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ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

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ABSTRACT

The interaction of an anchored flame with a low-level acoustic field is considered. The flame is considered to be 'held' by a gauze through which the reactants are allowed to flow. Small perturbations in velocity in the combustion zone are linked upstream and downstream with order O(M) acoustic disturbances at large distances from the source, (where M is the flow Mach number). This matching process yields a frequency condition governing the flame vibration and preliminary results for emitted acoustic waves are presented.

INTRODUCTION

The theory of flame vibration is a subject much addressed in the literature and it is impossible to summarise all the work being done in this area. This particular paper is a small contribution to the recent research being done on burner noise.

Jones [1,2] has considered the theory of vibrating flames in tubes and van Harten, Matkowsky and Kapila [3] have considered the effect of sound impinging on a flame front. These works are dealing essentially with adiabatic moving flame fronts. In this present work the flame is considered anchored with some heat loss to a gauze (this is considered to be of high conductance - see Clarke and McIntosh [4]). The physical set up is thus similar to that used by Madarame [5], but the mathematical model is somewhat different since only long-wave disturbances in the combustion zone are considered. Thus the wave equations dominate only on (the muchlarger) acoustic length scales outside this zone. Within the combustion zone the flame responds in a quasi-steady manner to the fluctuating velocity that it experiences, and in accordance with unsteady combustion equations including mass and thermal diffusion. The matching of values and gradients of temperature, pressure, density and velocity then lead to a frequency condition which the flame oscillations must obey.

This work is thus a contribution to the understanding of acoustical oscillations encountered in experiments and observed, in particular, by Schimmer and Vortmeyer [6] and Roberts [7].

In this paper, the chemistry (albeit along simple lines) is included and the activation energy of the combustion becomes a further parameter in the resulting equation. Preliminary results are shown here for emitted acoustic waves as a first stage to the modelling of more complicated burner port systems where resonance and acoustic forcing must be considered.

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ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

A statement of the basic equations is made in Section 2, with a summary of the analysis in Section 3 where the frequency relation is derived for small amplitude, emitted acoustic waves. Section 4 constitutes a brief discussion of the preliminary findings from this result.

BASIC EQUATIONS

The one dimensional unsteady equations in pressure p, density ρ, temperature T, gas velocity u and lean species mass fraction C_ℓ are given by:

$$p = \frac{\rho T}{T_{01}} \quad , \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \quad , \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} - \frac{1}{L_e \rho}\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = L_e\frac{(1+\sigma)}{\sigma}Q_1\Lambda_1 C_\ell (C_\ell + |\Delta_1|)\, e^{\theta_1(1-1/T)}$$
$$+ \frac{T_{01}(1-\gamma^{-1})}{\rho}\left[\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \frac{4}{3}\gamma M^2 S_c \lambda\left(\frac{\partial u}{\partial x}\right)^2\right] , \tag{3}$$

$$\frac{\partial C_\ell}{\partial t} + u\frac{\partial C_\ell}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}\left(\lambda\frac{\partial C_\ell}{\partial x}\right) = -L_e\Lambda_1 C_\ell (C_\ell + |\Delta_1|)\, e^{\theta_1(1-1/T)} \quad , \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\gamma M^2 \rho}\frac{\partial p}{\partial x} = \frac{4}{3}\frac{S_c}{\rho}\frac{\partial}{\partial x}\left(\lambda\frac{\partial u}{\partial x}\right) . \tag{5}$$

These equations are derived from the application of the gas law and the principles of continuity, energy, lean-species C_ℓ, and momentum. Pressure p, density ρ, and velocity u, are non-dimensionalised with respect to upstream values. Temperature T is non-dimensionalised with respect to its burnt value. M represents flow Mach number which will be treated as a small number, and with small variations in pressure, we adopt the assumption that thermal conductivity λ obeys the rule [8],

$$\rho\lambda = 1 \tag{6}$$

The assumption (for simplicty) of constant specific heat C_p' and constant (overall) mass diffusion leads to the definition of the following non-dimensional transport coefficients:

Lewis number, $$L_e \equiv \frac{\rho'\mathcal{D}'}{\lambda'/c_p'}, \tag{7}$$

Schmidt number, $$S_c \equiv \frac{\mu'}{\rho'\mathcal{D}'}, \tag{8}$$

where $\mathcal{D}'$ and μ' represent coefficients of diffusion and dynamic viscosity respectively and the dash $'$ denotes a dimensional quantity. The symbol γ represents the ratio of specific heats and T_{01} is the non-dimensionalised temperature in the initially steady unburnt stream (subscript "$_{01}$" signifies initial value at "$x = 0$", the gauze downstream surface - see Fig. 1). Thus T_{01}^{-1} is the ratio of the temperature jump across the flame, and for this simple idealised model downstream heat-losses are not considered. Referring to Fig. 1, the flame is considered anchored with some heat loss to a porous gauze (considered thin on an acoustic scale). Hirschfelder conditions are assumed to be obeyed at the downstream holder surface such that

$$C_\ell(0,t) - \frac{1}{m_0}\left(\frac{\partial C_\ell}{\partial x}\right)\Big|_{x=0} = \text{constant} , \tag{9}$$

$$T(0,t) = \text{constant} , \tag{10}$$

where

$$m_o \equiv \rho_c u_c \quad , \tag{11}$$

is the mass flux at the gauze surface. The two conditions (9) and (10) are effectively a statement that back diffusion of products is negligible and that the conductance of the gauze is very large [8]. Before and after the gauze, acoustic waves are permitted and at the gauze, continuity of velocity, pressure and density is assumed. Far upstream and downstream the boundary conditions are simply that any disturbances decay. Thus for this preliminary study, feedback mechanisms and gauze porosity effects are not included. (Nevertheless the theory can readily be modified at a later stage to include these).

It should be pointed out that characteristic diffusion lengths and times have been used to non-dimensionalise the equations and the initially steady system is completely described by the mathematical model described in [4]. The non-dimensional parameters σ, Q_1, Θ_1 and Λ_1 are respectively the ratio of molecular weights, the reduced heat of reaction, the activation energy and the so-called pre-exponential 'eigen' value. This latter quantity is typically (for far-from-stoichiometric conditions) proporitonal to Θ_1^2. Lastly the term $|\Delta_1|$ in equation (4) is related to the mixture strength [4].

ANALYSIS

In order to deal with the density changes in the combustion zone it is convenient to transform the spatial coordinate x to,

$$x_1 \equiv \int_0^x \rho \, dx \quad . \tag{12}$$

On this assumption, the differential equations become,

$$p = \frac{\rho T}{T_{01}} \quad , \tag{13}$$

$$\frac{\partial \rho}{\partial t} + m_o \frac{\partial \rho}{\partial x_1} + \rho^2 \frac{\partial u}{\partial x_1} = 0 \quad , \tag{14}$$

$$\frac{\partial T}{\partial t} + m_o \frac{\partial T}{\partial x_1} - \frac{1}{L_e} \frac{\partial^2 T}{\partial x_1^2} = \frac{L_e (1+\sigma) Q_1}{\sigma} \Lambda_1 C_\ell (C_\ell + |\Delta_1|) e^{\Theta_1 (1 - 1/T)}$$
$$+ T_{01}(1-\gamma^{-1}) \left[\frac{1}{\rho} \frac{\partial p}{\partial t} + \frac{m_o}{\rho} \frac{\partial p}{\partial x_1} + \frac{4}{3} \gamma M^2 S_c \left(\frac{\partial u}{\partial x_1} \right)^2 \right] \quad , \tag{15}$$

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

$$\frac{\partial c_e}{\partial t} + m_o \frac{\partial c_e}{\partial x_1} - \frac{\partial^2 c_e}{\partial x_1^2} = -L_e \Lambda_1 c_e (c_e + |\Delta_1|) e^{Q_1(1-1/T)} , \tag{16}$$

$$\frac{\partial u}{\partial t} + m_o \frac{\partial u}{\partial x_1} + \frac{1}{\gamma M^2}\frac{\partial p}{\partial x_1} = \frac{4}{3} S_c \frac{\partial^2 u}{\partial x_1^2} , \tag{17}$$

where (see equation (11)) m_o represents the mass flux at the gauze surface $(x_1=0)$.

We now consider small perturbations of the variables p, u, T, c_e and ρ ,

$$p = p_s + \delta p_u + \ldots , \tag{18}$$

$$u = u_s + \delta u_u + \ldots , \tag{19}$$

$$T = T_s + \delta T_u + \ldots , \tag{20}$$

$$c_e = c_{es} + \delta c_{eu} + \ldots , \tag{21}$$

$$\rho = \rho_s + \delta \rho_u + \ldots , \tag{22}$$

where δ is a small quantity and the subscript "s" refers to the steady values. Since the steady solution considered here is that of the flat anchored flame, we know that

$$p_s = 1 + M^2 p_{fs} + \ldots , \tag{23}$$

where p_{fs} is a flow induced pressure. On the assumption that

$$\delta >> M^2 , \tag{24}$$

the differential equations in the unsteady terms become, (at order δ):

$$T_{o1} p_u = \frac{T_{o1} T_u}{T_s} + \rho_u T_s , \tag{25}$$

$$T_s \frac{\partial p_u}{\partial t} - \frac{\partial T_u}{\partial t} - \frac{\partial T_u}{\partial x_1} - U_{u_o} \frac{dT_s}{dx_1} + T_{c_1} \frac{\partial u_u}{\partial x_1} + (p_u - p_{u_o}) \frac{dT_s}{dx_1} + T_s \frac{\partial p_u}{\partial x_1} = 0 \quad , \tag{26}$$

$$\frac{\partial T_u}{\partial t} + \frac{\partial T_u}{\partial x_1} + (p_{u_o} + u_{u_o}) \frac{dT_s}{dx_1} - \frac{1}{L_e} \frac{\partial^2 T_u}{\partial x_1^2} = \frac{L_e(1+\sigma)Q_1 R_u}{\sigma} + (1-\gamma^{-1}) T_s \left(\frac{\partial p_u}{\partial t} + \frac{\partial p_u}{\partial x_1} \right) , \tag{27}$$

$$\frac{\partial C_{\ell u}}{\partial t} + \frac{\partial C_{\ell u}}{\partial x_1} + (p_{u_o} + u_{u_o}) \frac{dC_{\ell s}}{dx_1} - \frac{\partial^2 C_{\ell u}}{\partial x_1^2} = -L_e R_u , \tag{28}$$

$$\frac{\partial u_u}{\partial t} + \frac{\partial u_u}{\partial x_1} + \frac{1}{T_{c_1}} (p_{u_o} + u_{u_o}) \frac{dT_s}{dx_1} + \frac{1}{\gamma M^2} \frac{\partial p_u}{\partial x_1} = \frac{4}{3} S_c \frac{\partial^2 u_u}{\partial x_1^2} , \tag{29}$$

where

$$R_u \equiv \Lambda_1 \left[\frac{\theta_1 T_u}{T_s^2} \left(|\Delta_1| + C_{\ell s} \right) C_{\ell s} + \left(|\Delta_1| + 2C_{\ell s} \right) C_{\ell u} \right] \exp\left[\theta_1 \left(1 - \frac{1}{T_s} \right) \right] , \tag{30}$$

and the term $(p_{u_o} + u_{u_o})$ in the above equations is the perturbation of the mass flux m_u at the gauze downstream surface $(x_1 = 0)$ and arises in this form because of condition (10) (i.e. $T_u(x_1 = 0) = 0$). It should further be pointed out the term R_u defined by equation (28) appears since [see Ref. 9] we assume

$$\delta << \theta_1^{-1} \quad , \tag{31}$$

and thus linearise the chemical term. Notice also that,

$$X_{u_o} \equiv X_u (x_1 = 0) \qquad (X = p, T, C_\ell, u) \, . \tag{32}$$

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

Acoustic Zones

At large distances from the gauze (order M^{-1} in terms of diffusion lengths), it is considered that a small acoustic field is present. Thus we write,

$$\hat{x} = M x, \tag{33}$$

to define distances on an acoustic scale. Time is not rescaled since for large wavelength acoustic oscillations, a typical unit of time can be considered to be comparable to x_c'/u_{o_1}', where x_c' is a typical diffusion length, and u_{o_1}' is the initial flow velocity. Under these conditions, we define variations in pressure, velocity, temperature, density and lean species as,

$$p_u = M p_u^{(a)}(\hat{x},t) + M^2 p_u^{(b)}(\hat{x},t) + \cdots \quad , \tag{34}$$

$$T_u = M T_u^{(a)}(\hat{x},t) + \cdots \quad , \tag{35}$$

$$\rho_u = M \rho_u^{(a)}(\hat{x},t) + \cdots \quad , \tag{36}$$

$$u_u = u_u^{(a)}(\hat{x},t) + \cdots \quad , \tag{37}$$

$$C_{\rho_u} = 0 \quad . \tag{38}$$

The latter equation emphasises that species variations (as well as $O(1)$ variations in temperature, as will be seen in the next section) decay on combustion length scales, and do not extend into the acoustic zone.

We now substitute equations (34) to (38) into equations (25) to (29) for the upstream case (where $T_s = 1$) and the downstream case (where $T_s = T_{o_1}$). Keeping leading order terms only, we obtain the classic acoustic equations:

Upstream

$$\left\{ \begin{aligned} & T_u^{(a_1)} = \left(p_u^{(a_1)} - \rho_u^{(a_1)} \right) T_{o_1} \quad , && (39) \\ & \frac{\partial \rho_u^{(a_1)}}{\partial t} + \frac{\partial u_u^{(a_1)}}{\partial \hat{x}} = 0 \quad , && (40) \\ & p_u^{(a_1)} = \gamma \rho_u^{(a_1)} \quad , && (41) \\ & \frac{\partial u_u^{(a_1)}}{\partial t} + \frac{1}{\gamma} \frac{\partial p_u^{(a_1)}}{\partial \hat{x}} = 0 \quad , && (42) \end{aligned} \right.$$

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

Downstream

$$\left\{\begin{array}{ll} T_u^{(a2)} = p_u^{(a2)} - \frac{1}{T_{c1}}\rho_u^{(a2)} , & (43) \\ \frac{1}{T_u}\frac{\partial \rho_u^{(a2)}}{\partial t} + T_{c1}\frac{\partial u_u^{(a2)}}{\partial \hat{x}} = 0 , & (44) \\ p_u^{(a2)} = \frac{\gamma}{T_{c1}}\rho_u^{(a2)} , & (45) \\ \frac{\partial u_u^{(a2)}}{\partial t} + \frac{1}{\gamma}\frac{\partial p_u^{(a2)}}{\partial \hat{x}} = 0 , & (46) \end{array}\right.$$

where superscripts "a1" and "a2" are used to denote the upstream and downstream acoustic zones respectively.

Combustion Zone

Near the flame, we define the following coefficient functions:

$$p_u = M p_u^{(1)}(x_1,t) + M^2 p_u^{(2)}(x_1,t) + \cdots , \qquad (47)$$

$$T_u = T_u^{(0)}(x_1,t) + M T_u^{(1)}(x_1,t) + \cdots , \qquad (48)$$

$$C_{\ell u} = C_{\ell u}^{(0)}(x_1,t) + M C_{\ell u}^{(1)}(x_1,t) + \cdots , \qquad (48a)$$

$$\rho_u = \rho_u^{(0)}(x_1,t) + M \rho_u^{(1)}(x_1,t) + \cdots , \qquad (49)$$

$$u_u = u_u^{(0)}(x_1,t) + M u_u^{(1)}(x_1,t) + \cdots , \qquad (50)$$

and keep distances measured on the diffusion length scale using the coordinate x_1.

Substitution of the series expansion (47) - (50) into equations (25) to (29) yield to leading order, the familiar combustion relations linking conductive, diffusive and reactive terms:

$$\rho_u^{(0)} = -\frac{T_{c1} T_u^{(0)}}{T_s^2} , \qquad (51)$$

$$\frac{\partial T_u^{(0)}}{\partial t} + \frac{\partial T_u^{(0)}}{\partial x_1} + u_{u_0}^{(0)}\frac{dT_s}{dx_1} - T_{c1}\frac{\partial u_u^{(0)}}{\partial x_1} = 0 , \qquad (52)$$

$$\frac{\partial T_u^{(0)}}{\partial t} + \frac{\partial T_u^{(0)}}{\partial x_1} + u_{u_0}^{(0)}\frac{dT_s}{dx_1} - \frac{1}{Le}\frac{\partial^2 T_u^{(0)}}{\partial x_1^2} = \frac{Le\,(1+\sigma)Q_1}{\sigma} R_u^{(0)} \quad , \tag{53}$$

$$\frac{\partial C_{\ell u}^{(0)}}{\partial t} + \frac{\partial C_{\ell u}^{(0)}}{\partial x_1} + u_{u_0}^{(0)}\frac{dC_{\ell s}}{dx_1} - \frac{\partial^2 C_{\ell u}^{(0)}}{\partial x_1^2} = -Le\, R_u^{(0)} \quad , \tag{54}$$

$$p_u^{(1)} = \text{function of } t \text{ only} \ . \tag{55}$$

where $R_u^{(0)}$ is as equation (30) with $T_u, C_{\ell u}$ replaced by $T_u^{(0)}, C_{\ell u}^{(0)}$ respectively. Note that equation (55) follows as a result of the necessity in the momentum equation (29), that pressure gradients are small over diffusion lengths. Note also that although we have not explicitly written them out, there will be a further set of equations for the next order terms in the combustion zone series expansion.

<u>Matching of Combustion and Acoustic Zones</u>

Using the principles of matched asymptotic expansions [19] for matching values and gradients, it can be shown that the following connections between the upstream acoustic zone and the combustion zone must apply:

Values:

$$p_u^{(1)}(-\infty, t) = p_u^{(a1)}(0, t) \quad , \tag{56}$$

$$T_u^{(0)}(-\infty, t) = 0 \quad , \tag{57}$$

$$C_{\ell u}^{(0)}(-\infty, t) = 0 \quad , \tag{58}$$

$$\rho_u^{(0)}(-\infty, t) = 0 \quad , \tag{59}$$

$$u_u^{(0)}(-\infty, t) = u_u^{(a1)}(0, t) \quad , \tag{60}$$

$$p_u^{(2)}(-\infty, t) = p_u^{(b1)}(0, t) + \left.\frac{\partial p_u^{(a1)}}{\partial \hat{x}}\right|_{\hat{x}=0} \quad , \tag{61}$$

$$T_u^{(1)}(-\infty, t) = T_u^{(a1)}(0, t) \quad , \tag{62}$$

$$C_{\ell u}^{(1)}(-\infty, t) = 0 \quad , \tag{63}$$

$$\rho_u^{(1)}(-\infty, t) = \rho_u^{(a1)}(0, t) \quad , \tag{64}$$

$$u_u^{(1)}(-\infty,t) = u_u^{(b_1)}(0,t) + \left.\frac{\partial u_u^{(m_1)}}{\partial \hat{x}}\right|_{\hat{x}=0} . \tag{65}$$

Gradients:

$$\left.\frac{\partial p_u^{(1)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial T_u^{(0)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial C_{\ell u}'^{(0)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial \varrho_u^{(0)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial u_u^{(0)}}{\partial x_1}\right|_{-\infty} = 0 , \tag{66-70}$$

$$\left.\frac{\partial p_u^{(2)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial p_u^{(a_1)}}{\partial \hat{x}}\right|_{0} \quad ; \quad \left.\frac{\partial u_u^{(1)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial u_u^{(a_1)}}{\partial \hat{x}}\right|_{0} , \tag{71,72}$$

$$\left.\frac{\partial T_u^{(1)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial C_{\ell u}'^{(1)}}{\partial x_1}\right|_{-\infty} = \left.\frac{\partial \varrho_u^{(1)}}{\partial x_1}\right|_{-\infty} = 0 . \tag{73-75}$$

In exactly the same way, matching conditions will apply on the downstream ($x_1 = +\infty$) side of the combustion zone. For the solution of the leading order equations (51)-(55) we shall only need matching conditions (56-60) and (66-70) but the second order conditions (61-65), (71-75) are included to indicate the nature of the next order solution, where gradients of acoustic terms begin to be important. However it is sufficient for our purposes only to consider leading order terms.

Harmonic Solutions

In this preliminary study, it is our purpose to seek harmonic solutions to the overall equations. In the outer zone we write

$$X_u^{(a_{\frac{1}{2}})} = \overline{X}_u^{(a_{\frac{1}{2}})}(\hat{x})\, e^{\omega t} \quad , (X = T, \varrho, p, u), \tag{76}$$

and restricting ourselves to emitted waves only, we obtain for equations (37) - (44), the solutions,

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

Upstream

$$\left\{\begin{aligned} \bar{p}_u^{(a1)} &= P^{(a1)} e^{\omega\hat{x}}, && (77)\\ \bar{\rho}_u^{(a1)} &= \gamma^{-1} P^{(a1)} e^{\omega\hat{x}}, && (78)\\ \bar{T}_u^{(a1)} &= (1-\gamma^{-1}) T_{01} P^{(a1)} e^{\omega\hat{x}}, && (79)\\ \bar{u}_u^{(a1)} &= -\gamma^{-1} P^{(a1)} e^{\omega\hat{x}}, && (80) \end{aligned}\right.$$

Downstream

$$\left\{\begin{aligned} \bar{p}_u^{(a2)} &= P^{(a2)} e^{-\omega\hat{x}/\sqrt{T_{01}}}, && (81)\\ \bar{\rho}_u^{(a2)} &= T_{01}\gamma^{-1} P^{(a2)} e^{-\omega\hat{x}/\sqrt{T_{01}}}, && (82)\\ \bar{T}_u^{(a2)} &= (1-\gamma^{-1}) P^{(a2)} e^{-\omega\hat{x}/\sqrt{T_{01}}}, && (83)\\ \bar{u}_u^{(a2)} &= \frac{P^{(a2)}}{\gamma\sqrt{T_{c1}}} e^{-\omega\hat{x}/\sqrt{T_{c1}}}, && (84) \end{aligned}\right.$$

where $P^{(a1)}$, $P^{(a2)}$ are amplitude factors for the two parts of the acoustic disturbance.

In the combustion zone we write

$$\left\{\begin{aligned} p_u^{(1)} &= \bar{p}_u(x_1)\, e^{\omega t}, && (85)\\ Y_u^{(1)} &= \bar{Y}_u(x_1)\, e^{\omega t}, \quad (Y = T, c_\ell, \rho, u, R), && (86) \end{aligned}\right.$$

The differential equations (51) to (55) then yield ,

$$\bar{\rho}_u = -\frac{T_{c1}}{T_s^2}\bar{T}_u, \qquad (87)$$

$$\omega\bar{T}_u + \frac{d\bar{T}_u}{dx_1} + \bar{u}_{u0}\frac{dT_s}{dx_1} = T_{c1}\frac{d\bar{u}_u}{dx_1}, \qquad (88)$$

$$\omega \overline{T}_u + \frac{d\overline{T}_u}{dx_1} + \overline{u}_{uo}\frac{dT_s}{dx_1} - \frac{1}{Le}\frac{d^2\overline{T}_u}{dx_1^2} = \frac{Le(1+\sigma)Q_1\overline{R}_u}{\sigma} , \tag{89}$$

$$\omega \overline{C}_{\ell u} + \frac{d\overline{C}_{\ell u}}{dx_1} + \overline{u}_{uo}\frac{dC_{\ell s}}{dx_1} - \frac{d^2\overline{C}_{\ell u}}{dx_1^2} = -Le\overline{R}_u , \tag{90}$$

$$\overline{p}_u = P \quad (\text{constant}) , \tag{91}$$

with boundary conditions upstream

$$\overline{p}_u(-\infty) = \overline{p}_u^{(a1)}(0) , \tag{92}$$

$$\overline{T}_u(-\infty) = 0 , \tag{93}$$

$$\overline{C}_{\ell u}(-\infty) = 0 , \tag{94}$$

$$\overline{\rho}_u(-\infty) = 0 , \tag{95}$$

$$\overline{u}_u(-\infty) = \overline{u}_u^{(a1)}(0) , \tag{96}$$

$$\frac{d\overline{p}_u}{dx_1} = \frac{d\overline{T}_u}{dx_1} = \frac{d\overline{C}_{\ell u}}{dx_1} = \frac{d\overline{\rho}_u}{dx_1} = \frac{d\overline{u}_u}{dx_1} = 0 , \tag{97-101}$$

and downstream,

$$\overline{p}_u(+\infty) = \overline{p}_u^{(a2)}(0) , \tag{102}$$

$$\overline{T}_u(+\infty) = 0 , \tag{103}$$

$$\overline{C}_{\ell u}(+\infty) = 0 , \tag{104}$$

$$\overline{\rho}_u(+\infty) = 0 , \tag{105}$$

$$\overline{u}_u(+\infty) = \overline{u}_u^{(a2)}(0) , \tag{106}$$

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

$$\frac{d\bar{p}_u}{dx_1} = \frac{d\bar{T}_u}{dx_1} = \frac{d\bar{c}_{\ell u}}{dx_1} = \frac{d\bar{\rho}_u}{dx_1} = \frac{d\bar{u}_u}{dx_1} = 0 \quad . \tag{107-111}$$

Note that relations(77), (81), (91), (92) and (102) immediately imply that

$$P^{(a1)} = P^{(a2)} = P \quad . \tag{110}$$

It is found that conditions(9), (10) at the gauze surface are compatible with conditions (93) and (94). Restated these become

$$\bar{T}_u(0) = 0 \quad , \tag{113}$$

$$\bar{C}_{\ell u c} - \left.\frac{d\bar{C}_{\ell u}}{dx_1}\right|_{x_1=0} = -\bar{u}_{u_0}\left.\frac{dC_{\ell s}}{dx_1}\right|_{x_1=0} \quad . \tag{112}$$

The solution to equations (87)-(90) has received much attention in the literature and we do not repeat the details here. By again using the principles of matched asymptotic expansions based on the largeness of the activation energy Θ_1, the chemical term $\bar{R}_u$ can be replaced by jump conditions in pre-heat and equilibrium soltuions across the flame considered to be at the position $x_1 = x_{1f_1}$ [9].

If we define the amplitude of the temperature disturbance at the flame to be $\bar{T}_u^*$ then this can be connected to the upstream velocity fluctuation $\bar{u}_{u_0}$, through,

$$\bar{T}_u^* = -\frac{A}{G}\,\bar{u}_{u_0} \quad , \tag{115}$$

where

$$A \equiv \frac{k_e B_1}{\omega(\frac{1}{2}-S')}\left(\frac{1}{2} - S' + \frac{s e^{-\frac{1}{2}k_e x_{1f_1}}}{\operatorname{sh} k_e s x_{1f_1}}\right)\left(G - \frac{\omega}{k_e}\right) + B_1\left(1 + \frac{r e^{-\frac{1}{2}x_{1f_1}}}{\operatorname{sh} r x_{1f_1}}\right) , \tag{116}$$

$$G \equiv D + \frac{2}{\Theta_1 B_1}\left[(\tfrac{1}{2}+R)(s+S') + D\right](\tfrac{1}{2}-S') \quad , \tag{117}$$

$$D \equiv s^2 - \frac{1}{2}S' + \frac{1}{2}R - RS' \quad , \tag{118}$$

$$R \equiv \frac{r\,\mathrm{ch}\,r x_{i_{f1}}}{\mathrm{sh}\,r x_{i_{f1}}} \quad ; \quad S \equiv \frac{s\,\mathrm{ch}\,Le\,s x_{i_{f1}}}{\mathrm{sh}\,Le\,s x_{i_{f1}}} , \tag{119a,b}$$

and

$$r \equiv \sqrt{\omega + 1/4} \quad ; \quad s \equiv \sqrt{\frac{\omega}{Le} + 1/4} , \tag{120a,b}$$

with

$$B_1 \equiv (1 - T_{01})\left(1 - e^{-Le x_{i_{f1}}}\right)^{-1} . \tag{121}$$

The term G defined in (117) is effectively (other than $O(\Theta_1^{-2})$ terms) that which appears as the LHS of the non-acoustic dispersion relation in ref [9], equation (150).

We can also find an expression for the downstream velocity $\overline{u}_{u_\infty}$. This is given by

$$\overline{u}_{u_\infty} = -\frac{\overline{u}_{u_0} \mathcal{V}}{T_{01}} , \tag{122}$$

where,

$$\mathcal{V} = -\frac{\omega \mathcal{B}}{Le\, G(1/2 - s)} - T_{01} - \frac{\mathcal{A}}{G}\left(1/2 - S + \frac{s e^{-Le x_{i_{f1}}/2}}{\mathrm{sh}\,Le\,s x_{i_{f1}}}\right)$$
$$+ \frac{Le\,B_1}{\omega}\left[\left(\frac{1}{2} - S + \frac{s e^{-Le x_{i_{f1}}/2}}{\mathrm{sh}\,Le\,s x_{i_{f1}}}\right) - e^{-Le x_{i_{f1}}}\left(\frac{1}{2} + S - \frac{s e^{Le x_{i_{f1}}/2}}{\mathrm{sh}\,Le\,s x_{i_{f1}}}\right)\right] , \tag{123}$$

and

$$\mathcal{B} = \frac{2}{\Theta_1}\left[\left(\frac{1}{2} - S + \frac{s e^{-Le x_{i_{f1}}/2}}{\mathrm{sh}\,Le\,s x_{i_{f1}}}\right) - \left(\frac{1}{2} - S\right)\left(1 + \frac{r e^{-x_{i_{f1}}/2}}{\mathrm{sh}\,r x_{i_{f1}}}\right)\right] . \tag{124}$$

But from equations (80), (84), (96), (106) and the knowledge that

$$\overline{u}_{u_0} = \overline{u}_{u_{-\infty}} \left[\equiv \overline{u}_u \;(x_1 = -\infty)\right] , \tag{125}$$

we know that,

$$\overline{U}_{u_e} = -\frac{P}{\gamma} \quad ; \quad \overline{U}_{u_\infty} = \frac{P}{\gamma\sqrt{T_{c_1}}} \tag{126a,b}$$

Using equation (122), this then leads to the condition,

$$\ell = \sqrt{T_{c_1}} \quad , \tag{127}$$

with, from (115),

$$\overline{T}_u^* = \left(\frac{A}{G\gamma}\right)P \tag{128}$$

Condition (127) constitutes the main result of this analysis and results (127) and (128) form the basis for the discussion in the next section.

DISCUSSION

Equation (127) is a relation determining the complex frequency ω, given Lewis number L_e initial stand-off distance x_{if_1} , activation energy Θ_1 and temperature ratio T_{c_1} . If the real part of ω is positive (negative), then oscillations are linearly unstable (stable). The relationship thus becomes a dispersion relation of the form derived in [9] but with small emitted acoustic waves present. It is found (Fig. 2) that for large x_{if_1} (small heat loss) the difference in predictions of neutral stability is small. However for x_{if_1} below approximately 4, the predictions depend much on the temperature ratio T_{o_1} Using the parameter $\Theta_1 B_1^* \left(\equiv \Theta_1(1-T_{o_1})\right)$ to compare with previous analyses one observes that for $\Theta_1 B_1^* = 10,\ T_{o_1} = 0{\cdot}2$, the neutral stability point for pulsating plane flames is moved to a lower value of Lewis number indicating that for flames with acoustic emmission the stability band will be increased. Since hydrocarbon flames with Lewis numbers less than unity generally correspond to fuel-lean flames, it is apparent the fuel-lean pulsating instability is suppressed with acoustic emmission present.

Using equation (126) or equation (115), amplitude and phase relationships can be derived between the acoustic and combustion disturbances. We use relation (115) to deduce expressions for the relative amplitude and phase lag of the upstream velocity fluctuation to the temperature disturbance. This then effectively measures the relative amplitude of the acoustic disturbance (upstream and downstream) to the flame temperature (and stand-off) fluctuation.

The amplitude and phase lag functions are given by

$$\mathrm{Ampl}\left(\frac{\overline{U}_{u_e}}{\overline{T}_u^*}\right) = \left|-\frac{G}{A}\right| = \sqrt{Z_r^2 + Z_i^2} \quad , \tag{129}$$

$$\mathrm{Phse}\left(\frac{\overline{u}_{uo}}{\overline{\overline{T}}_u^*}\right) = \tan^{-1}\left(\frac{Z_i}{Z_r}\right) \quad , \tag{130}$$

where

$$Z_r \equiv \mathrm{Real}\left(-\frac{G}{A}\right) \quad , \tag{131}$$

$$Z_i = \mathrm{Imag}\left(-\frac{G}{A}\right) \quad . \tag{132}$$

Figs. 3 and 4 indicate typical values of amplitude and phase. They show the variaton of amplitude (Fig. 4) and phase (Fig. 5) with stand-off distance for $\Theta_1 B_1^* = 10$, $L_e = 0.7$ and a range of T_{o_1} values. The cases $T_{o_1} = 0.2, 0.4$ represent more realistic values of gas expansion ($T_{o_1}^{-1} = 5$ and $T_{o_1}^{-1} = 2.5$ respectively). The Lewis number parameter L_e roughly measures mixture strength [11] and for hydrocarbon flames represents a moderate fuel-lean condition. Fig. 3 shows a large dependence of the relative amplitude of acoustic emission (to flame disturbance) with stand-off distance. The greater the heat loss (i.e. the smaller the value of x_{if_1}) the greater the relative amplitude predicted. Reducing the gas expansion ratio ($T_{o_1}^{-1}$) from 5 reduces this effect (see Fig. 3 for $T_{o_1} = 0.4, 0.6, 0.8$).

Turning to Fig. 4, we observe that the acoustic emission is predicted to be almost 90° out of phase with the combustion fluctuations for x_{if_1} below approximately 4. Above this value the phase lag drops until the acoustic fluctuations become only slightly out of phase, for x_{if_1} very large. In fact (see Fig. 3) the relative amplitude drops theoretically to zero for $x_{if_1} = \infty$. Thus in the absence of feedback mechanisms acoustic emission is only predicted when a significant heat loss is present and this highlights the principle involved. Schimmer and Vortmeyer [6] discussing the acoustical oscillation of a flat flame came experimentally to the same conclusions concerning phase lag at moderate x_{if_1} values attributing it to oscillating heat transfer at the burner. In their experimental observations, they found the relative phase lag of 'acoustic particle velocity' to 'flame displacement' fluctuations was about 90° (see Ref. 6, Fig. 5). This confirms the results of the theory presented here, since typical flame stand-off distances are represented by x_{if_1} values of between 3 and 4 [see Ref. 8]. As x_{if_1} increases the flame approaches adiabatic conditions and the lack of heat transfer to the burner removes the cause of acoustic emission.

From equations (113) and (126) it is evident that

$$\frac{P}{\gamma \overline{\overline{T}}_u^*} = -\frac{\overline{u}_{uo}}{\overline{\overline{T}}_u^*} \quad , \tag{133}$$

so that in this model, the fluctuations in pressure will be 180° behind (i.e. out of phase) those of velocity. This lies in between the two possible cases of Schimmer and Vortmeyer and the idealised result (here) is mainly due to not taking account of the acoustic characteristics of the upstream flow which will normally have an effect in a real burner port system.

However the qualitative agreement of some of these findings with experimental data gives confidence to the extension of the theory to include acoustic feedback which is usually the characteristic of practical burner systems.

CONCLUDING REMARKS

The interaction of an anchored flame with a low level acoustic field has been considered such that at large distances from the combustion zone a small acoustic field is coupled with the local combustion field. Matching between the two zones leads to a complex frequency relation which must be obeyed by small (linearised) disturbances. The findings from this new relation are as follows:

(i) Given moderate gas expansion through the flame, the range of stable Lewis numbers (linked to mixture strength) is greater when acoustic emission is included, than in the non-acoustic case.

(ii) The relative amplitude of emitted acoustic fluctuations to combustion fluctuations increases with the heat loss to the gauze.

(iii) The relative phase lag between upstream velocity disturbances and flame temperature fluctuations is about 90° for moderate non-dimensional stand-off distance (x_{f_1}) values. The phase lag diminishes as x_{f_1} increases beyond 4.

No acoustic forcing or feedback mechanisms have been included at this stage. The theory is presented here as a basis upon which to add these effects. It is to be expected that these effects will alter some of the results of this work which therefore should be regarded as of a preliminary nature.

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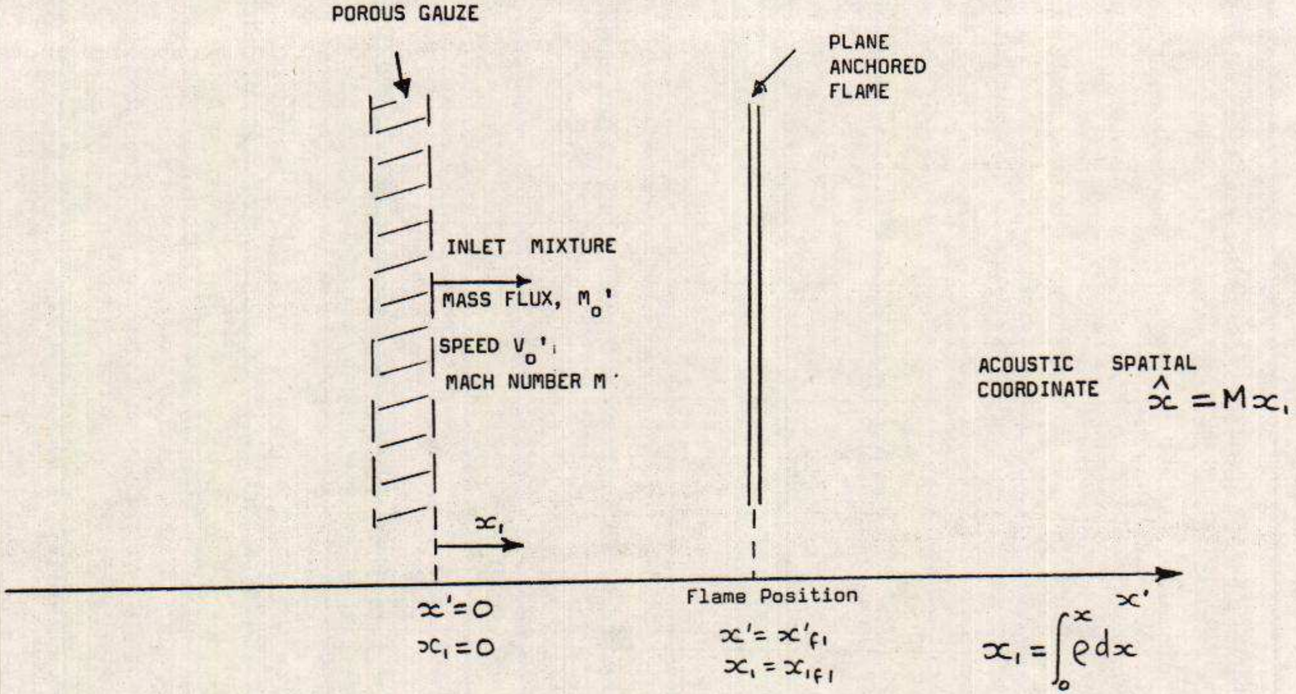

FIG. 1. SCHEMATIC OF PLANE FLAME ANCHORED ON A POROUS GAUZE.

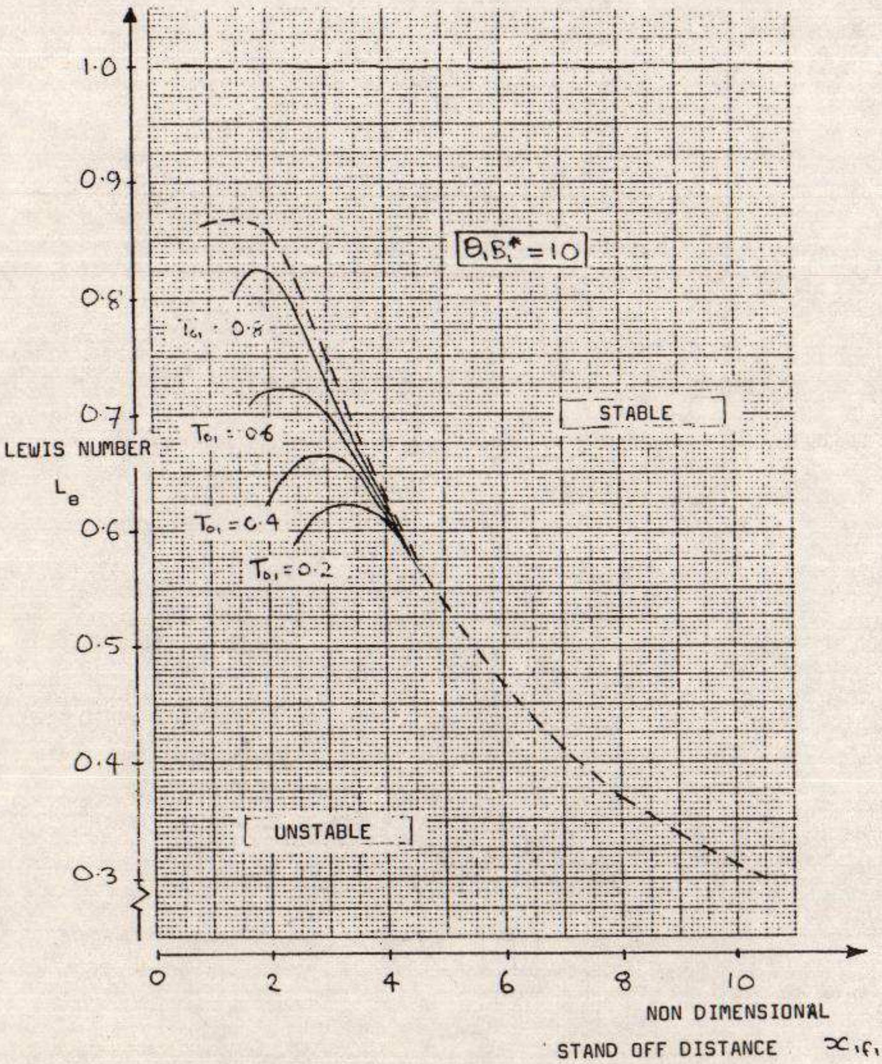

FIG. 2
CRITICAL LEWIS NUMBER FOR NEUTRAL STABILITY (WITH ACOUSTIC EMISSION PRESENT) PLOTTED AGAINST STAND OFF DISTANCE OF FLAME. THE DOTTED CURVE IS THE NON-ACOUSTIC CURVE FOR COMPARISON. THE FAMILY OF CURVES ARE FOR VARYING TEMPERATURE RATIOS (T_o1) ACROSS THE FLAME.

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

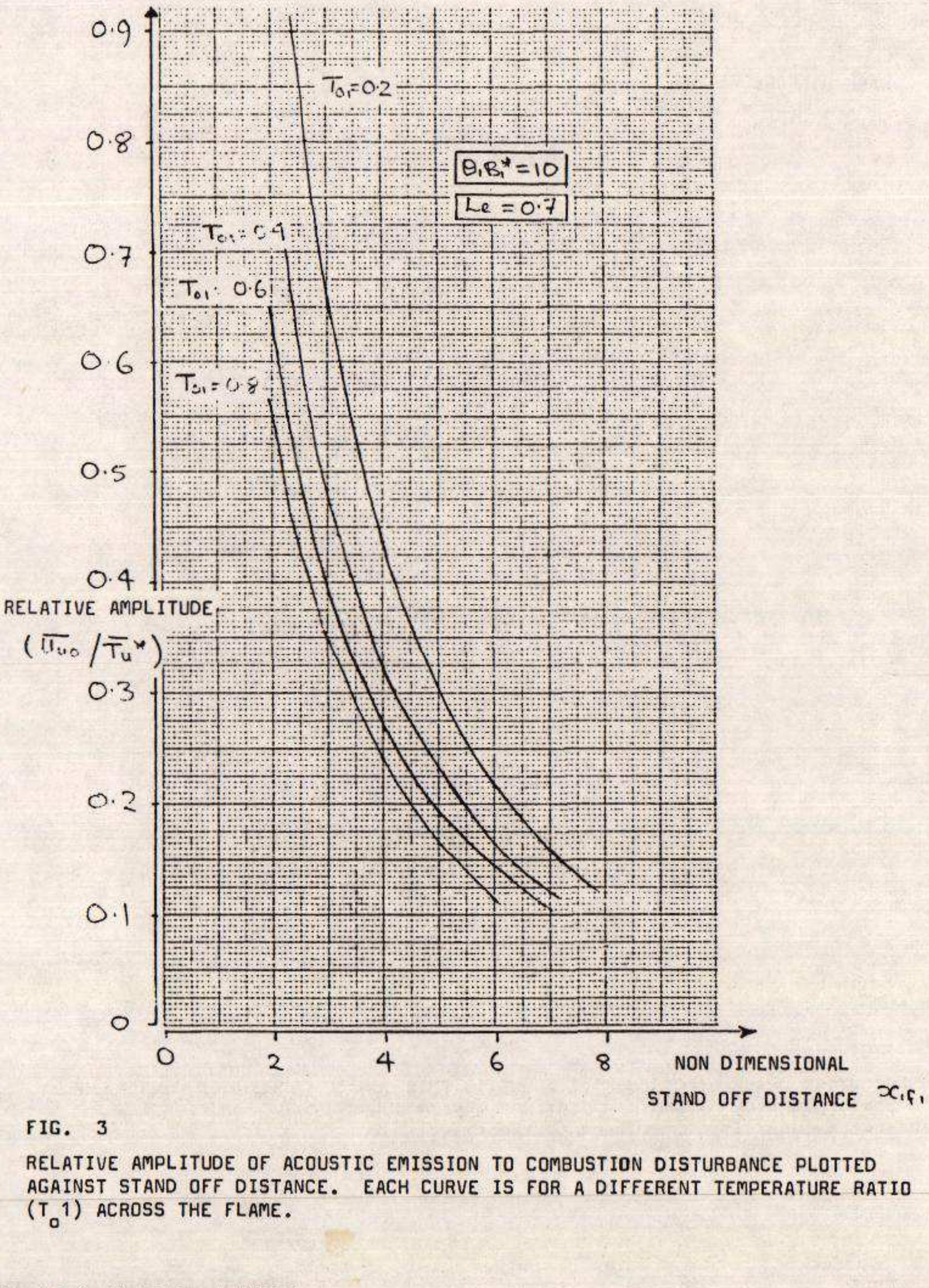

FIG. 3

RELATIVE AMPLITUDE OF ACOUSTIC EMISSION TO COMBUSTION DISTURBANCE PLOTTED AGAINST STAND OFF DISTANCE. EACH CURVE IS FOR A DIFFERENT TEMPERATURE RATIO (T_o1) ACROSS THE FLAME.

ON THE COUPLING OF AN ANCHORED FLAME WITH AN ACOUSTIC FIELD

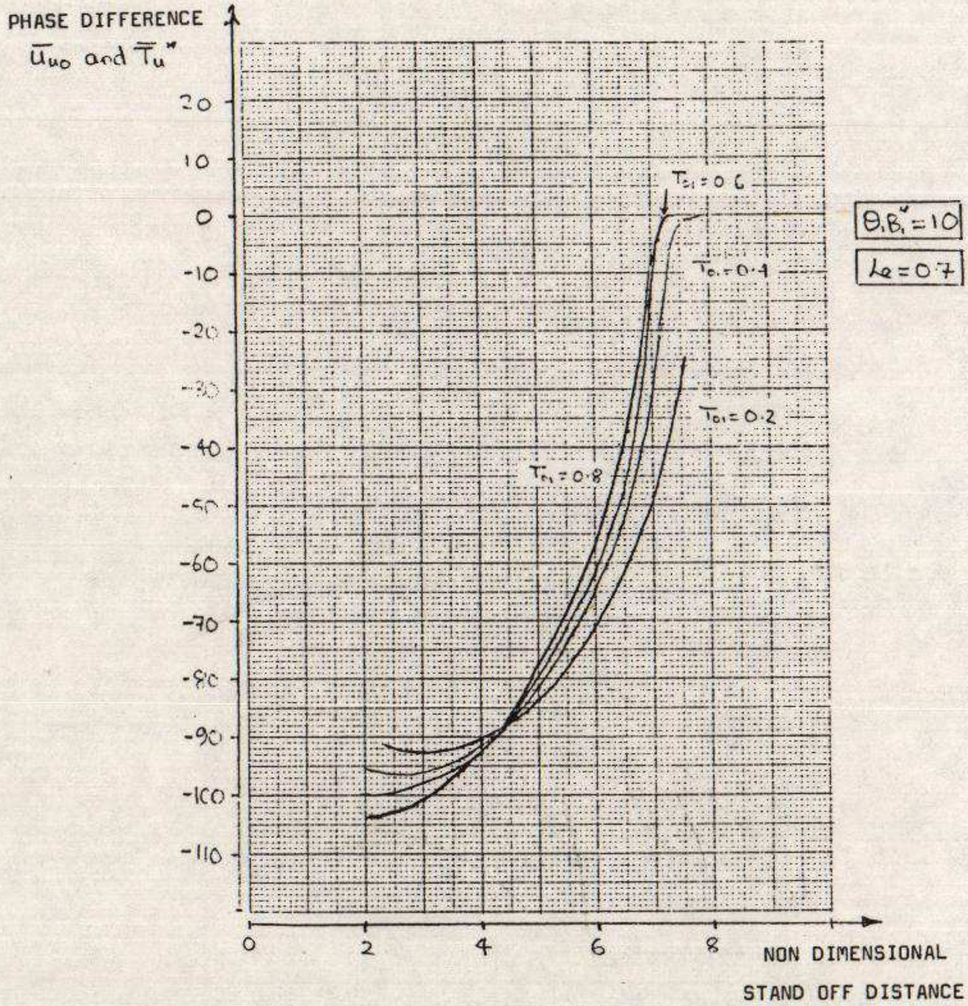

FIG. 4

RELATIVE PHASE DIFFERENCE OF ACOUSTIC EMISSION TO COMBUSTION DISTURBANCE PLOTTED AGAINST STAND OFF DISTANCE. EACH CURVE IS FOR A DIFFERENT TEMPERATURE RATIO (T_o1) ACROSS THE FLAME.