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A COMPARISON OF CURVED BEAM ELEMENTS

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A study has been made (Ashwell, Sabir and Roberts, Refs.1,2) into the effectiveness of certain curved finite elements when applied to arches. The object of the work was to gain understanding of the problem of selecting and devising suitable shape functions for curved structures, particularly cylindrical shells. The present paper summarises this work and extends the assessment of the shape functions to their use in the fields of stability and vibrations.

The solution of cylindrical shells by "rectangular" finite elements has been considered by a number of authors, and several shape functions have been suggested. The simplest is obtained by combining the usual plane stress polynomial expressions for displacements u, v parallel to the shell surface with the rectangular plate bending shape function of Zienkiewicz and Cheung (Ref.3) for normal displacement w. This leads to a 20 X 20 element stiffness matrix (Connor and Brebbia; Sabir and Ashwell; Sabir, Refs. 4, 5, 6). An extension of this element to 24 X 24, by adding terms to the polynomial for w is given by Bogner, Fox and Schmit (Ref.7) and Cantin and Clough (Ref. 8). Bogner et al increase the stiffness matrix to 48 X 48 by taking more terms in the polynomials for u and v. Cantin and Clough point out that polynomial shape functions do not allow the element to undergo rigid-body displacements, and show how the 24 X 24 element can satisfy this requirement if certain trigonometrical terms are included in the shape function. Applied to a pinched cylinder of particular dimensions the result element is superior to the simple 20 X 20 element, but not quite so good as the 48 X 48 element of Bogner et al.

The investigation carried out by Ashwell, Sabir and Roberts (Ref.1) began with an attempt to assess the element of Cantin and Clough when applied to arches. The problem of a circular arch was chosen because (i) it is the simplest such problem which introduces curvature and thus gives rise to the difficulty associated with need for rigid-body modes, and (ii) exact analytical solutions to a number of problems can be readily obtained. Several problems were solved, for arches of different radius/thickness ratios, and subtending different angles; and curves were plotted showing convergence with increasing numbers of elements. As was expected, the Cantin and Clough element was very superior to the simple 20 X 20 element, but, unexpectedly, the success of both elements (and also a 'shallow shell' modification of Cantin and Clough's element) was strongly dependent on the proportions of the arch considered. Thus, while all the shape functions gave reasonable convergence for a thick (radius/thickness ratio 40) and moderately shallow (subtending 40^{o}) arch, none of the shape functions converged (for up to 36 elements) if the arch was thin (radius/thickness ratio 320) and deep (subtending 180^{o}).

This result suggested that it was necessary to consider other shape functions if it was required to solve arches (and therefore cylindrical shells) of all proportions. This was done in the second part of the work (Ref.2) in which (i) the 48 X 48 shape function of Bogner et al was applied to arches, and (ii) a new shape function was developed from simple strain functions. Both these shape functions converged satisfactorily for arches of all proportions, but the new shape function was appreciably better than that of Bogner et al if the arch was thin and deep and a small number of elements was used. The shape functions were also used to calculate internal stress resultants (thrust, shearing force, bending moment) and it was found that neither shape function gave universally satisfactory results, if nodal values of the stress resultants were calculated by the normal procedure using a stress matrix (Ref.5) - although mid-element values could be obtained accurately.

Accurate values of the nodal stress resultants could be obtained by using the element stiffness matrix instead of the stress matrix.

This work is now being extended to determine whether the shape functions found to be satisfactory for linear static problems are also satisfactory for stability and vibration problems. The problems chosen are (i) the in-plane stability of a thin circular ring under a uniform inward radial load, and (ii) the in-plane free vibrations of such a ring. It is hoped to report the results of this work at the conference.

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