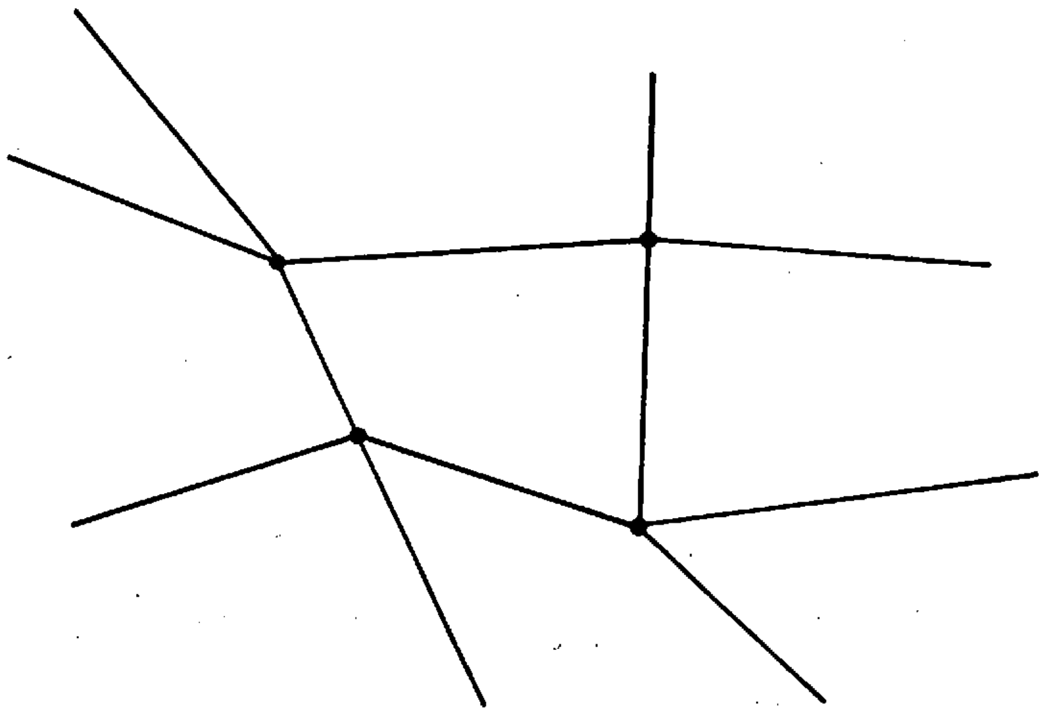


FIGURE 3.12

Triangular elements used to confine the high density of elements to the stack.

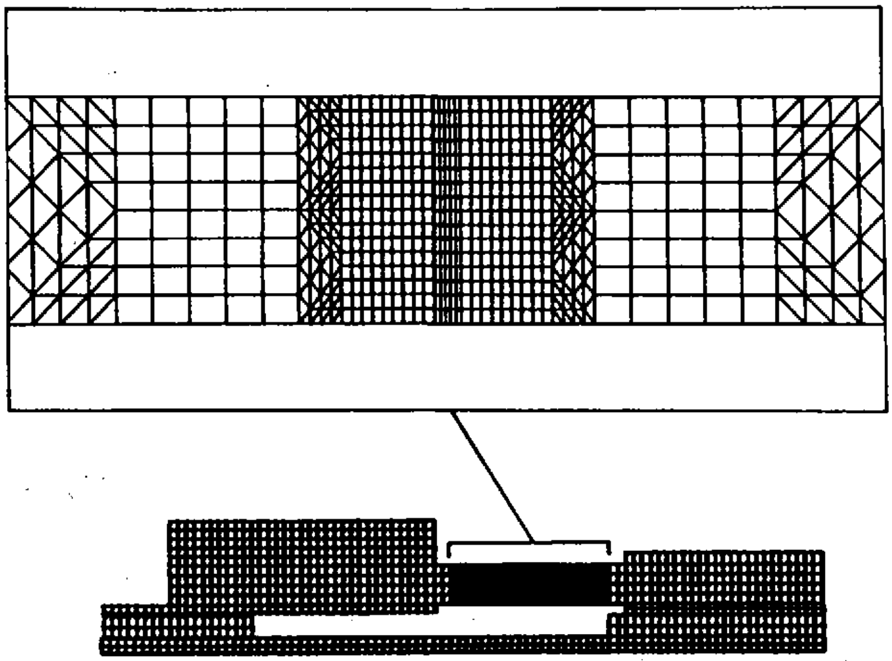
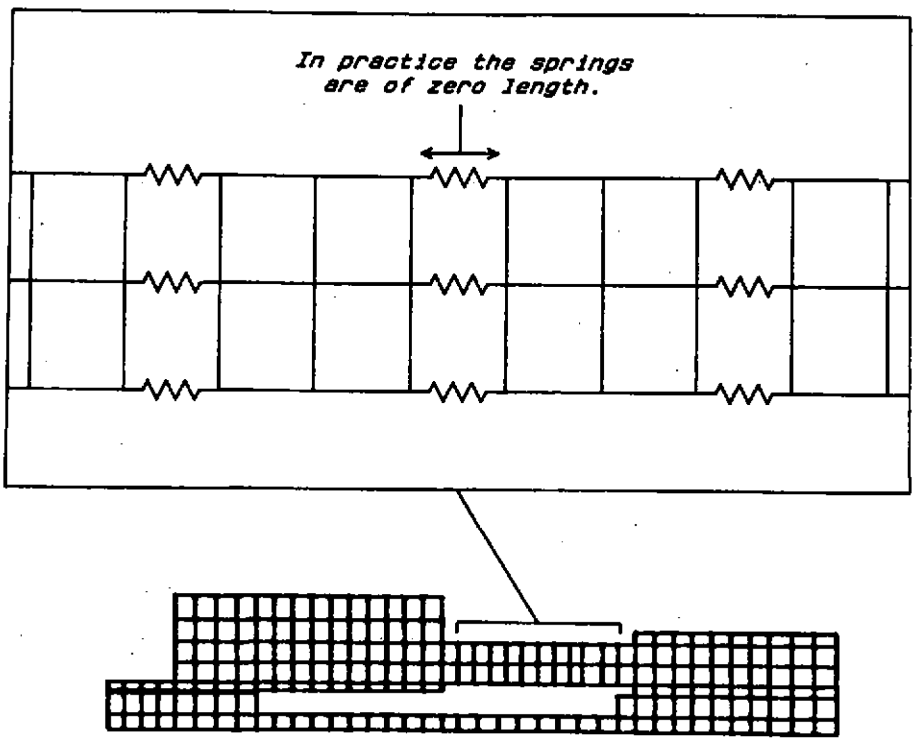


FIGURE 3.13

Epoxy joints modelled using spring elements.



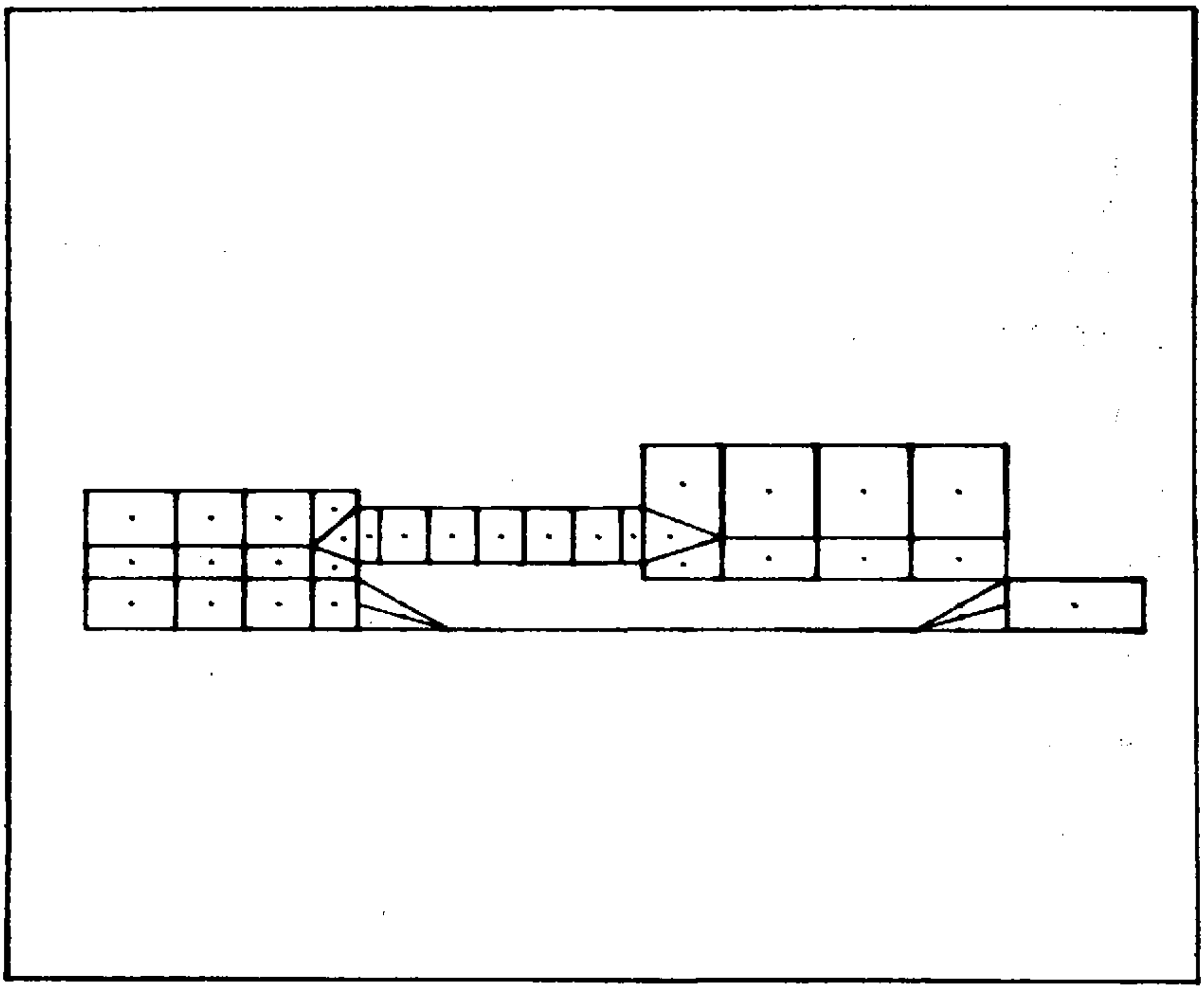


Figure 3.14

New Mesh for Transducer L109A.

1-D Rod Elements are used to represent the Centre Bolt.

TABLE 3.1 : Comparison of PAFEC results with PETPAN and FJEUXB.

Resonance Frequency from Experiment		Results from PETPAN	ERROR*	Results from FJEUXB	ERROR*	Results from PAFEC	ERROR*
L104	6.134 kHz	6.542 kHz	6.6%	5.871 kHz	-4.3%	6.527 kHz	6.4%
L109S	14.810 kHz	15.542 kHz	4.9%	13.300 kHz	-10.2%	17.204 kHz	17.5%
L109A	18.810 kHz	21.617 kHz	14.9%	18.120 kHz	-3.7%	21.498 kHz	14.3%
L108	29.384 kHz	36.478 kHz	24.1%	30.890 kHz	1.0%	32.292 kHz	9.9%

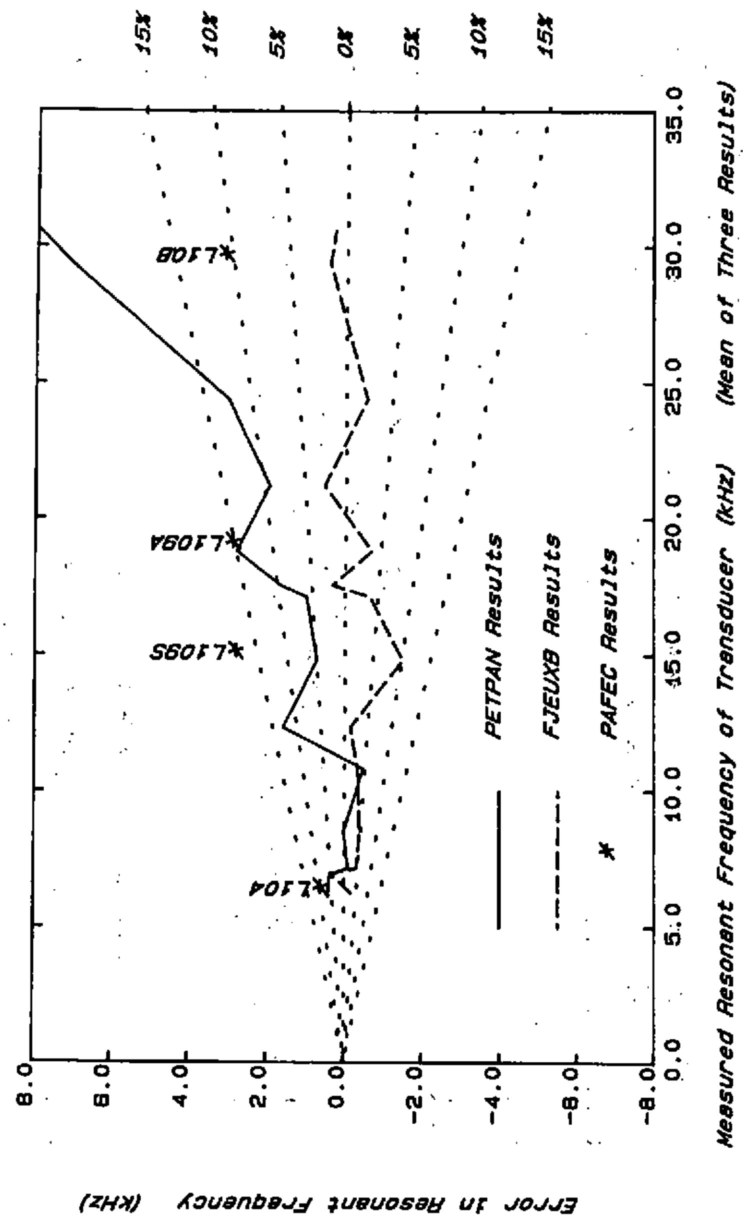
* ERROR : This is defined as the difference between the experimental value for the resonance frequency and that calculated by the program as a percentage of the experimental value.

TABLE 3.2 : Comparison of PAFEC results using spring elements with those from the simple model.

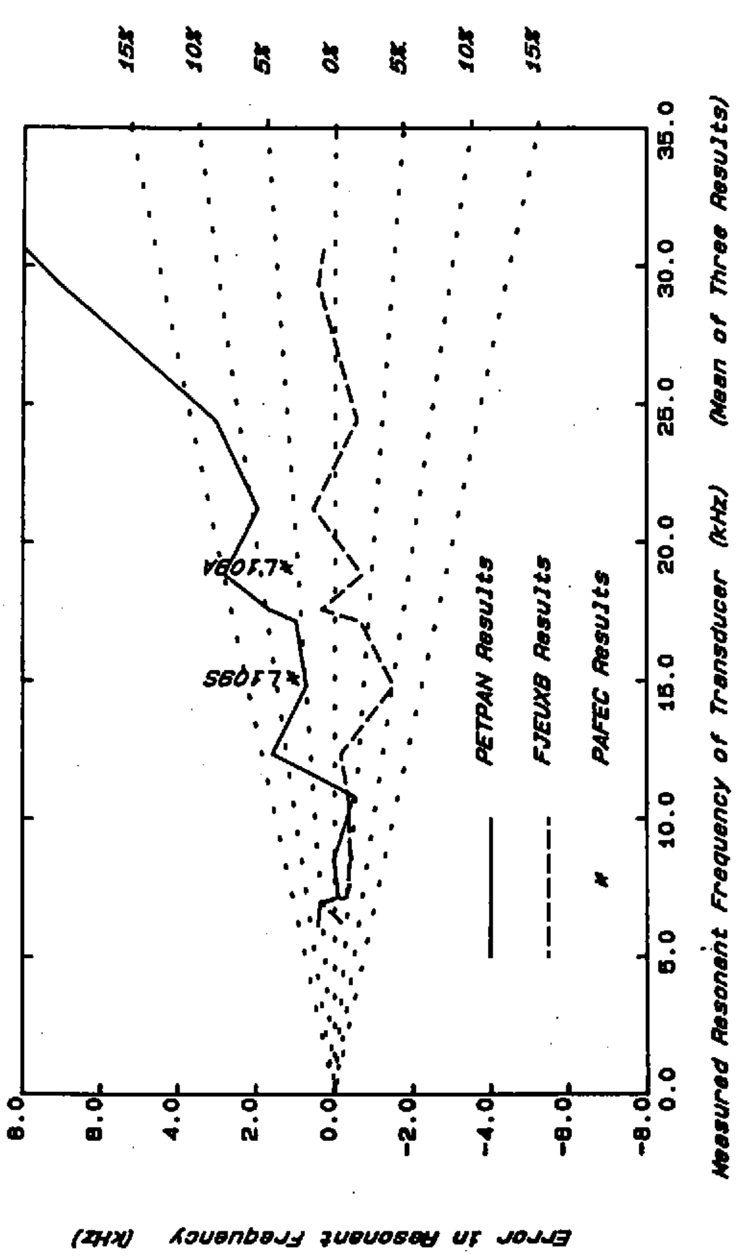
	Resonance Frequency from Experiment	PAFEC Results using the Simple Model	ERROR*	PAFEC Results using Spring Elements	ERROR*
L104	6.134 kHz	6.527 kHz	6.4%	6.535 kHz	6.53%
L109S	14.810 kHz	17.204 kHz	17.5%	16.203 kHz	9.41%
L109A	18.810 kHz	21.498 kHz	14.3%	20.430 kHz	8.61%
L108	29.384 kHz	32.292 kHz	9.9%	32.247 kHz	9.74%

* ERROR : This is defined as the difference between the experimental value for the resonance frequency and that calculated by the program as a percentage of the experimental value.

GRAPH 3.1 : Comparison of PAFEC results with PETPAN and FJEUXB.



GRAPH 3.2 : Comparison of PAFEC results, using a single layer of epoxy to represent the joints, with PETPAN and FJEUXB.



GRAPH 3.3 : Comparison of PAFEC results, using spring elements to represent the joints, with PETPAN and FJEUXB.

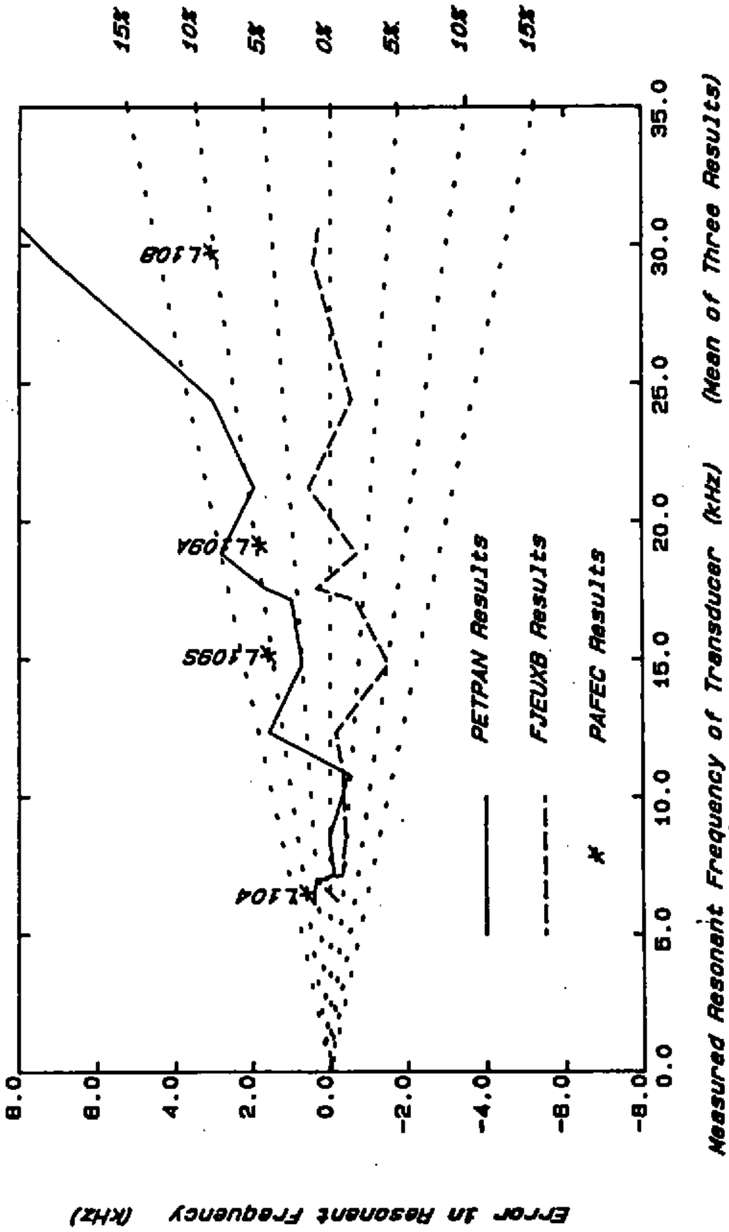


FIGURE A.1

Admittance Loop of a Transducer.