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A STUDY OF ERRORS IN SOURCE RECONSTRUCTION USING NEARFIELD ACOUSTIC HOLOGRAPHY

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1. INTRODUCTION

The inclusion of the evanescent-wave field can produce high resolution images, removing the $\lambda/2$ limitation of acoustical reconstructions. By definition, evanescent waves can only be successfully detected in the very close nearfield of the source. The technique using this field is known as Nearfield Acoustic Holography (NAH).[1]

The measurement of sound pressure alone around a source does not determine how energy is delivered into the sound field. The acoustic intensity vector field is the important factor for understanding how the source radiates sound, and it is calculated using the pressure and particle velocity fields. The holographic calculation of the velocity is equivalent to taking the spatial derivative of the pressure data which greatly magnifies any inaccuracy in the data or in the field; thus, the evaluation of velocity rather than of pressure determines the limitation for the intensity mapping.[2] [3] [4]

On practice, the inclusion of this evanescent field has been associated with a k-space filter which, at the very least, truncates the spectrum shape in order to eliminate its higher order components. These k-space filters have been used in addition to the guard bands of zeros before Fourier Transformation (FT) of the real-space data (from convolution theory) and the truncation of the Green Function. Also other types of filter, which are designed to solve specific problems, have been used. Thus, reconstructions which do not correspond to the expected source have appeared during the development of NAH; and empirical methods have been developed to minimize this problem. The difference between the reconstructed and expected (or real) source is referred to as 'distortion'.[4] [5] [6] [7]

2. SOFTWARE

The two monopole source reconstruction reported in [2] was used as a standard, keeping the parameters 'a' normalized (where 'a' is half the distance between monopoles). These parameters are summarized in table 1 and the schematic representation of the experiment is given in fig. 1.

$\lambda = 16$ a	λ = wavelength of the source signal
H = 32 a	H = aperture of the planar array
$z = z_s = 0.0$ a	z_s = source plane distance in z-axis
$z_h = 0.96$ a	z_h = hologram plane distance in z-axis
$x_{inc} = \Delta x = 0.5a$	Δx = distance between sampled points

Table 1 Standardized parameters for the 2 monopole experiment in [2], with 'a' being half the distance between the monopole.

Five programmes were developed working at different precisions. Two programmes calculated the hologram fields for two different sources, two programmes were used to evaluated the source plane velocities, and a plotting programme produced magnitude hard copies. An example of these plots is given in Fig. 2.

The first programme -which works in quadruple precision (16 bytes ≈ 32 decimal digits)- calculates the hologram field of two monopole sources in phase at the hologram plane distance z_h=0.96a from the source plane (where 'a' is half the monopole separation), using the monopole expression given in [8]. The second programme works in double precision (8 bytes ≈ 15 decimal digits), and calculates the hologram field for a circular piston at the distance $z_h=0.06\lambda$ using the expression given by [9]. The third programme can calculate the velocity at the source plane z_s=0, using FFT routines that work in double precision. Thus, for the two-monopole reconstructions, the data is truncated from 16 bytes to 8 bytes just before using the FFT routine (NAG routine CO6FJF), reducing any numerical error in the hologram data to the least significant bit. The fourth programme, which works in single precision (4 bytes ≈ 6 decimal digits), plots the magnitude of the reconstruction, the vertical displacement representing the velocity magnitude. Another reconstruction programme was developed working in single precision, using a different routine to perform the FFT (IBM Engineering and Scientific Subroutines, ESS routine SCFT2) reducing the dynamic range with respect to the NAG programme; thus, the error introduced in the input data is due to truncation from 16 or 8 bytes to 4.

The NAH reconstruction equation used in the NAG and ESS programmes is given in [3], with a complex k_z (shifted by $\Delta k/2$ in k_x and k_y), there is no allowance for attenuation in the medium. This formulation -k-space formulation of the Green Function from analytical FT- corresponds to $g^{(5)}$ from Veronesi [5], which should yield a more accurate reconstruction of the source, and the problems of coincidence for k_z should be minimized.

3. SIMULATIONS

It was decided to study the influence of some of the array parameters in the 'quality' of the reconstruction process. In [6] it was mentioned that for better reconstructions, it is necessary to keep z_h as close to the source as possible. Having it λ dependent could cause 'distortion'. [10]

The influence of sample spacing was investigated by keeping the same aperture in the hologram plane and simulating arrays of 32x32, 64x64 and 128x128; i.e. Δx is changed to keep H constant (where H is the aperture of the array). All the other parameters were as in Table 1. To test the influence of H, Δx was kept constant with the other parameters as in Table 1; the number of points was again changed (32x32 and 128x128 points).

The influence of λ (the wavelength) was investigated by increasing its 'a' normalized value (N=128, and keeping the other parameters constant); values of λ=32a, and λ=64a were used.

To investigate the origin of the distortion, which has been attributed to noise in the input data, the dynamic range of the system and errors introduced by the FFT; we compared the results given by the NAG and ESS programmes run with the same values.

A two dimensional window in real-space was used to investigate the 'distortion' phenomenon (instead of the k-space filter or adding attenuation to the complex k_z), as suggested [7]. A Hanning window was chosen because it eliminates the non-zero values at the border, gives a better spatial resolution than the rectangular window (inherent to the process) and it is commonly used in one dimensional signal processing.

4. DISCUSSION OF RESULTS

It should be clear that source reconstruction using the evanescent field can lead to 'distortion' in the reconstructions, and that the requirement of *a priori* knowledge of the source under study makes of the technique a case-by-case approach. It is interesting to point out that for a complicated form of a k-space filter this *a priori* knowledge is indispensable for avoiding a preconceived reconstruction of the source that could not be related to its physical characteristics.

It was found that this 'distortion' increased when Δx has reduced (i.e. increasing the number of measurement points in the hologram plane). Thus, augmenting the components of the evanescent field taking into account for reconstructing the source, reduces the possibilities of a 'good' reconstruction. Variations in λ have a lesser effect on the quality of the reconstruction at a given $z=z_s=0$. Simulations with $z_h=0.06\lambda$ (instead of $z_h=0.96a$) show that, although being considered close enough to the source for detecting the evanescent field, the increase of λ, keeping the rest of the parameters constant, produces distortion.

The standard way to deal with the 'distortion' is by means of a k-space filter, although alternative methods have been suggested which depend on the position of the origin, such as using a complex wavenumber, Wiener filter or windowing in real-space. For complicated sources, the k-space filter does not seem the best solution, because of the subjectivity in the choice of the cut-off frequency.

If we consider that a k-space filter is a numerical way to increase Δx, it is possible to argue that for a system at a given distance z_h from the source plane, there is a minimum Δx that could not require a k-space filter. Thus, there is a limit for Δx, reducing it further (by adding more scanning points to the hologram plane) does not improve the quality of the reconstruction or the resolution in real-space; and possibly, reducing Δx after this limit degrades the reconstruction due to the effects of the k-space filter in the rest of the real-space. This Δx is source dependent. Thus, in practice, NAH seems still frequency dependent, although it has exceeded the $\lambda/2$ resolution limit.

The origin of the 'distortion' was investigated by comparing the reconstruction of the ESS and NAG programmes. The main difference between them is the scale factor, which is caused by the differences between the FFT routines used in the programmes.

Using a Hanning window gives a better reconstruction shape, although the magnitude of the source has been attenuated. With the k-space filter the magnitude should be better represented at expenses of the form. The main advantage of the use of a window in real-space is that its properties and effects on the data have been largely studied for signal processing. But if Δx is decreased, it is possible to arrive to a distorted condition in spite of the window.

A possible explanation for this 'distortion' could be the inherent aliasing caused by the impossibility of having a spatial anti-alias filter at the hologram plane. This aliasing affects the highest frequencies, which are multiplied by the highest values of the exponential evanescent wave, resulting in large errors in the reconstructions. Also, the smaller Δx, the higher the value of the Green Function and the more important the aliasing. All these results in the reconstruction process are related to the ill-conditioning inherent to inverse problems (ill-posed problems).

5 INVERSE PROBLEM

The inverse problems relevant to standard AH and NAH are related to the solution of the Fredholm integral equation of the first kind

$$\varphi(p) = \int_a^b K(p,t)\ s(t)\ dt \qquad c \le p \le d \qquad (1)$$

Where K(p,t) is an operator (or kernel) which relates the unknown function space s(t), describing the state of the object of the measurement, to some other function $\varphi(p)$ which is known or can be measured. A special case of this type of equations is

$$\tilde{\varphi}(\mathbf{r}) = \int_{-\infty}^{\infty} s(\alpha)\ G(\mathbf{r}-\alpha)\ d\alpha \qquad (2)$$

Where $\mathbf{r}$ is a multidimensional position vector.

$s(\alpha)$ is the source distribution.

$\tilde{\varphi}(\mathbf{r})$ is the complex amplitude of the resultant wave field

G is the integration kernel or Green function associated with the wave propagation.

Inverting operator equations of the first kind are generally ill-posed (I-P) or ill-conditioned. This situation arises when $\tilde{\varphi}$ is not sensitive to certain perturbations in s, i.e., there exist functions ∂s such that

$$Gs \approx G(s+\partial s) \qquad (3)$$

i.e., G smooths out changes in s. Under such circumstances, the solution can not be sharply defined and may not even be unique; thus small errors in the data, which are inevitable, can lead to large errors in the solution unless suitable stabilizing constraints are imposed (i.e., unless additional prior knowledge can be taken for granted), or the least-squares approach is used to 'regularize' the solution. Pike, et. al., [11] comment that for extracting the maximum amount of reliable information about the solution, without introducing arbitrary components, it is necessary to use either explicitly or implicitly the method of eigenvalue decomposition, or singular value decomposition (SVD) if s and φ lie in different functional spaces.[12] [13] [14]

All I-P problem can be treated as effectively not fully defined. The meaning of this is that any function satisfying the condition of small deviation from the expected function can be considered as a 'candidate', which could be a 'good' or a 'bad' one. Thus, it is necessary to define a class of 'admissible' solutions based on different ideas of *a priori* information about the desirable solution.[12] [13]

6. CONCLUSIONS

NAH source reconstruction can present I-P problems. Due to the I-P problem, there is not a unique solution. Choosing the 'best' solution could be time consuming from just a mathematical approach or a very subjective technique is there is not a formal approach. More important than the resolution of the reconstruction is its closeness to the 'true' source.

The different simulations done to identify the effects of the important parameters in NAH and the origin of the 'distortions' show, in the light of the I-P approach, that there is not a specific noise which provokes the 'distortion' to occur. It would seem logical to suppose that the largest noise will trigger the occurrence of 'distortion', but as it is shown in some early experiments (see for example [15]), uncalibrated systems working in 'noisy' environments can manage to reconstruct some type of sources without the I-P problem arising. The simulations have shown that the sensitive parameters for having a 'good' or 'bad' reconstruction in a given system are the distance $d = z_s - z_h$, Δx, and H. The distance d should be the minimum possible and it should not be function of λ. The sampling distance Δx could be used to minimize the I-P problem. For a well-posed reconstruction, the variation of λ has a minor role; i.e., for a well-behaved system λ/a is not a critical factor in the quality of the reconstruction, although very large λ/a could cause 'distortion' by truncation due to the hologram plane (H being small with respect to λ).

Truncation of the field and aliasing ,which are source dependent, also can trigger the I-P problem and they could be an important factor that deviates the reconstruction from the 'real' source.

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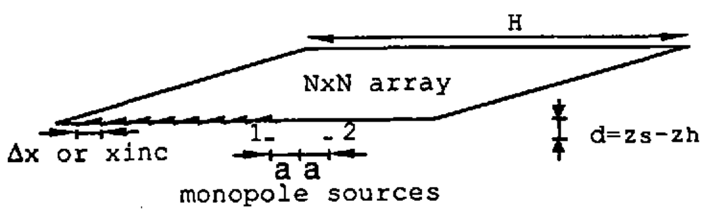

Fig. 1 Representation of the experiment.

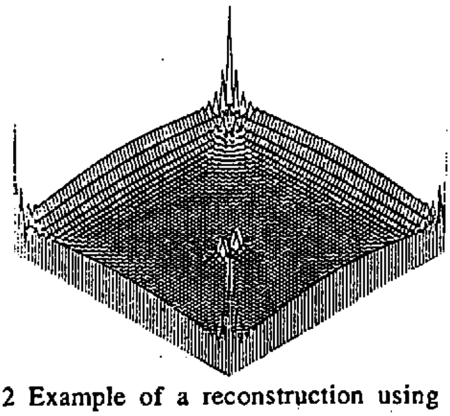

Fig. 2 Example of a reconstruction using NAH.