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HYBRID BOUNDARY ELEMENT-FINITE ELEMENT APPLICATIONS IN UNDERWATER ACOUSTICS

A.F. Seybert, T.W. Wu & W.L. Li

Department of Mechanical Engineering
University of Kentucky
Lexington, KY 40506-0046, USA

1. INTRODUCTION

Coupled fluid-structure interaction is a problem of practical importance in acoustics. When an elastic structure, especially a thin-walled structure, is submerged in a heavy fluid such as sea water, the vibration of the structure will be affected by the surrounding fluid, and the whole system becomes a coupled structural acoustic problem.

The boundary element method has been used extensively in acoustic radiation and scattering from bodies with known velocity, pressure, or impedance distribution [1-4]. This method is based on a set of integral equations defined on the boundary surface of the fluid, thereby reducing the computational dimension of the problem by one. Thus, discretization is confined to the boundary surface only. For an exterior radiation or scattering problem, the Sommerfeld radiation condition is automatically incorporated in the boundary integral equation. The BEM has also been applied to fluid-structure interaction problems [5,6]. Using the BEM to model the structure as well as the fluid is impractical when the structure is a thin shell. The reason is that nearly singular behavior may occur when the inner and outer surfaces of the thin shell are very close.

A more general way to model the structure is to use the finite element method. A coupled technique, using the FEM for the structure and the BEM for the fluid, is a natural choice for numerical solution of fluid-structure interaction problems. Most solution strategies for fluid-structure interaction are in the category of structural-variable methodology, that is, the pressure acting on the fluid-solid interface is treated as a structural loading, leading to a perturbation of the original structural dynamic equations *in vacuo*. Wilton [7] developed a fluid-variable methodology in which the finite element equations are substituted into the acoustic equations on the surface, resulting in a system of equations much smaller than that of the structural equations. In Ref. [8], Mathews and Hitching applied the boundary integral formulation due to Burton and Miller [9] to the fluid-structure interaction problem. For further applications of the coupled method, the reader is referred to Refs. [10,11].

In all the above formulations, either the wet surface of the structure is assumed to be smooth enough so that a unique normal velocity is defined at any nodal point on the surface or planar (constant) elements are adopted to facilitate numerical manipulation. The inconsistency between the finite element model of the structure and the boundary element model of the fluid makes it impossible to match exactly compatibility (or continuity) conditions on the interface surface. Usually, the continuity conditions are enforced at a selected set of collocation points. For example, Schenck and Benthien [12] enforced approximately the continuity condition by equating the BEM normal velocity on a constant element to the average of the FEM normal velocity across the wet surface of the corresponding finite element.

In this paper, the coupled technique is used to solve acoustical radiation and scattering problems. Quadratic isoparametric elements are used in both the FEM and the BEM. Continuity conditions are enforced on the fluid-structure interface on which the normal may not be uniquely defined (e.g., at a corner). To reduce the dimension of the final system of equations, the structural displacement is expressed as a linear combination of either Ritz vectors or eigenvectors. Several numerical examples are examined including radiation of sound from vibrating structures and scattering of sound waves from submerged elastic obstacles.

2. DESCRIPTION OF THE COUPLED FLUID-STRUCTURE SYSTEM

Consider an elastic structure submerged completely in an inviscid fluid of density ρ_0 and speed of sound c. Suppose the structure is subjected to a time-harmonic loading of angular frequency ω either from inside of the structure (radiation) or from a surrounding fluid (scattering). The sound pressure p in the fluid must satisfy the Helmholtz differential equation

$$\nabla^2 p + k^2 p = 0 , \tag{1}$$

where $k=\omega/c$ is the wave number in the fluid. For numerical treatment, it is preferred to reformulate Eq. (1) into an integral form. In the BEM, the commonly used integral representation of Eq. (1) is the Helmholtz integral equation

$$C(P)p(P) = \int_S [G(Q,P)\frac{\partial p}{\partial n}(Q) - \frac{\partial G}{\partial n}(P,Q)p(Q)]\, dS(Q) + 4\pi p_I(P) , \tag{2}$$

where $G(Q,P) = \exp(-ikr)/r$, $r=|Q-P|$, S is the boundary surface of the acoustic fluid, n is the normal on S, and p_I represents the sound pressure of the incident wave. In the case of a radiation problem $p_I = 0$, and for a scattering problem p is the total pressure (incident plus scattered). The leading coefficient C(P) is 4π and zero, respectively, for P in and out of the fluid. When P is a boundary point at which there exists a unique tangent plane, C(P) is equal to 2π. Otherwise, C(P) must be evaluated [4]. It should be noted that the Sommerfeld radiation condition has been used in deriving Eq. (2).

On the boundary, the normal derivative of sound pressure is related to the normal displacement through the momentum equation

$$u_n = \frac{1}{\rho_0 \omega^2}\frac{\partial p}{\partial n} . \tag{3}$$

Using Eq. (3), Eq. (2) may be rewritten as

$$C(P)p(P) = \int_S [G\rho_0\omega^2(\underline{u}.\underline{n}) - \frac{\partial G}{\partial n}p]\, dS + 4\pi p_I(P) . \tag{4}$$

In discretized form, Eq. (4) becomes

$$Ap = Bu + 4\pi p_I , \tag{5}$$

where p, u, and p_I are the nodal value vectors of the sound pressure, displacement, and incident sound pressure, respectively; and A and B are the assembled coefficient matrices.

It should be noted that for an exterior problem the previous formulation based on the surface Helmholtz integral equation may fail to yield a unique solution at certain wave numbers, called critical or characteristic wave numbers. To avoid this difficulty, several modified integral formulations have been developed. An enhanced CHIEF method is used in the present work.

If u_n is specified on the boundary, as in a Neumann problem, Eq. (5) alone will determine the pressure p on the boundary, or *vice versa*. In the fluid-structure interaction problem, however, neither u_n nor p is known *a priori*.

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Thus, an additional set of equations governing the motion of the elastic structure must be provided.

Applying the finite element method to the structure results in

$$Ku - \omega^2 Mu = f^m + f^a , \tag{6}$$

where the loading is divided into two parts, the mechanical loading f^m and the acoustic loading f^a due to the fluid. A damping matrix $i\omega C$ may be added to Eq. (6) to include the effect of structural damping, but in the current study it is neglected for simplicity. The dynamic condensation technique is used to reduce the dimension of Eq. (6) by decomposing the displacements into two groups, the "master" displacements to be retained and the "slave" displacements to be eliminated. Its usage with coupled fluid-structure problems can be found in Ref. [13]. The acoustic load f^a in Eq. (6) represents the dynamic coupling between the structure and the fluid,

$$f^a = -Lp , \tag{7}$$

where L is a coupling matrix. Combining Eqs. (5), (6), and (7) leads to the following coupled structural acoustic equations

$$\begin{pmatrix} K-\omega^2 M & L \\ -B & A \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f^m \\ 4\pi p_I \end{pmatrix} . \tag{8}$$

Equation (8) can be solved simultaneously (in a least-squares sense if the enhanced CHIEF method is used) for both the displacement and pressure, or the pressure may be eliminated, resulting in a structural version solution of the coupled problem. However, the more widely used approach falls in the category of fluid-variable methodology, in which the final equation takes the form

$$[A + B(K - \omega^2 M)^{-1} L]\ p = 4\pi p_I + B(K - \omega^2 M)^{-1} f^m . \tag{9}$$

Once the boundary pressure is determined, the structural displacements and other field quantities can be found with little difficulty.

3. EIGENVECTOR SYNTHESIS TECHNIQUE

Modal synthesis (or superposition) is an approximation whereby the structural displacements are expressed as a linear combination of the *in vacuo* normal modes of the structure, i.e.,

$$u = S\lambda , \tag{10}$$

where S is a matrix whose columns are the eigenvectors of the structure *in vacou*, and λ is an unknown coefficient vector. Under this transformation, Eq. (9) becomes

$$\{ A + B_s(\Lambda-\omega^2 I)^{-1} L_s \}\ p = 4\pi p_I + B_s(\Lambda-\omega^2 I)^{-1} f_s^m , \tag{11}$$

where

$$\begin{aligned} \Lambda &= S^T KS = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_n) , \\ B_s &= BS , \end{aligned} \tag{12}$$

$$L_s = S^T L ,$$
$$f_s^m = S^T f^m .$$

In the above, the eigenvectors are mutually orthonormal with respect to the mass matrix M, that is,

$$S^T M S = I . \tag{13}$$

4. RITZ VECTOR SYNTHESIS TECHNIQUE

The eigenvectors can be generated through a standard procedure which will not be discussed here. However, it should be mentioned that if only part of the eigenvectors, generally the lower modes, is chosen to be used in Eq. (10), the solution may be inaccurate. The reason is that for a given loading only a few modes dominate the response, and these may not be the lower modes. The modes that are not excited by the load will make little or no contribution to the displacement. This suggests the construction of other sets of basis vectors that depend on the loading pattern. For this purpose, Ritz vectors are commonly used as a substitute for the eigenvectors in dynamics analysis [14]. The Ritz vectors are generated from the recurrence relation [14]

$$K q_{i+1}^{*} = M q_i , \qquad i = 1, 2, ..., N , \tag{14}$$

where q_i is the i-th Ritz vector and q_{i+1}^{*} is the initial solution of the i+1-th Ritz vector. Equation (14) does not apply to the first Ritz vector. The first Ritz vector q_1 should be obtained from the static equaiton

$$K q_1^{*} = f^m , \tag{15}$$

where q_1^{*} is the first Ritz vector before normalization.

As mentioned before, the vector q_{i+1}^{*} in Eq. (14) is only an initial estimate of the i+1-th Ritz vector and is subject to an orthogonalization procedure. This is done by using the Schmidt orthogonalization scheme which takes a special form due to Eq. (14):

$$q_{i+1}^{**} = \beta_{i+1} q_{i+1} = q_{i+1}^{*} - \alpha_i q_i - \beta_i q_{i-1} , \tag{16}$$

where $\beta_{i+1} \equiv \| q_{i+1}^{**} \|_M$ and $\alpha_i \equiv (q_i , q_{i+1}^{*})_M$. Here the inner product and its corresponding norm are defined as follows:

$$(a,b)_M = \bar{a}^T M b$$

and $$\| a \|_M = (a,a)_M^{1/2} , \tag{17}$$

where a and b are any two vectors and $\bar{a}$ is the complex conjugate of the vector a.

5. NUMERICAL RESULTS

In the present implementation, quadratic isoparametric elements are used in both the FEM and BEM models. This consistency is necessary in order to enforce the continuity conditions exactly on the interface. The K and M matrices, the structural load vector f^m, and the eigenvectors, if needed, are generated by the ANSYS program [15]. All of the other computations and the main algorithms described above are implemented in the BEM program BEMAP [16]. Throughout this section, the acoustic fluid is sea water with density ρ_0=1026 kg/m^3 and speed of sound c=1500 m/s.

First, the modal synthesis method is examined for a scattering problem consisting of a plane wave impinging on a spherical shell of outer and inner radii a and a/2, respectively. By taking advantage of symmetry, only one-half of the shell is modeled by 12 solid elements with 90 nodes, resulting in a total of 238 degrees of freedom after constraining 32 displacements in the z-direction on the plane of symmetry. Figure 1 shows the magnitude ratio of the scattered pressure to the incident pressure at a distance of 3a from the center of the sphere for two cases, an aluminum and a steel sphere. The coupled FEM/BEM results in Fig. 1 agree well with that of the pure boundary element method [6].

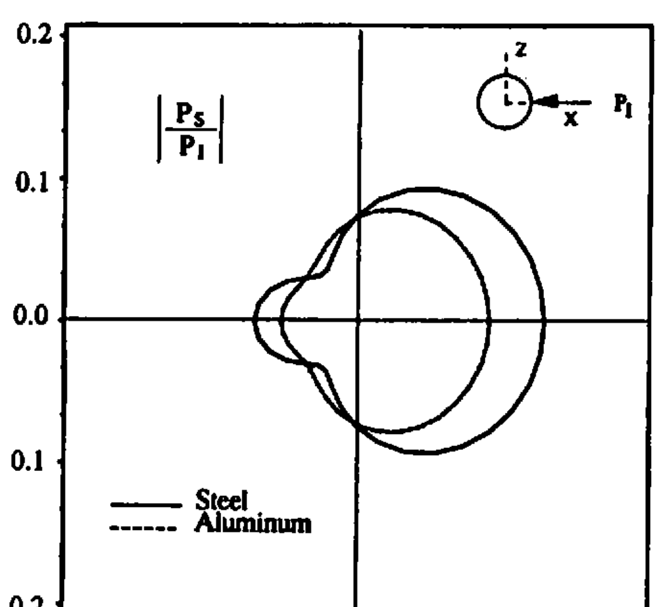

Fig. 1 Ratio of the scattered to the incident pressures at a distance of 3a from the center of a sphere when ka=1 by full modal expansion.

In the above example, all of the mode shapes are included in the displacement expansion, Eq. (10). Usually, only a portion of the modal vectors is chosen in the synthesis procedure, resulting in a significant reduction in the dimension of the final system of equations. However, problems arise as to how many and which mode shapes should be included. In structural analysis, as a rule of thumb, the modes selected are the lowest modes or those whose eigenvalues are near the frequency of the exciting force. For coupled fluid-structure interaction, this procedure may not work well.

Now consider an aluminum cylindrical shell having ends that are excited internally by a uniform pressure p_0. The length of the cylinder is 4a and the thickness is a/4 where a is the mean radius. Plotted in Fig. 2 is the pressure directivity for ka = 1 at a radius 50a from the cylinder.

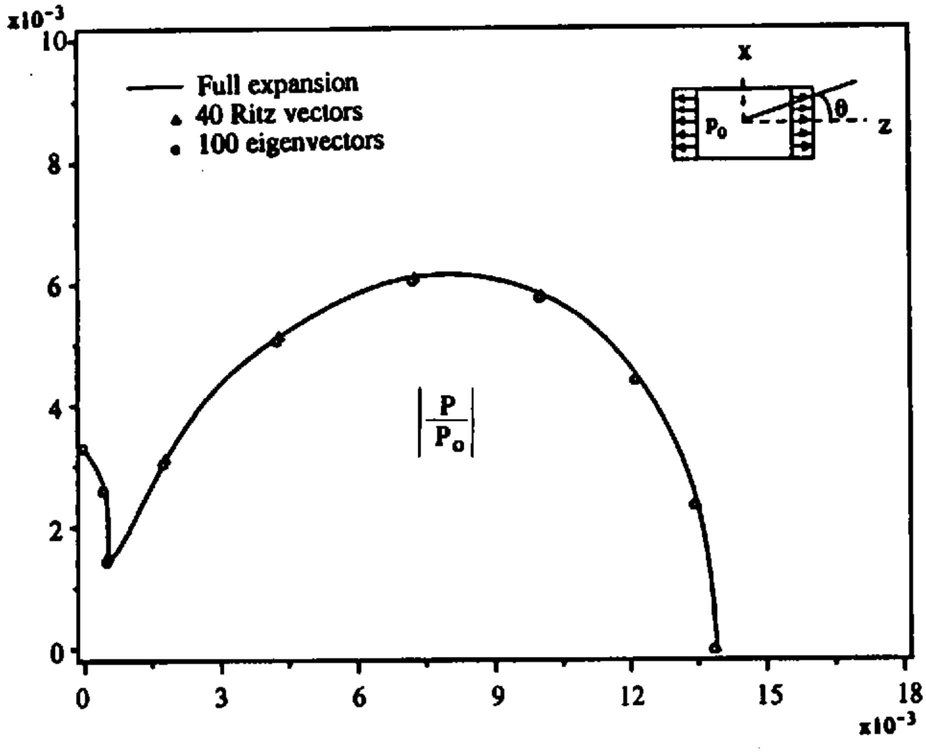

Fig. 2 Normalized pressure magnitude at a distance of 50a from the center of the cylindrical shell by using 40 Ritz vectors and 100 eigenvectors, respectively.

It can be seen from the previous example that the Ritz vector synthesis converges faster than the modal synthesis method, i.e., to obtain a certain numerical accuracy, fewer Ritz vectors are needed. In addition, the cost of generating a set of Ritz vectors is less than that of producing the same number of eigenvectors. Approximately $n^3/6$

operations are needed to construct n Ritz vectors from an nxn matrix as compared to $2n^3/3$ operations for n eigenvectors.

As mentioned above, some basis vectors in the displacement expansion do not contribute significantly to the solution and may be omitted without significant loss of numerical accuracy. The Ritz vectors are usually generated in a descending order of the participation factors, i.e., the first Ritz vector has the largest participation factor, etc. However, the situation is quite different for the modal synthesis method in which the eigenvectors with large participation factors may be sparsely spaced. Therefore, selecting the eigenvectors with large participation factors [17] will improve the efficiency of the modal synthesis method. As an example, consider the scattering of a plane wave from the cylindrical shell considered above. Figure 3 shows the scattered sound pressure level at a distance of 3a from the end of the cylinder as a function of frequency for 30 selected eigenvectors. Also shown in Fig. 3 is the result obtained using the first 60 eigenvectors. It can be seen that the improvement obtained by using the selection procedure is significant.

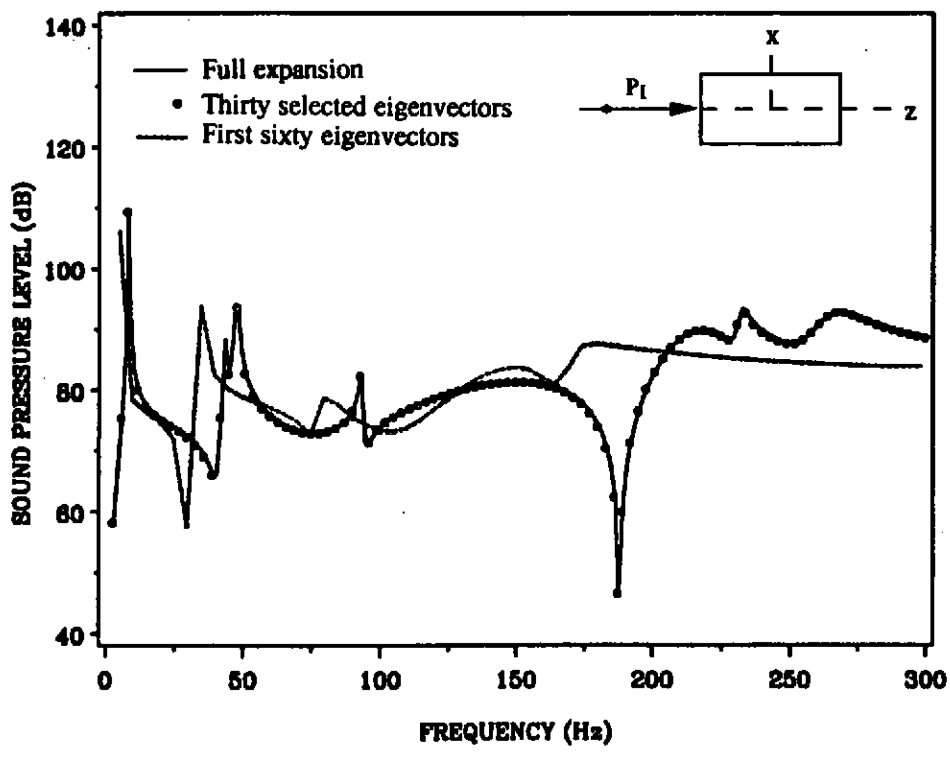

Fig. 3 Back scattering from a circular aluminum cylinder.

6. CONCLUSIONS

The coupled FEM/BEM technique is used to solve fluid-structure interaction by superposition of Ritz vectors or eigenvectors. Comparison of the two approaches shows that the Ritz vector synthesis is much more efficient than the eigenvector synthesis. A participation factor based on the loading condition is used so that the basis vectors which have the most contribution to the solution can be selected even before the problem is really solved. A consistent formulation in which quadratic isoparametric elements are used in both the FEM and the BEM permits the exact enforcement of the continuity conditions on the interface.

7. ACKNOWLEDGEMENTS

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