HIGHER ORDER CUMULANTS FOR BROADBAND SIGNAL ARRIVAL ESTIMATION

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1. INTRODUCTION

A new method for the adaptive estimation of the time delay between two signals obtained from measurements at spatially detached sensors is proposed. This technique builds on the algorithm first proposed by Chiang and Nikias [1]. Fourth-order cumulants are employed in order to deal with signals which have symmetric probability density functions. Simulations are included to validate the potential of this new technique.

The estimation of the time delay between two signals obtained from measurements at spatially detached sensors has important application in the fields of geophysics, radar, and sonar [2]. Within such applications the sensor signal measurements are generally corrupted by some form of additive noise. Moreover, the unknown time delay may be time-varying, and therefore an adaptive approach is necessary for the time delay estimation problem.

A classical approach to the time delay estimation problem is based on the use of the Least Mean Square (LMS) algorithm [3,4] which estimates the required second-order statistics from the received input data. The performance of this method, however, is limited by the additive noise present at the sensors, particularly when there is some measure of spatial correlation between the noise measurements. To overcome this limitation a migration to the use of an adaptive algorithm based on higher order cumulants has been proposed by Chiang and Nikias [1]. The basic assumption for the application of higher order cumulants is that the probability density function of the measurement noise is such that its cumulants are zero for orders greater than two, as for Gaussian noise, whilst for the signal these cumulants are non-zero.

The algorithm due to Chiang and Nikias is specialised to third-order cumulants and therefore their assumption is that the probability density function of the signal is not symmetric. To overcome this limitation the contribution of this paper is to propose a new adaptive algorithm based on fourth-order cumulants. This new algorithm is designed to operate on measurements of a signal with a symmetric non-Gaussian probability density function corrupted by additive Gaussian noise. The additive Gaussian noise present at the two sensors can be correlated. The new algorithm employs a parametric model and a stochastic gradient algorithm to adjust the time-delay estimate at each iteration.

2. SIGNAL AND MODEL ASSUMPTIONS

The measurements, x(k) and y(k), at the sensors are assumed to be of the form

$$x(k) = s(k) + n_1(k) \quad (1)$$

$$y(k) = s(k-D) + n_2(k) \quad (2)$$

where s(k) is the signal and s(k-D) is a delayed version of the same signal, D is the delay to be estimated which may be time-varying, and $n_1(k)$ and $n_2(k)$ are unknown but assumed zero-mean Gaussian noises. The signal s(k) is assumed to be a zero-mean non-Gaussian stationary random process with nonzero kurtosis, namely $E\{s^4(k)\} \neq 0$. A zero mean signal with a symmetric probability density function has zero skewness i.e. $E\{s^3(k)\} = 0$. The new algorithm must estimate the time delay D, therefore a parametric model is developed. The substitution of eqn. (2) into eqn. (1) yields

$$y(k) = x(k-D) - n_1(k-D) + n_2(k) \quad (3)$$

which as in [1] can be generalised to

$$y(k) = \sum_{i=-L}^{L} b_i x(k-i) - n_1(k-D) + n_2(k) \quad (4)$$

where all the b_i values except $i = D$ are zero and $b_D = 1$. The limits of the summation in eqn. (4) are chosen for practical reasons since both positive and negative delays could occur and to be much greater than the maximum expected delay. Next, multiply both sides of eqn. (4) by x(k+l)x(k+m)x(k+n) and take expectations to yield

$$\begin{aligned} &E\{y(k)x(k+l)x(k+m)x(k+n)\} = \\ &\sum_{i=-L}^{L} b_i E\{x(k-i)x(k+l)x(k+m)x(k+n)\} \\ &- E\{n_1(k-D)x(k+l)x(k+m)x(k+n)\} \\ &+ E\{n_2(k)x(k+l)x(k+m)x(k+n)\} \end{aligned}$$

or

$$\begin{aligned} m^{(4)}{}_{yxxx}(l,m,n) = &\sum_{i=-L}^{L} b_i m^{(4)}{}_{xxxx}(l+i,m+i,n+i) \\ &- m^{(4)}{}_{n_1xxx}(l+D,m+D,n+D) + m^{(4)}{}_{n_2xxx}(l,m,n) \end{aligned} \quad (5)$$

where the superscripts are used to identify the order of the moments. The final term on the right-hand-side of eqn. (5) is zero for all (l,m,n) since the noise is zero mean and independent from the signal. The fourth-order moments in eqn. (5) are related to fourth-order cumulants in the following manner [5]

$$\begin{aligned} m^{(4)}{}_{yxxx}(l,m,n) = &\, c^{(4)}{}_{yxxx}(l,m,n) + m^{(2)}{}_{yx}(m)m^{(2)}{}_{xx}(n-l) \\ &+ m^{(2)}{}_{yx}(n)m^{(2)}{}_{xx}(m-l) + m^{(2)}{}_{yx}(l)m^{(2)}{}_{xx}(n-m) \end{aligned} \quad (6)$$

and

$$m^{(4)}_{xxxx}(l+i,m+i,n+i) = c^{(4)}_{xxxx}(l+i,m+i,n+i)$$
$$+ m^{(2)}_{xx}(m+i)m^{(2)}_{xx}(n-l) + m^{(2)}_{xx}(n+i)m^{(2)}_{xx}(m-l)$$
$$+ m^{(2)}_{xx}(l+i)m^{(2)}_{xx}(n-m) \tag{7}$$

where $c^{(4)}_{xxxx}(l,m,n)$ and $c^{(4)}_{yxxx}(l,m,n)$ are respectively the auto and cross fourth-order cumulants. It is straightforward to show that by substitution of eqns. (6) and (7) into eqn. (5) and with the use of the assumed parametric model given in eqn. (4) the required cumulant model becomes

$$c^{(4)}_{yxxx}(l,m,n) = \sum_{i=-L}^{L} b_i c^{(4)}_{xxxx}(l+i,m+i,n+i) \tag{8}$$

For the adaptive algorithm designed to estimate D the cumulant terms contained within eqn. (8) are estimated from the measurement data. Therefore these cumulant terms become time dependent, i.e. $c^{(4)}_{yxxx}(k;l,m,n)$ and $c^{(4)}_{xxxx}(k;l,m,n)$.

3. THE NEW TIME DELAY ESTIMATION ALGORITHM

In order to simplify the cumulant calculation and to make the algorithm suitable for time delay estimation the region of the fourth-order cumulant plane is specialised to the line l=m=n, hence eqn. (8) becomes

$$c^{(4)}_{yxxx}(k;l) = \sum_{i=-L}^{L} b_i c^{(4)}_{xxxx}(k;l+i) \tag{9}$$

where the simplification in the three indices is used as a shorthand notation to represent the line in question. The required cumulants are therefore calculated from

$$c^{(4)}_{xxxx}(k;l) = m^{(4)}_{xxxx}(k;l) - 3m^{(2)}_{xx}(k;l)m^{(2)}_{xx}(0) \tag{10}$$

$$c^{(4)}_{yxxx}(k;l) = m^{(4)}_{yxxx}(k;l) - 3m^{(2)}_{yx}(k;l)m^{(2)}_{xx}(0) \tag{11}$$

The configuration of the adaptive time delay estimation method based on fourth-order cumulants is shown in Figure 1. In a manner similar to Chiang and Nikias [1] a performance measure is defined. With the assumption that the $c^{(4)}_{xxxx}(l)$ terms are the fourth-order cumulants of x(k), the output of the FIR filter in Figure 1 is given by

$$c^{(4)}_{zxxx}(l) = \sum_{i=-L}^{L} b_i c^{(4)}_{xxxx}(l) \tag{11}$$

where the $\{b_i\}$ values are the coefficients of the FIR filter. The performance measure, E, is defined to be the sum of the squared errors between the desired output $c^{(4)}_{yxxx}(l)$ and the output of the FIR filter $c^{(4)}_{zxxx}(l)$, i.e.

$$E = \sum_{l=-L}^{L} [c^{(4)}_{xxxx}(l) - c^{(4)}_{yxxx}(l)]^2 \tag{12}$$

which can also be written as

$$E = \sum_{l=-L}^{L} [\sum_{i=-L}^{L} b_i c^{(4)}_{xxxx}(l+i) - c^{(4)}_{yxxx}(l)]^2 \tag{13}$$

and more neatly in matrix notation as

$$E = (\underline{C}^{(4)}_{xxxx}\underline{b} - \underline{C}^{(4)}_{yxxx})^T(\underline{C}^{(4)}_{xxxx}\underline{b} - \underline{C}^{(4)}_{yxxx}) \tag{13}$$

where $(.)^T$ is the transpose operator, and

$$\underline{C}^{(4)}_{yxxx} = [c^{(4)}_{yxxx}(-L), c^{(4)}_{yxxx}(-L+1), \ldots, c^{(4)}_{yxxx}(L)]^T$$

$$\underline{b} = [b_{-L}, b_{-L+1}, \ldots, b_L]^T$$

$$\underline{C}^{(4)}_{xxxx} = \begin{bmatrix} c^{(4)}_{xxxx}(-2L) & c^{(4)}_{xxxx}(-2L+1) & \cdots & c^{(4)}_{xxxx}(0) \\ c^{(4)}_{xxxx}(-2L+1) & c^{(4)}_{xxxx}(-2L+2) & \cdots & c^{(4)}_{xxxx}(1) \\ \vdots & \vdots & & \vdots \\ c^{(4)}_{xxxx}(0) & c^{(4)}_{xxxx}(1) & \cdots & c^{(4)}_{xxxx}(2L) \end{bmatrix}$$

which is a $(2L+1)(2L+1)$ Hankel matrix.

To minimise the performance measure an expression for the gradient of E with respect to $\underline{b}$ is required, viz.

$$\frac{\partial E}{\partial \underline{b}} = 2(\underline{C}^{(4)}_{xxxx}\underline{C}^{(4)}_{xxxx}\underline{b} - \underline{C}^{(4)}_{xxxx}\underline{C}^{(4)}_{yxxx}) \tag{14}$$

When this gradient function is employed within the adaptive algorithm the exact values required for the cumulants are replaced by their instantaneous values calculated recursively in time from the input data. The peformance measure E therefore becomes a function of the time index k, E(k), and is defined as

$$E(k) = (\underline{C}^{(4)}_{xxxx}(k)\underline{b}(k) - \underline{C}^{(4)}_{yxxx(k)})^T \times (\underline{C}^{(4)}_{xxxx}(k)\underline{b}(k) - \underline{C}^{(4)}_{yxxx(k)}) \tag{15}$$

which is only an estimate of E at the k-th time instant and $\underline{b}(k) = [b_{-L}(k), b_{-L+1}(k), \ldots, b_L(k)]^T$ is the parameter vector a time k and similarly $\underline{C}^{(4)}_{yxxx}(k)$ and $\underline{C}^{(4)}_{xxxx}(k)$ are respectively the estimates of $\underline{C}^{(4)}_{yxxx}$ and $\underline{C}^{(4)}_{xxxx}$. The necessary gradient estimate at time k is given by

$$\nabla(k) \equiv \frac{\partial E(k)}{\partial \underline{b}(k)} = 2[\underline{C}^{(4)}_{xxxx}(k)\underline{C}^{(4)}_{xxxx}(k)\underline{b} - \underline{C}^{(4)}_{xxxx}\underline{C}^{(4)}_{yxxx}] \tag{16}$$

To update the FIR filter parameter vector at each iteration a stochastic gradient algorithm is used

$$\underline{h}(k+1) = \underline{h}(k) - \mu(k)\nabla(k) \tag{17}$$

To control the rate of adaptation $\mu(k)$ is chosen to lie in the range

$$0 < \mu(k) < \frac{1}{trace^2\{\underline{C}^{(4)}{}_{xxxx}(k)\}} \tag{18}$$

so as to meet the stability requirement of the adaptation algorithm independent of the eigenvalue spread of the input data matrix.

To complete the algorithm it is necessary to estimate the moments from the input data. Calculation of the moment estimates are based on simple integrators of the form

$$m^{(4)}{}_{xxxx}(k;l) = \frac{1}{k+1}\sum_{i=0}^{k} \lambda^{k-i} x(i+2L)x^3(i+2L+l)$$

$$m^{(4)}{}_{yxxx}(k;l) = \frac{1}{k+1}\sum_{i=0}^{k} \lambda^{k-i} y(i+2L)x^3(i+2L+l)$$

$$m^{(2)}{}_{xx}(k;l) = \frac{1}{k+1}\sum_{i=0}^{k} \lambda^{k-i} x(i+2L)x(i+2L+l)$$

$$m^{(2)}{}_{yx}(k;l) = \frac{1}{k+1}\sum_{i=0}^{k} \lambda^{k-i} y(i+2L)x(i+2L+l)$$

where $0 < \lambda \leq 1.0$, and λ is termed as a forgetting factor [6] because it introduces an exponential windowing of the data. Such a window is necessary when the time delay is time-varying and the above statistics must track such changes. The 2L factor is included so that all the required moments are generally nonzero from k=0. When the time delay is assumed to be fixed λ is chosen to be equal to unity.

4. SIMULATIONS

The first simulation of the adaptive time delay estimation method (SIMULATION 1) is based on a fixed time delay D = 7. The initial parameter vector of the adaptive filter is set to zero at time k=0. The signal to noise ratio equals 0dB, that is the variance of the signal and noise are both set to unity. The signal is chosen to be a zero mean white process with a symmetric probability density function, unity variance and $E\{s^4(k)\} = 6$. The choice of a Laplace probability density function is only one example and the algorithm can be equally well applied to other signals which have nonzero fourth-order cumulants. The additive noise signals $n_1(k)$ and $n_2(k)$ are chosen to be uncorrelated white zero mean Gaussian processes. The length of the adaptive filter L=10 and since the time delay is fixed, λ, the forgetting factor, is selected as unity. The adaptation gain parameter μ is 0.05. The simulation is run for 3000 time samples and the instantaneous parameter vector is shown in Figure 2.

The choice of such a simulation length is so that the adaptive algorithm has sufficient time to converge. Clearly, the proposed adaptive algorithm converges to the true time delay.

The second set of simulations (SIMULATION 2.1 and 2.2) is the same as the first simulation except that the additive noises $n_1(k)$ and $n_2(k)$ are spatially correlated. This correlation is introduced by passing $n_1(k)$ through a length 10 FIR filter to form $n_2(k)$. To show the advantage of the new algorithm the parameter vector at time 3000 which is shown in Figure 3 is the result from the application of a time delay estimation algorithm based on second order statistics [1] to the same problem. This correlation based algorithm is unable to differentiate between the delay in the signal from that in the additive noise. However, as shown in Figure 4, the new adaptive time delay estimation algorithm is not affected by cross correlation present between the two adative Gaussian noise signals.

This new algorithm can also operate on sinusoidal signals and when the forgetting factor λ is chosen to be less than unity, nonstationary environments. These possibilities for the algorithm are currently under investigation.

5. CONCLUSION

A new time delay estimation algorithm based on fourth-order cumulants has been introduced. This algorithm was targeted at signals which have symmetric probability density functions. The use of fourth-order cumulants was motivated by their robustness to additive Gaussian noise. The algorithm development included a specialisation to a single line in the fourth-order cumulant space in order to reduce the number of moments which must be estimated to calculate the required cumulants. Simulations have been included to verify the potential of the new algorithm for time delay estimation.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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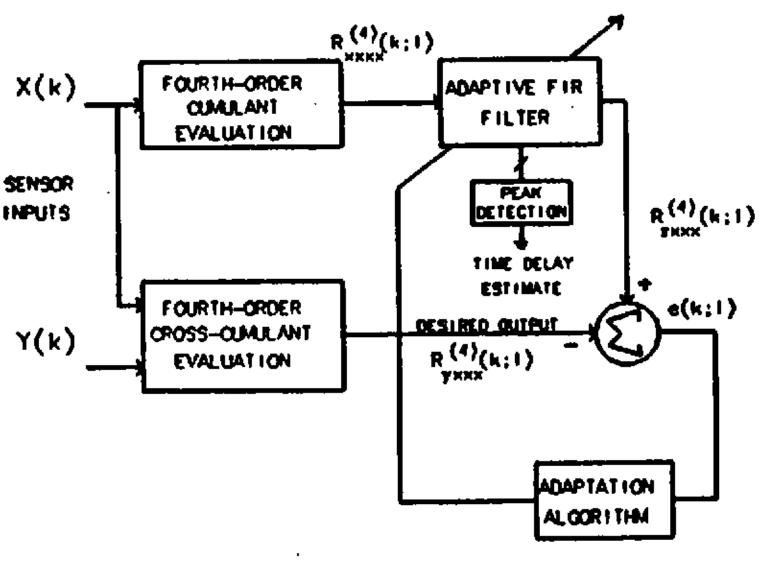

Figure 1 Structure for the adaptive time delay estimation method based on fourth-order cumulants.

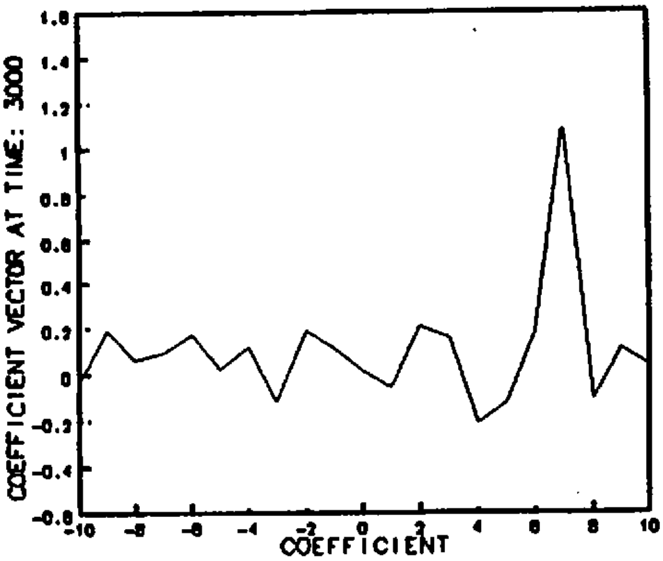

Figure 2 SIMULATION 1 PARAMETER VECTOR

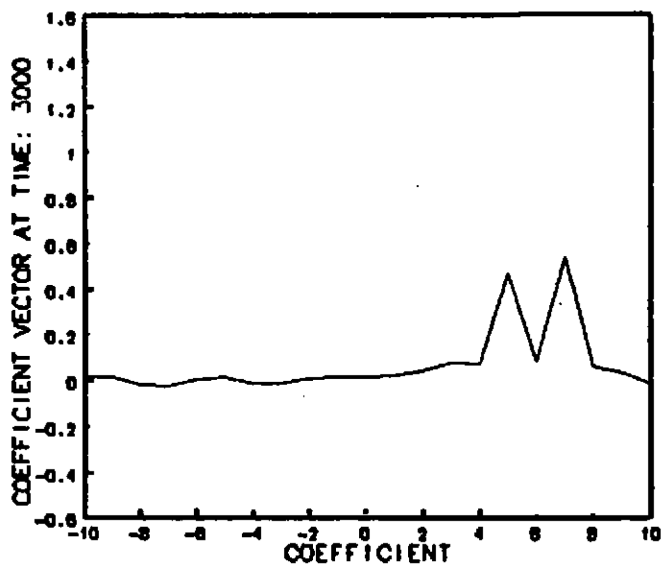

Figure 3 SIMULATION 2.1 PARAMETER VECTOR

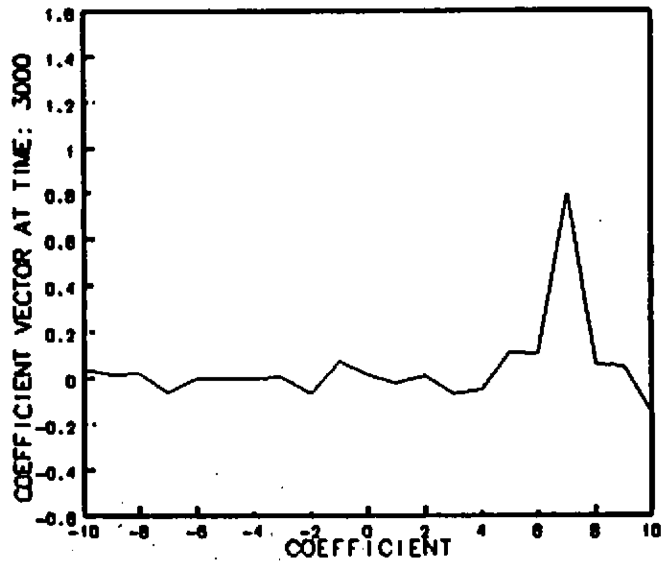

Figure 4 SIMULATION 2.2 PARAMETER VECTOR