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FAST TRANSVERSAL FILTERS FOR ADAPTIVE NOISE ATTENUATION

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1. INTRODUCTION

Active attenuation is a technique for reducing noise by the superposition of a secondary acoustic signal on the original signal such that they interfere destructively. However, in most applications the sound to be attenuated or other parameters such as temperature or air-flow rates vary with time making it difficult to maintain effective attenuation. To compensate for the effect of these variations requires a controller whose characteristics change with time so as to maintain a maximum level of attenuation; an adaptive digital controller. This paper shows how an exact least squares filter, the Fast Transversal Filter (FTF) [1] can be applied to the reduction of band limited noise in a section of air conditioning duct and compares it's performance against the gradient based Filtered-X algorithm [2].

2. THE MONOPOLE ACTIVE ATTENUATOR

A typical monopole duct control system is shown in FIG 1 with it's simplified equivalent block diagram in FIG 2. The primary noise, in most duct systems usually generated by a fan, is detected by a microphone placed upstream of a secondary source (loudspeaker). A measure of the primary noise x(t) is processed by the system with the aim of producing an output from the loudspeaker equivalent to -y(t). Where y(t) is the signal after allowing for the acoustic path between microphone and loudspeaker. For cancellation to occur we need:-

$$0 = o(t) + y(t) \qquad - (1)$$

thus

$$x(t)*w(t)*l(t) = -x(t)*p(t) \qquad - (2)$$

where :-

- w(t) is the impulse response of the controller.
- l(t) is the impulse response of the loudspeaker.
- p(t) is the impulse response of the acoustic path between microphone and loudspeaker.
- o(t) is the output from the loudspeaker.
- * denotes convolution in time.

Taking Laplace transforms we get that for cancellation :-

$$W(s) = -P(s).L^{-1}(s) \qquad - (3)$$

Equation (3) shows that the required controller transfer function depends on the both the acoustic transfer function between the detector and loudspeaker and the loudspeaker characteristics. Since P(s) depends on both the air flow rate and temperature to provide effective attenuation the system must adjust for these variations as well as for any change in the loudspeaker transfer function due to ageing or speaker replacement. Such an adaptive system is represented by FIG 1 with the inclusion of the error microphone, indicated by the dotted line. This microphone gives a measure of the difference between the desired performance and the actual system performance which is used to adjust the digital controller coefficients so as to produce and sustain maximum attenuation.

2.1 The Fast Transversal Filter.
If we assume that the acoustic transfer function between the sensor microphone and the secondary loudspeaker can be represented by a finite impulse response filter of length m we can represent the signal we wish to cancel in vector form [3] as :-

$$Y_m(n) = X_{0,m-1}(n)P_m(n) \qquad - (4)$$

where :-

$$X_{0,m-1}(n) = \begin{vmatrix} x(1) & x(0) & \ldots & .. \\ \ldots & \ldots & \ldots & .. \\ x(n-1) & x(n-2) & \ldots & x(n-m) \\ x(n) & x(n-1) & \ldots. & x(n-m+1) \end{vmatrix}$$

and

$$P_m(n) = [\ p_1(n), p_2(n) \ldots\ldots \qquad p_m(n)]$$

and represents the vector of coefficients describing the acoustic transfer function. The output vector of the digital controller is given by :-

$$O_m(n) = X_{0,m-1}(n)W_m(n) \qquad - (5)$$

Since the two signal are superimposed in the vicinity of the loudspeaker the sampled pressure (assuming linearity of the loudspeaker and any required amplification) at a microphone located at the point of superposition will be:-

$$e(n) = y(n) + o(n) \qquad - (6)$$

Thus from (5) and (6) we get :-

$$E_m(n) = Y_m(n) + X_{0,m-1}(n)W_m(n) \qquad - (7)$$

Since this pressure is the function we wish to minimise it will be called the error to accord with common terminology. The aim of the controller is to minimise this error, a measure of which can be obtained from the inner trace of the error vector :-

$$\varepsilon(n) = \langle E^T(n|n), E(n|n) \rangle$$

$$= E^T(n|n)E(n|n) \qquad - (8)$$

The minimum wrt $W_m(n)$ will occur when :-

$$\frac{\partial \varepsilon(n)}{\partial W_m(n)} = 0 \qquad -(9)$$

Solving (10) produces the value of $W_m(n)$ which minimizes $\varepsilon(n)$ as :-

$$W_m(n) = -\langle X_{0,m-1}(n), X_{0,m-1}(n) \rangle^{-1} X^T_{0,m-1}(n) Y_m(n) \qquad -(10)$$

The FTF is a method, derived using vector space techniques, which finds $W_m(n)$ with a minimum number of calculations; approx seven to ten times the length of the filter depending on the type of implementation. It's main advantages over simple gradient based methods such as the Filtered-X is that it exhibits a near optimal convergence rate which is not affected by the correlation properties of the input signal. There are however problems relating to the numerical stability of the FTF [4]. A solution to this in practice is to switch to a more robust method after initial convergence and only switch back to the FTF when the error signal increases above a preset limit.

2.2 Modified FTF Algorithm

In practise it is not possible to use the error signal at the output of the adaptive algorithm since it is modified by the path between the speaker and the error microphone. This can be represented by forming a new error vector E'(n) from the original using :-

$$E'(n) = AE(n) \qquad -(11)$$

$$E'(n) = A[Y_m(n) + X_{0,m-1}(n)W_m(n)] \qquad -(12)$$

where A is a square matrix representing the effect of the modification. In this case we wish to minimise $\langle E'(n),E'(n)\rangle$ wrt to the controller coefficients $W_m(n)$ i.e

$$\frac{\partial \varepsilon'(n)}{\partial W_m(n)} = 0 \qquad -(13)$$

Expanding and differentiating $\varepsilon'(n)$ gives us that :-

$$\frac{\partial \varepsilon'(n)}{\partial W_m(n)} = 0 + [A^TAX^T_{0,m-1}(n)]^TY_m(n) + X^T_{0,m-1}(n)A^TAY_m(n) + [X^T_{0,m-1}(n)A^TAX_{0,m-1}(n)]^TW_m(n) \qquad -(14)$$

The minimum will be when this equates to zero i.e

$$0 = 2X^T_{0,m-1}(n)A^TAY_m(n) + 2X^T_{0,m-1}(n)A^TAX_{0,m-1}(n)W_m(n) \qquad -(15)$$

thus the coefficients which will minimise $\varepsilon'(n)$ are given by :-

$$W_m(n) = -[X^T_{0,m-1}(n)A^TAX_{0,m-1}(n)]^{-1}X^T_{0,m-1}(n)A^TAY_m(n)$$

this can be seen to be directly analogous to the previous solution (10) if the equation is rearranged in the form :-

$$W_m(n) = -\langle AX_{0,m-1}(n),AX_{0,m-1}\rangle^{-1}[AX_{0,m-1}(n)]^TAY_m(n) \qquad -(16)$$

For the output to be analogous to that previously we must pre-filter the input to the FTF algorithm by A as shown in FIG 3 so that the output with the optimal coefficients is given by :-

$$O_m(n) = [AX_{0,m-1}(n)]W_m(n) \qquad -(17)$$

2.3 The Filtered-X Algorithm

Rather than attempt an exact solution to minimising the error criterion $\varepsilon'(n)$ above the Filtered-X method attempts to minimise another measure of the error given by :-

$$\varepsilon(n) = E\{e^2(n)\} \qquad -(18)$$

where :-

E{ } denotes the expectation operator.
$e(n)$ is given by (6).

The properties of the algorithm regarding rates of convergence, stability, etc. have been investigated by Widrow and Stearns [2]. The block diagram for an adaptive system based on the Filtered-X algorithm is the same as FIG 3 except in the way that the coefficients $W_m(n)$ are updated.

The above discussion ignored the propagation of the secondary signal upstream as well as downstream. This has the effect of introducing feedback into the system making the system effectively of the Infinite Impulse Response (IIR)/pole-zero type. Some possible solutions to this feedback effect include the use of directional transducers or adaptive IIR filters. Although there are difficulties associated with the convergence of such filters, results have been obtained which appear to justify their commercial viability [5].

3. EXPERIMENTAL RESULTS

To compare the convergence rates of the FTF and Filtered-X algorithms a computer simulation was performed on a model system with 16 coefficient filters. The input in both cases was effectively white noise and the faster convergence of the FTF is easily seen from FIG 4. To investigate the operation of the FTF as an adaptive noise canceller an experimental rig was set up similar to that shown in FIG 1. The modified FTF algorithm was implemented using a combination of fixed precision and floating point arithmetic on a DSP56001 processor. The primary noise signal was generated using a signal generator connected via amplification to a loudspeaker at one end of the duct. The output of the generator was pseudo random noise with a flat spectrum in the range 0 - 200 Hz. This output was also used as the input to the adaptive system i.e x(n). The error path matrix A was set to a time delay equal to that measured between the loudspeaker and error microphone. The levels obtained at the error microphone with and without the attenuator on are shown in FIG 5.

4. CONCLUSIONS

The results presented here show that it is possible to achieve significant reductions in band limited noise within an air conditioning duct under experimental conditions. The improved convergence rate of the FTF has implications for situations where this is a critical factor, possibly in the design of broad band three dimensional noise attenuation systems where the convergence time must be within the response time of the room or in situations where the signal to be attenuated is changing rapidly.

5. REFERENCES

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[3] Alexander S T. Adaptive Signal Processing Theory and Applications. Springer-Verlag New York 1986.

[4] Cioffi J M. Limited Precision effects in Adaptive Filtering. IEEE Trans Circuit Syst. Vol CAS-34 pp1097-1110 , July 1987.

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FIGURES

FIG 1 Monopole Attenuator

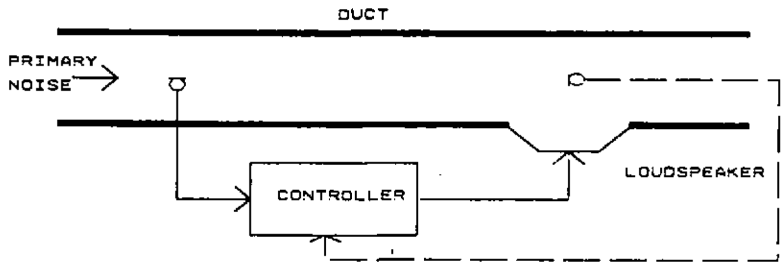

FIG 2 Simplified Equivalent Block Diagram
Of Monopole Attenuator.

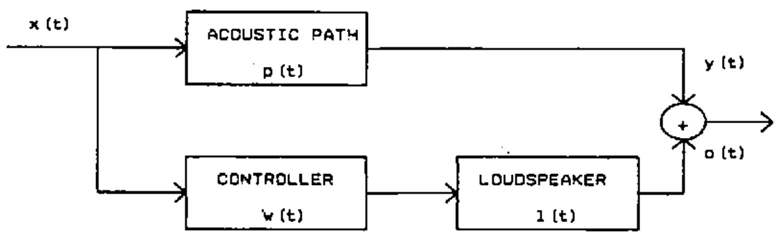

FIG 3 Adaptive Filter Block Diagram.

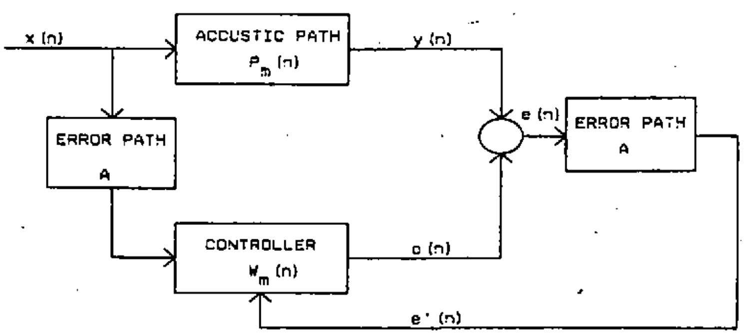

FIG 4 Comparison of FTF —— and Filtered-X --- Convergence Rates.

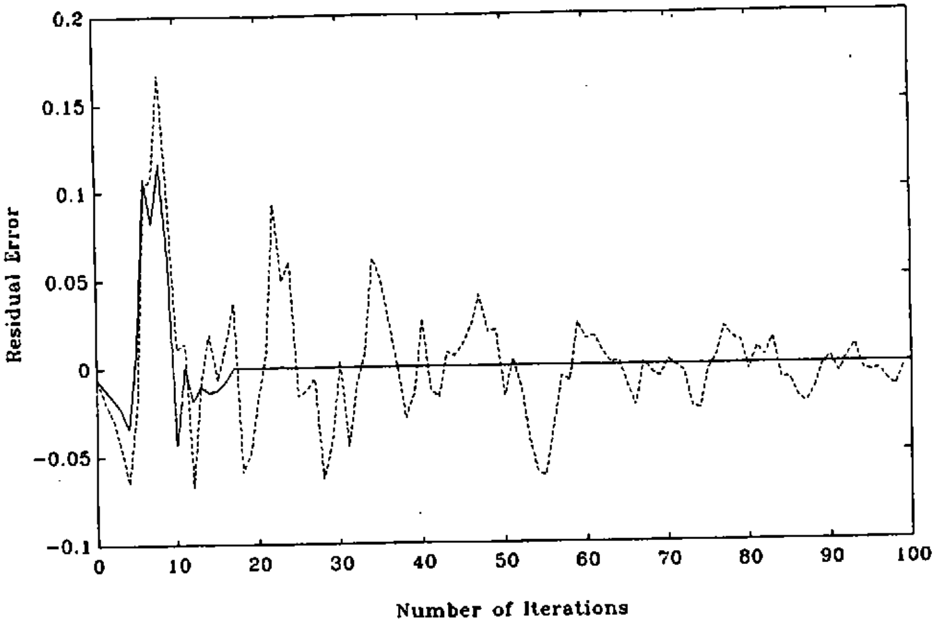

FIG 5 Levels with and without Active Attenuation.