

THE DESIGN AND TESTING OF NON LINEAR ACOUSTIC SYSTEMS

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1 INTRODUCTION

This paper is concerned with the design, testing and performance of parametric sonar arrays in which the non-linear acoustic interaction in the water between two overlapping primary beams of different frequencies can be used to generate secondary acoustic beams at the *difference frequency*, the *sum frequency* and also at the *harmonic frequency* of each primary signal. The strength of these secondary signals depends upon the design of the parametric array, but at best are 20 to 30 dB lower than the primary signals which generated them, and a failure to follow fundamental principles of sonar transducer array design and measurement technique will result in a much more significant loss of power. Detailed experimental verification using both high and low frequency arrays of different geometries is reported and confirm the theories of H.O. Berkay over the far field regime.

2 DISCUSSION

The chief result of the non linear sound transmission is that a secondary acoustic beam is generated at the difference frequency as the primary frequencies propagate through the medium, ie the difference frequency is continually being formed with increasing range from the source. For this reason a parametric array is often described as an array of "virtual" sources which extends some distance into the water medium, with each source boosting the difference frequency sound pressure. The length of this "virtual" array ensures that the difference frequency signal has narrow beamwidth and low sidelobes.

Within the zone of interaction of the primary waves, four main secondary frequencies are produced. These are :-

1. The "difference frequency" between the two primary waves.
2. The "sum frequency" of the two primary waves.
3. The "harmonic frequency" of each primary wave.

Higher order intermodulation products are also produced by the non linear medium but they are at a similarly reduced level from the secondary components and are ignored in the subsequent discussion.

Figure 1 shows the Demonstrator Parametric Array Sonar (DPAS) (which is described later) transmitting primary frequencies of 17.5 kHz and 19.5 kHz. At a distance of 300 metres it was possible to measure a 2 kHz difference frequency, a 37 kHz sum frequency, and the harmonics

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of each primary frequency at 35 and 39 kHz.

Once generated, these secondary components proceed independently of the primary waves, suffering from the appropriate spreading loss and the appropriate frequency dependent attenuation loss. For sonar applications it is usually the production of the difference frequency which is of interest because it has several interesting properties and its formulation has been much studied.

The advantages of Parametric Arrays can be summarized :-

1. Low frequencies can be generated from relatively small high frequency arrays.
2. Narrow beams can be produced at low frequencies from small arrays.
3. The difference frequency beam either does not exhibit side lobes, or the side lobes are at an extremely low level.
4. The difference frequency can be varied over a wide range with only a small change in beamwidth.

The disadvantages of Parametric Arrays can be summarized :-

1. Poor efficiency of the conversion to the difference frequency.
2. Non linear distortion relies upon the physical characteristics of the medium which may or may not remain constant over the primary wave field, and also may vary from day to day.

The generation of the sum and harmonic frequencies has been largely ignored because being of high frequency, they are more rapidly absorbed by the medium, and are therefore of little interest. However, if the primary frequencies are low but at extremely high power, then this triplet of high frequencies will be generated at a sufficiently low frequency and high power level to travel considerable distances in the water medium (Figure 1). This raises the question as to whether these components are a useful adjunct, or a liability to a parametric sonar system.

Berktag, Reference 1, shows that the expected difference frequency source level for a parametric array can be calculated using the following equation :-

$$SL_D = SL_H + SL_L + \{20 \times \log_{10}(F_D)\} + \left\{20 \times \log_{10}\left(\frac{|V|}{\alpha_T r_0}\right)\right\} - 286.5 \quad 1$$

where :-

- | | |
|--------|---|
| F_D | Difference Frequency in kHz |
| F_C | Central Primary Frequency in kHz |
| SL_H | rms High Primary Source Level dB re 1 micro-Pascal at 1 metre |

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SL_L rms Low Primary Source Level dB re 1 micro-Pascal at 1 metre
286.5 Coefficient for sound transmission through sea water

Berkay, Reference 2, shows that similar expressions can be used to determine the source levels of the sum and harmonic frequencies.

A derivation of the parameter "V" for the difference, sum and harmonic signals is included in the Appendix.

3 DESIGN OF PARAMETRIC ARRAYS

There are two general categories of parametric array :-

1. Electrically mixed arrays in which the two primary excitation frequencies are generated and mixed electrically (generating amplitude modulation) before application to the transducer elements which would all be of the same type.
2. Discrete arrays in which the primary frequencies are generated independently and applied separately to two independent sets of transducer elements forming two arrays which may or may not be interleaved. In this case the elements themselves may or may not be of the same type in each array.

If the bandwidth of a single transducer is not large enough to yield the necessary primary frequencies at the required power level, then transducers of different types must be used to form the parametric array.

CHOICE OF ELECTRICAL AMPLIFIER SYSTEM

It has already been stated that the formation of the difference frequency is a very inefficient process. The best success a parametric array designer can hope to achieve is a difference frequency source level which is approximately 25dB down on the primary frequency source levels. As a general rule the acoustic energy present at the difference frequency is of similar magnitude to the energy which would have been present had the primary frequency array been excited at the difference frequency directly. In this case the acoustic energy would have been spread over a wide beamwidth instead of the narrow beamwidth associated with a parametric array.

Because the secondary components are generated at such a low level compared to the primary signals careful choice of the electrical amplifier system is required and, in particular, the electrically mixed array demands the use of amplifiers capable of delivering high power with the minimum of distortion if measurements of the linearity / non-linearity of the electrical system employed is to be avoided. This is one of the main causes of failure in the design of parametric arrays.

In the DPAS system the amplifiers and matching units were situated immediately behind the
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transmitting arrays in order to prevent transmission of high power signals along long lengths of cable and eliminate as much as possible any "cross-talk" between the signal cables.

RECEIVE ARRAY SYSTEMS

The previous section discussed non linearity of the electrical amplifier systems used to drive the array and showed that this must be reduced to a minimum in order to measure the secondary acoustic signals. It is just as important to eliminate non linearity from the "receiving" measurement system.

It is common practice when attempting to measure very low level signals to amplify the signal from the receive hydrophone or array, and then apply high and low pass filters to obtain the desired frequency signal. When attempting measurements on parametric arrays it is necessary to filter the incoming signals *before* amplification in order to eliminate measuring the non linearity of the receiving amplifiers. Figure 2 shows the experimental setup used to measure the DPAS system. The incoming signal from the hydrophone is split into three components - primary, difference and sum frequency signals before passage to appropriate bandpass filters employing high roll off outside of the required frequency band. The signals were then passed to pre-amplifiers / cable driving amplifiers before measurement. Figure 3 shows the hydrophone sensitivities achieved by this system for each frequency band.

SPACING OF THE ELEMENTS

Another common failure of parametric arrays concerns the spacing of the elements. This is vitally important for all transducer arrays, but a parametric array requires some extra consideration, if discrete primary frequency arrays are used. Consider the upper drawing of Figure 4 and imagine it to be a "normal transducer array"; i.e. all the elements are identical. It is standard practice to space the elements such that the centre to centre spacing is half a wavelength at the main frequency of operation. Generally speaking this yields the optimum array in terms of interaction effects and beam patterns. This Figure actually describes a parametric array with the high and low frequency elements arranged in vertical staves and raises the question "How should the elements be spaced?"

If the spacing between adjacent low frequency staves is set at half a wavelength, then the spacing between each high and low frequency element is then one quarter of a wavelength. Each element is obviously very small and a large number will be required to fill the array, which in turn increases the costs of the array. On the other hand if the spacing between adjacent high and low frequency staves is set at half a wavelength, which would give sensibly sized elements, the spacing between adjacent low frequency staves would then be one whole wavelength. This in turn, yields problems if steering of the array is contemplated. The principle discussed here applies equally well to "hexagonal packing" or any other way of arranging the transducers to form the array. A parametric array with the transducers arranged in this format has been built and tested with satisfactory results (see later). The spacing between the staves was in excess of half a wavelength at the central primary frequency but electronic steering of the beams was not a

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requirement for this array.

The lower drawing of Figure 4 demonstrates a possible solution to the problem. The high and low frequency arrays are completely separated and manufactured as "normal transducer arrays" in their own right. For each array the centre to centre spacing of the elements is one half wavelength at the chosen frequency of operation of that array. In this case the high frequency array is smaller than the low frequency array to ensure that each array has an identical beam pattern at particular primary frequencies. This need not be the case for all parametric arrays. Since most of the mixing of the two frequencies occurs in the far field this arrangement of transducers should not be detrimental. However, in practice, the separation of the arrays is responsible for some loss of difference frequency source level which can be accounted for by the application of a "correction factor" (Reference 7) to the theoretical models.

The configuration is sketched in Figure 5 where the two primary arrays are rectangular and comparable in size; the first of width $2p$, the second of width $2r$, and their edges are separated by a distance q . If $q = 0$, then the system is not spatially disjoint and the formulae for parametric action developed elsewhere apply, with the sound levels at the array faces the correct values to take to determine the difference frequency field. If q is non zero, however, there is a region out to some unknown range R within which the primary beams do not overlap and no parametric interaction occurs. As the arrays are "similar" the geometrical development of the primaries with distance from the array faces will be similar and at range R two plane virtual arrays of semi width kp and kr , for some k , will just touch. If the source levels generated by these virtual arrays is known the conventional theories can be applied to find the secondary source level. This assumes that there is no energy loss between each true primary array and its virtual image - all that happens is that the energy in each is spread over k times the area so each effective Source Level falls by the factor k , so their product (and hence the parametric source level) is reduced by $20 \log_{10}(k)$ and k is given :-

$$k = 1 + \left(\frac{q}{(p+r)} \right)^2 \quad 2$$

CRITICAL SOURCE LEVELS

It is erroneous to believe that a specified difference frequency power can be achieved simply by increasing the primary frequency source level. Not only must the well known effect of "cavitation" be considered but the parametric array designer must be aware of the effect of "saturation" of the primary frequencies. Increasing the primary source levels beyond a certain limit actually results in a decrease in primary source level, as energy is transferred from the primary frequencies to the harmonic frequencies, thus lowering the energy which is available for difference frequency formation. This "Critical Source Level" of the primary frequencies has been discussed by Willette and Moffett (Reference 3) and is described by Equation 3. Up to this critical source level the array is said to be "absorption limited" because the primary frequencies are

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predominantly affected by the well known "small signal" absorption coefficient. Beyond the critical source level the array is then said to be "saturation limited". Figure 6 sketches the situation.

$$SL_C = 281.0 - (20 \times \log_{10}(F_C)) \quad 3$$

Where :-

F_C	Central Primary Frequency (kHz)
SL_C	Critical Source Level (dB re 1 micro-Pascal @ 1 metre)

The models discussed are valid provided the array is "absorption limited" and should not be used with primary frequency source levels within 3dB of the critical source level. In the "absorption limited" region, changing the primary frequency power results in a change in the difference frequency power of twice the magnitude. For example, increasing each primary source level by 3dB results in an increase in difference frequency source level of 6dB, which is evident from Equation 1. From an overall power point of view this is to be expected since the total primary input power has increased by 6dB and so has the difference frequency power.

RANGE OF MEASUREMENT

There are two rules which the parametric array designer should consider. These are particularly important if source level at the difference frequency is an important parameter.

1. For any given range of measurement there should be as large a number of primary frequency wavelengths as possible i.e. make the central primary frequency as high as possible. Low primary frequencies mean that in any given range, there will be relatively few primary frequency wavelengths available for mixing and therefore a low difference frequency source level will result.
2. The "Step-Down Ratio" is defined as the ratio of central primary frequency to difference frequency. The higher the "Step-Down Ratio" the poorer the efficiency of conversion to the difference frequency. If the primary frequencies are set too high compared to the desired difference frequency then a low difference frequency source level will result.

These two rules work in opposition to each other, and the parametric array designer must find a suitable compromise. Generally speaking "Step-Down Ratios" of between six and twelve to one offer the best solutions.

The effect of the "Step-Down Ratio" on difference frequency generation is clearly shown in Figure 7 where DPAS is transmitting at a central primary frequency of 18.5 kHz to achieve difference frequency signals in the 1 to 5 kHz band and each primary signal is maintained at a constant level. The 1 kHz difference frequency signal is approximately 20 dB lower than the 5

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kHz signal.

The formation of the difference frequency is a far field effect and varies with range from the array. This is a direct consequence of the two rules discussed above. Below a particular range from a parametric array it will be impossible to measure any difference frequency signal. The difference frequency becomes prominent as the range from the parametric array increases, and its source level will then build up to a maximum at a certain range. This is discussed in more detail in later sections but it is important to note that one of the most common failures of parametric arrays to date has been the lack of consideration of the distance from the array at which detection of the difference frequency is attempted.

In the DPAS system the conventional near field / far field boundary occurs less than 6 metres from the primary source arrays but attempts to measure a 3 kHz difference frequency signal at ranges of 7 metres and 49 metres on board RDV Crystal and at Loch Goil failed. It is shown later that the minimum range for measurement of this signal is approximately 150 metres and a very powerful 3 kHz signal was measurable at a range of 300 metres from the source arrays.

MEASUREMENT OF SECONDARY SIDE-LOBE LEVELS

Low sidelobe levels at the secondary frequencies are a direct result of this fact. Consider a rectangular array for which the first sidelobe is approximately 13dB lower than the main beam. If the first sidelobe of each primary frequency coincide, and each is 13dB lower than its main beam, then the first sidelobe of the secondary wave will be 26dB lower than the secondary main beam. Since the primary frequencies originate from the same array or from arrays which are designed to make their main beams coincide, it is unlikely that the primary sidelobes will coincide. In which case the secondary sidelobes will be that much lower again and losses of 50 to 60dB between secondary main beam and secondary first sidelobe would not be uncommon.

Good design of the parametric array results in a difference frequency main beam source level approximately 25 to 30 dB lower than the primary source levels, with its first sidelobe AT LEAST 26dB lower again and hopefully, buried in the noise background.

Good design of the measurement system should enable both the primary main beam levels and the secondary sidelobe levels to be measured.

4 DIFFERENCE FREQUENCY THEORETICAL MODELS

Various theoretical models exist for the prediction of difference frequency source level and they can be classified as either "range dependent" or "non range dependent" solutions.

Two range dependent solutions are :-

1. Berklay Solution (Equations 6, 7 and 8)
2. Fenlon Matched Asymptotic Solution (Equation 12)

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Two non range dependent solutions are :-

1. Mellen and Moffett Matched Solution (Equation 14)
2. Fenlon Ordinary Solution (Equation 11)

Theoretical calculations show that as the range of measurement is increased, the expected difference frequency source level increases to the values predicted by the "non range dependent models". The "limiting value" of Fenlon's Matched Asymptotic Solution is, in fact, Fenlon's Ordinary Solution.

Since it is only within the region where the two primary frequencies are mixing that the difference frequency signals are produced, it follows that the beamwidth of the difference frequency is the same as the beamwidth of the narrowest primary frequency.

A small percentage change in the primary frequencies can vary the difference frequency by large amounts with only a small change in difference beamwidth.

The bandwidth of a parametric array can be considerable provided the "step down ratio" (ratio of central primary frequency to difference frequency) is kept between six and twelve to one. Higher step down ratios result in considerable loss of source level at the difference frequency.

5 SUM AND HARMONIC FREQUENCY THEORETICAL MODELS

The sum and harmonic frequency equations are compared to the difference frequency equations in the Appendix. If Equation 16 describes the difference frequency source density function, then Equation 17 describes the sum frequency source density function and Equation 18 can be used to calculate the sum frequency source level, with the parameter "V" calculated by Equations 19, 20, 21 and 22.

Figure 7 shows theoretical results which are indicative of the behaviour of parametric arrays in general. The results shown are expected from DPAS operating in a sea temperature of twelve degrees Celsius, a salinity of thirty five parts per thousand, and the range of measurement is 300 metres.

The sum frequency is always a constant, at twice the central primary frequency; and that the sum frequency source level remains constant (for constant primary source levels) across the difference frequency band of interest. There is no "step-up ratio" affecting sum frequency generation analogous to "step-down ratio" which affects difference frequency generation.

For an "absorption limited" parametric array any change in primary source levels doubles the change in difference frequency source level. For instance a 5dB increase in each primary frequency source level results in a 10dB increase in difference frequency source level. The theory predicts that sum frequency generation behaves in exactly the same way, as shown in Figure 7.

Figure 8 shows the effect of varying the central primary frequency but maintaining a constant difference frequency. This is another way of displaying the effect of the "step-down ratio" on difference frequency generation. As the central primary frequency is lowered relative to the

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difference frequency, the efficiency of generation of the difference frequency improves. The efficiency of generation of the sum frequency however, is lowered by decreasing the central primary frequency.

It has been suggested (Reference 8) that knowledge of the sum frequency formation could lead to a calibration system for low frequency parametric sonars such as DPAS, where a large interaction distance is required to generate measurable difference frequency signals. Measurable sum frequency signals are generated within a much shorter distance than measurable difference frequency signals, making their measurement possible at short range sites such as RDV Crystal in Portland Harbour even though a difference frequency cannot be measured. Since the sum frequency is generated by non linear interaction in the water medium, the inference is that the difference frequency is in the process of being generated. It is therefore only necessary to make measurements on the sum frequency to prove that the parametric array is operating as expected.

The sum frequency parameter "V" defined by Equation 19 is identical to that used for difference frequency calculation except for the parameter ALPHA TS. This forces a change in the calculation of the Exponential Integral with result that this expression for "V" does not converge to a maximum value with increasing range "R". The effect of this is shown in Figures 9 and 10 where DPAS is shown transmitting at 20 kHz and 17 kHz with source levels of 235 dB re 1 micro-Pascal at each of these frequencies. The theoretical source level of the 3 kHz difference frequency maximizes approximately 2000 metres from the source, whereas the theoretical 37 kHz sum frequency source level continues to increase, eventually generating more source level than the primary frequency source levels. At approximately 20000 metres from the array the sum frequency source level equals the primary frequency source level of 235 dB re 1 micro-Pascal; and continues to increase indefinitely.

Figures 9 and 10 show that the minimum range of measurement for validity of the sum frequency theory in this particular case is 12.5 metres as compared to 154.5 metres for the difference frequency measurement. Below the quoted "minimum range" the theoretical predictions become invalid, although it *may* still be possible to measure a secondary source level.

Practical measurements show that as the range of measurement is decreased the difference frequency does not have the required range to build up into a measurable signal and disappears into the noise background. Since the range from the parametric array for which the sum theory becomes valid is so different from that for which the difference theory becomes valid it is interesting to compare predictions. Figure 9 shows that as the range is decreased, the secondary source level (sum, difference and harmonics) decreases to a minimum value and then begins to increase. For instance, at 7 metres from the array, the difference frequency theory indicates that a difference frequency should have been present at approximately the same level as measured at 300 metres. At Loch Goil, no difference frequency was present 7 metres from the array, and this is interpreted as further validation of the theoretical model because the theory is invalid at this low range.

These latter Figures also show the expected source levels of the harmonic frequencies of the two primary frequencies. For spatially disjoint arrays such as DPAS, the correction factor postulated

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by Roebuck is not required because these are produced by the individual arrays themselves and are not influenced by the separation between them.

The harmonic frequency source levels are calculated in the same way as the sum frequency source levels and are defined by Equations 23 to 26. It is interesting to note that in this case the harmonic frequency is caused by a modulation of the appropriate primary frequency with itself and not by modulation with another, totally different, primary frequency. The primary source level used to calculate the harmonic source level is therefore half of that used to calculate the sum and difference source levels, (i.e. 6dB less).

The sum and harmonic frequency beampatterns behave in exactly the same way as the difference frequency beampatterns, in that at large distances from the source, their beamwidths are controlled by the beamwidth of the narrowest primary frequency, and their sidelobes, if present at all, are at an extremely low level.

6 EFFECT OF THE MEDIUM

As stated above the non linear distortion relies upon the physical characteristics of the medium which may or may not remain constant over the primary wave field, and may also vary from day to day. In general any effect which reduces the small signal absorption coefficient at the primary frequencies such as reduction in salinity of sea water or increase in temperature of either sea or fresh water acts to increase the primary frequency source level and hence has a beneficial effect upon difference frequency generation. The small signal absorption coefficient is a frequency dependent parameter and care must be taken over choosing its value for use in the theoretical models.

The difference frequency source level is calculated using Equation 1 which introduced a coefficient of -286.5 dB for non linear sound transmission through sea water. This is a "medium dependent" coefficient whose value decreases to -291.5 dB for transmission through glycerine and increases to -272 dB for transmission through ethyl alcohol, thus improving difference frequency generation by 14 dB in the latter case.

The parametric array should be designed for transmission into a particular medium and care should be taken over the spacing of the elements for that medium as described above. If transmission of sound is required from one medium to another it is important to remember that production of high power secondary signals relies upon transmission of high power primary signals for considerable distance from the parametric array. The impedance change which occurs at any interface between mediums may be detrimental to the passage of the primary signals and thus reduce the far field secondary signal levels.

7 EXPERIMENTAL STUDIES

In order to verify these parametric sonar models, both high and low frequency parametric arrays have been manufactured and tested on board RDV Crystal and at DRA Loch Goil as described

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in Reference 7.

EXPERIMENTAL STUDIES OF A HIGH FREQUENCY PARAMETRIC ARRAY

The high frequency array is designed to operate at a central primary frequency of 100 kHz, generating difference frequencies in the range 5 to 20 kHz.

The array was made up from one hundred and forty three elements arranged in thirteen staves each containing eleven elements. The elements are of a Barium Titanate / Aluminium sandwich construction shown in Figure 11. This type of construction is capable of quite high power levels at frequencies around 100 kHz. Electrical connections were made to alternate staves using screened cable to provide the two high primary frequencies. The elements were mounted in a square array of side 13 cm. within a 100 kHz Calibration Standard Transducer Body of the type used on board RDV Crystal. Layers of pressure release rubber approximately 1.5 mm thick were placed between adjacent elements of a staff making the centre to centre spacing of the elements 1.15 cm. This represents 0.76 of one wavelength at a frequency of 100 kHz. Layers of pressure release rubber approximately 2.2 mm thick were placed between adjacent staves. The centre to centre spacing between elements in adjacent staves was 0.98 cm., which represents 0.65 of one wavelength at the central primary frequency of 100 kHz. The spacing between adjacent high frequency staves and between adjacent low frequency staves was therefore greater than one wavelength. As a consequence, no attempt was made to steer the primary beams electronically.

Measurements were performed at the maximum test distance available on board RDV Crystal at the time, approximately 28 metres.

Centred upon a primary frequency of 100 kHz, difference frequencies between 5 and 20 kHz were considered for a range of primary source levels. Figures 12 and 13 compare practical results with the various theoretical models for primary source levels of 208 and 205 dB re 1 micro-Pascal at one metre respectively. This was well below the Critical Source Level for this array which is 241 dB re 1 micro-Pascal at one metre using Equation 3. The Figures show excellent agreement between the practical results and the Fenlon Matched and Berkay theories.

The 3dB drop in each primary source level from 208 to 205 dB causes the received difference frequency level to drop by 6dB, as expected.

Fenlon's "Ordinary Solution" and the "Matched Solution" of Mellen and Moffett appear to over estimate the difference frequency source level by approximately 6dB at 28 metres. The Berkay Model and the Fenlon "Matched Asymptotic Model" agree well at this distance because they can be described as "range dependent models". As the range of measurement increases the difference frequency source level increases to the value predicted by the non range dependent models. Figure 14 shows the received 9 kHz difference frequency level at 28 metres, and that this level would increase to a maximum about 200 metres from the source.

Mellen and Moffett did produce a "Basic Solution", which is "range dependent", but for square and circular arrays it yields the same results as the Berkay Model, and so is not included in this discussion.

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At a test distance of 28 metres the measurements of difference frequencies less than 8 kHz were technically invalid for this array. However, inspection of Figures 12 and 13 do not show any serious discrepancy between theory and practice at these low difference frequencies. The difference frequency level did not fall below the background noise level until attempts were made to measure a 4 kHz signal.

The parametric array designer should therefore use the minimum range of measurement discussed by Berkay as a guideline. It may be possible to measure difference frequency signals within this range but there may or may not be deviations from the theoretical predictions.

Figures 2 and 3 depict the system used to receive signals from the DPAS system which is described in the next section. A similar system was used in this case with the primary frequencies measured using a 100 kHz Standard Calibration Transducer and the difference frequency using an 8 kHz Standard Calibration Transducer commonly used on RDV Crystal, mounted side by side. The received signals were then passed through appropriate filters before amplification (if required) and measurement.

Figures 15 and 16 describe beam patterns at the primary frequencies of 105 kHz and 95 kHz respectively and Figure 17 shows the resultant 10 kHz difference frequency. Notice that the difference frequency exhibits no sidelobes and its beamwidth is identical to the primary frequencies.

In Transducer Design the 3dB beamwidth for a rectangular array is approximately given by Equation 4 where λ represents the wavelength and D represents the length or diameter of the transducer.

$$\theta = \left(\frac{\lambda}{D} \right) \times 50.76 \quad 4$$

For a transducer of side 13cm. and operating at 100 kHz the expected 3dB beamwidth is 5.9 degrees which is confirmed in Figures 15 and 16. Operating at 10 kHz the expected beamwidth for a transducer of this size is 59 degrees, but the difference frequency shown in Figure 17 has a 3dB beamwidth of approximately 6 degrees. To produce this beamwidth at 10 kHz would require a transducer of side 1.3 metres.

EXPERIMENTAL STUDIES OF A LOW FREQUENCY ARRAY

This section describes the design of the low frequency parametric array "DPAS" (Demonstrator Parametric Array Sonar), which is designed to operate at a central primary frequency of 18.5 kHz, generating a difference frequency beam between 1 and 5 kHz.

DPAS differs from the commonly used parametric configuration by employing two separate but adjacent arrays for generation of the primary beams. The arrangement is shown in the lower diagram of Figure 4. The advantage of this configuration is that it imposes fewer constraints on the linearity of the electrical driving system, and it enables transducer designs optimised for

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different frequencies to be used in the two primary arrays, providing optimum effective bandwidth.

The array consists of two 11 by 11 element rectangular arrays located side by side. The elements are conventional piston transducers of two sizes, located in a thick metal baffle plate. The low frequency array is designed for operation between 16 and 18 kHz, with resonance at 17 kHz. The high frequency array is designed for operation between 19 and 21 kHz, with resonance at 20 kHz. The centre to centre spacing of the elements of each array is one half wavelength at the appropriate resonant frequency, allowing electronic steering of unshaded beams up to forty five degrees without aliasing.

The low frequency array is 495mm square, and the high frequency array is 420mm square, with spacing between the arrays of 85mm. At the design frequency with each array operating at resonance (generating a 3 kHz difference frequency), each array has an identical beam pattern providing maximum physical overlap of the primary beams and maximum efficiency of non linear conversion.

The critical source level (Equation 3) for this array is 255 dB re 1 micro-Pascal at one metre, well above the design maximum source level of 235dB. The array is therefore designed to operate well within absorption limited as opposed to saturation limited conditions, and the theoretical models discussed should apply.

Measurements of source level and beam pattern in both the primary and secondary beams were carried out at ARE Loch Goil using the experimental setup shown in Figure 18. The DPAS array was suspended from a raft at the mid-water depth of approximately 45 metres (well below the maximum full power cavitation depth of 28 metres) using a test station which permitted the array to be trained in the horizontal plane. The transmitted signals were recorded on a fixed hydrophone via the "receive system" shown in Figure 2, which was suspended below a second raft at a range of approximately 300 metres from the source. Pulsed CW transmissions were used throughout.

The test station used at Loch Goil enabled measurements to be made in the horizontal plane every 2.5 degrees for angles up to 45 degrees to the array normal, and then at 5 degree intervals up to 90 degrees. The beam patterns are therefore relatively coarse, and are also subject to some uncertainty in angle due to relative motion of the rafts and backlash in the training gear. The beam patterns are nevertheless adequate to illustrate the salient features of the array acoustic performance.

Figures 19 and 20 compare difference frequency results with and without correction factor for two different array geometries. In the first instance, Figure 19, the geometry is as described; a low frequency array 495 mm square, a high frequency array 420 mm square, with a spacing between the arrays of 85 mm; for which the correction factor is -1.5 dB. In the second case, the widths of the arrays remain the same, but the length of the low frequency array reduces to 360 mm, and the length of the high frequency array reduces to 305 mm with an array separation of 217 mm. This change of geometry changes the correction factor from -1.5 dB to -4 dB. As can be seen the agreement between theory after the correction has been applied and measurement is

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very good.

Figures 21, 22, and 23 show the beam patterns of the sum and harmonic signals which are generated along with the 3 kHz difference frequency. The theoretical sum frequency source level has been corrected (reduced) by the appropriate factor. There is no need to apply this correction to the harmonic signals because these are generated by the individual arrays themselves and are not affected by their separation.

Figure 24 summarizes the source level measurements as function of difference frequency. The Figure clearly shows the strong dependence of difference frequency source level on step-down ratio. As the difference frequency is decreased from 5 kHz to 1 kHz, the difference frequency source level decreases by 25 dB whereas the 37 kHz sum frequency source level remains constant for constant primary source level.

The discrepancy between theory and practice for the 1 kHz difference frequency is thought to be due to the short range of measurement (300 metres) available at Loch Goil. A range of 300 metres is adequate for DPAS generating difference frequencies above 2 kHz, but for 1 kHz, the minimum range for validity of the theory is 492 metres. There is no discrepancy between theory and practice for the 37 kHz sum frequency which only requires 12.5 metres for validity of the theory.

Figure 25 repeats these measurements at a range of 49 metres on board RDV Crystal in Portland Harbour. The Figure shows good agreement between theory and practice for the sum frequency and the lack of difference frequency yields credence to the idea of using the sum frequency as a calibration tool for low frequency parametric sonars.

Figure 26 shows the effect of electronic steering of the primary frequencies. The secondary signals are all shifted by the appropriate angle. In practice the simple beamformer employed did not shift each primary frequency beam by precisely equal angles, resulting in a loss of secondary source level and a narrowing of sum and difference frequency beamwidth.

8 CONCLUSION

This paper provides the non specialist with a powerful tool for the design and testing of parametric arrays. The theoretical models discussed in the Appendix have been experimentally verified for both high and low frequency devices.

9 ACKNOWLEDGEMENT

The theoretical basis of this paper was the work of Berkta, described in References 1 and 2. The author also gratefully acknowledges the contribution of Roebuck (Reference 7).

The Demonstrator Parametric Array System (DPAS) was manufactured at DBE Technology by L. Lipscombe and R. Morris.

THE DESIGN AND TESTING OF NON LINEAR ACOUSTIC SYSTEMS

10 REFERENCES

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2. H.O. Berktaý Private Communication September 1988
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4. H.O. Berktaý Journal of Sound and Vibration (1965) 2(4),435-461 "Possible Exploitation of Non Linear Acoustics in Underwater Transmitting Applications"
5. F.H. Fenlon J.Acoust.Soc.Am., Vol 55, No.1 January 1974 35-46 "On the Performance of a Dual Frequency Parametric Source via Matched Asymptotic Solutions of Burgers' Equation."
6. Kinsler and Frey Fundamentals of Acoustics Publisher: J. Wiley and Sons
7. A.G. Elliott & I. Roebuck (to be published) "Spatial Development of Non Linear Fields from Parametric Arrays"
8. I. Roebuck Private Communication

11 APPENDIX

The relevant sum and difference frequency equations which are used in the theoretical models are summarized below :-

Difference Frequency Equations

$$\begin{aligned}
 R_{01} &= l^2 / \lambda_c \\
 R_{02} &= m^2 / \lambda_c \\
 R_0 &= S / \lambda_c \\
 R_0^2 &= R_{01} * R_{02} \\
 r_{01} &= (K_c / K_D) * R_{01} \\
 r_{02} &= (K_c / K_D) * R_{02} \\
 r_0 &= (K_c / K_D) * R_0 \\
 \alpha_T &= \alpha_H + \alpha_L - \alpha_D
 \end{aligned}$$

Sum Frequency Equations

$$\begin{aligned}
 R_{01} &= l^2 / \lambda_c \\
 R_{02} &= m^2 / \lambda_c \\
 R_0 &= S / \lambda_c \\
 R_0^2 &= R_{01} * R_{02} \\
 r_{01} &= (K_c / K_S) * R_{01} \\
 r_{02} &= (K_c / K_S) * R_{02} \\
 r_0 &= (K_c / K_S) * R_0 \\
 \alpha_{TS} &= \alpha_H + \alpha_L - \alpha_S
 \end{aligned}$$

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Berkay, Reference 4, deduced an expression for the 3 dB beamwidth of a parametric array :-

$$2 \times \theta_D = 4 \times \sqrt{\left(\frac{\alpha_T}{2 \times K_D} \right)} \quad 5$$

$$\text{Provided :- } \alpha_T / (2 \times K_D) \ll 1.0$$

The expected difference frequency source level can be calculated using Equation 1 and it is with the interpretation of the parameter "V" that the models disagree.

Small signal absorption coefficients are evaluated for sea temperatures of five and fifteen degrees Celsius using the expressions derived by Kinsler and Frey (Reference 6).

DIFFERENCE FREQUENCY THEORETICAL MODELS

Berkay (Reference 1) deduced the following expression for this parameter :-

Berkay Range Dependent Solution :-

$$V = 1 - \exp\left(\frac{-\alpha_T r_0}{N}\right) + T - jZ \quad 6$$

Where :-

$$Z = T + \alpha_T r_0 [E_1(\alpha_T r_0 N) - E_1(\alpha_T R)] \quad 7$$

$$T = \sqrt{\left(\frac{\pi \alpha_T r_0}{2N}\right)} \times \left[\operatorname{erf} \sqrt{(\alpha_T r_0 N)} - \operatorname{erf} \sqrt{\left(\frac{\alpha_T r_0}{N}\right)} \right] \quad 8$$

Provided that the range of measurement $R > 2r_{01}$

The Exponential Integral is given :-

THE DESIGN AND TESTING OF NON LINEAR ACOUSTIC SYSTEMS

$$E_1(x) = \int_x^{\infty} \left(\frac{1}{t} \right) \times \exp(-t) dt \quad 9$$

The Error Function is given :-

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \times \int_0^x \exp(-t^2) dt \quad 10$$

Fenlon deduced a range dependent solution (Matched Asymptotic), and a non range dependent solution (Ordinary) for the parameter "V".

Fenlon Ordinary Solution :-

$$V = \alpha_T r_0 \times \exp(\alpha_T r_0) \times E_1(\alpha_T r_0) \quad 11$$

Fenlon Matched Asymptotic Solution :-

$$V = \alpha_T r_0 \times \exp(\alpha_T r_0) \times \{ E_1(\alpha_T r_0) - E_1[\alpha_T (R + r_0)] \} \quad 12$$

Provided that $\alpha_T R$ is finite

The Exponential Integral $E_1(x)$ is defined by Equation 9 above.

Mellen and Moffett deduced three solutions for the parameter V; termed Basic, Matched Asymptotic and Range Dependent.

Mellen and Moffett Basic Solution :-

$$V = 1 - \exp(-\alpha_T r_0) + j\alpha_T r_0 E_1(\alpha_T r_0) \quad 13$$

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Mellen and Moffett Matched Asymptotic Solution :-

$$V = \left(\frac{\alpha_T r_0 \pi}{2} \right) \times [H_0(\alpha_T r_0) - Y_0(\alpha_T r_0)] \quad 14$$

Where :-

H_0 denotes the Zero Order Struve Function

Y_0 denotes the Zero Order Bessel Function of the Second Kind

Mellen and Moffett Range Dependent Solution :-

$$V = 1 - \exp(-\alpha_T r_0) + j\alpha_T r_0 [E_1(\alpha_T r_0) - E_1(\alpha_T R)] \quad 15$$

Provided that the range of measurement $R > r_0$ and $\alpha_T R$ is finite.

SUM FREQUENCY THEORETICAL MODELS

The above expressions calculate the parameter V which is required by Equation 1 in order to determine the difference frequency source level. A difference frequency source density function can be defined :-

$$q_D = j \left(\frac{\beta K_D P_H P_L}{\rho_0^2 C_0^3} \right) \quad 16$$

Where :-

P_H and P_L represent the appropriate primary frequency pressure

ρ_0 and C_0 denote the density and velocity of sound for the medium

K_D difference frequency wavenumber

It is therefore possible to define a sum frequency source density function :-

THE DESIGN AND TESTING OF NON LINEAR ACOUSTIC SYSTEMS

$$q_s = j \left(\frac{\beta K_s P_H P_L}{\rho_0^2 C_0^3} \right) \quad 17$$

Equation 1 may now be re-defined to calculate the source level of the sum frequency :-

$$SL_s = SL_H + SL_L + \{ 20 \times \log_{10} (F_s) \} + \left\{ 20 \times \log_{10} \left(\frac{|V|}{\alpha_{TS} r_0} \right) \right\} - 286.5 \quad 18$$

The parameter V is evaluated :-

$$V = 1 - \exp \left(- \frac{\alpha_{TS} r_0}{N} \right) + T - jZ \quad 19$$

Where :-

$$Z = T + \alpha_{TS} r_0 \left[E_1 \left(\frac{\alpha_{TS} R_0 N}{2} \right) - E_1 (\alpha_{TS} R) \right] \quad 20$$

$$T = \sqrt{\left(\frac{\pi \alpha_{TS} r_0}{2N} \right)} \times \left[\operatorname{erf} \sqrt{(\alpha_{TS} r_0 N)} - \operatorname{erf} \sqrt{\left(\frac{\alpha_{TS} r_0}{N} \right)} \right] \quad 21$$

Provided that the range of measurement $R > 2 r_{01}$

and $-\alpha_{TS} = \alpha_H + \alpha_L - \alpha_S$ with $\alpha_{TS} > 0$

In this case the Exponential Integral (Equation 9) is given by the expression :-

$$E_1(x) \approx 0.577 + \ln(x) + x \quad 22$$

THE DESIGN AND TESTING OF NON LINEAR ACOUSTIC SYSTEMS

HARMONIC FREQUENCY THEORETICAL MODELS

The harmonic frequency equations can be summarized :-

High Primary Harmonic Equations

$$\begin{aligned} R_{01} &= l_H^2 / \lambda_H \\ R_{02} &= m_H^2 / \lambda_H \\ R_0 &= S / \lambda_H \\ R_0^2 &= R_{01} * R_{02} \\ r_{01} &= (K_H / K_{HH}) * R_{01} \\ r_{02} &= (K_H / K_{HH}) * R_{02} \\ r_0 &= (K_H / K_{HH}) * R_0 \\ \alpha_{TH} &= \alpha_H + \alpha_H - \alpha_{HH} \end{aligned}$$

Low Primary Harmonic Equations

$$\begin{aligned} R_{01} &= l_L^2 / \lambda_L \\ R_{02} &= m_L^2 / \lambda_L \\ R_0 &= S / \lambda_L \\ R_0^2 &= R_{01} * R_{02} \\ r_{01} &= (K_L / K_{LH}) * R_{01} \\ r_{02} &= (K_L / K_{LH}) * R_{02} \\ r_0 &= (K_L / K_{LH}) * R_0 \\ \alpha_{TL} &= \alpha_L + \alpha_L - \alpha_{LH} \end{aligned}$$

The harmonic frequency source density functions are identical to the sum and difference source density functions except for the substitution of the appropriate wavenumbers. The harmonic frequency source levels are calculated in the same way as the sum frequency source levels except that the primary frequency source levels used must be 6dB lower than used to calculate the sum and difference frequency source levels.

The source level of the High Primary Harmonic Frequency is given :-

$$SL_H^{HH} = SL_H - 6.0 \quad 23$$

$$SL_{HH} = SL_H^{HH} + SL_H^{HH} + \{20 \times \text{Log}_{10}(F_{HH})\} + \left\{20 \times \text{Log}_{10}\left(\frac{|V|}{\alpha_{TH} r_0}\right)\right\} - 286.5 \quad 24$$

Similarly, the source level of the Low Primary Harmonic Frequency is given :-

$$SL_L^{LH} = SL_L - 6.0 \quad 25$$

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$$SL_{LH} = SL_L^{LH} + SL_L^{LH} + \{20 \times \text{Log}_{10}(F_{LH})\} + \left\{20 \times \text{Log}_{10}\left(\frac{|V|}{\alpha_{TL} r_0}\right)\right\} - 286.5 \quad 26$$

12 GLOSSARY OF TERMS

F_H	High Primary Frequency
F_L	Low Primary Frequency
F_C	Central Primary Frequency
F_D	Difference Frequency
F_S	Sum Frequency
F_{HH}	High Primary Harmonic Frequency
F_{LH}	Low Primary Harmonic Frequency
α_H	Small signal absorption coefficients at the indicated frequency in nepers / metre
α_L	
α_D	
α_S	
α_{HH}	
α_{LH}	
K_H	Wavenumbers at the indicated frequency
K_L	
K_C	
K_D	
K_S	
K_{HH}	
K_{LH}	
λ_C	Central Primary Wavelength

l Length of transducer (metres)

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m	Width of transducer (by convention $l > m$)
S	Area of the transducer ($l * m$)
N	Aspect ratio of the transducer (l / m)
C_0	Velocity of sound in the medium
SL_H	Source Level at appropriate frequency

DPAS SETUP FOR 2kHz TRANSMISSION
AT LOCH GOIL APRIL/MAY 1988
ELECTRONIC STEERING 0 DEGREES

Fig 1

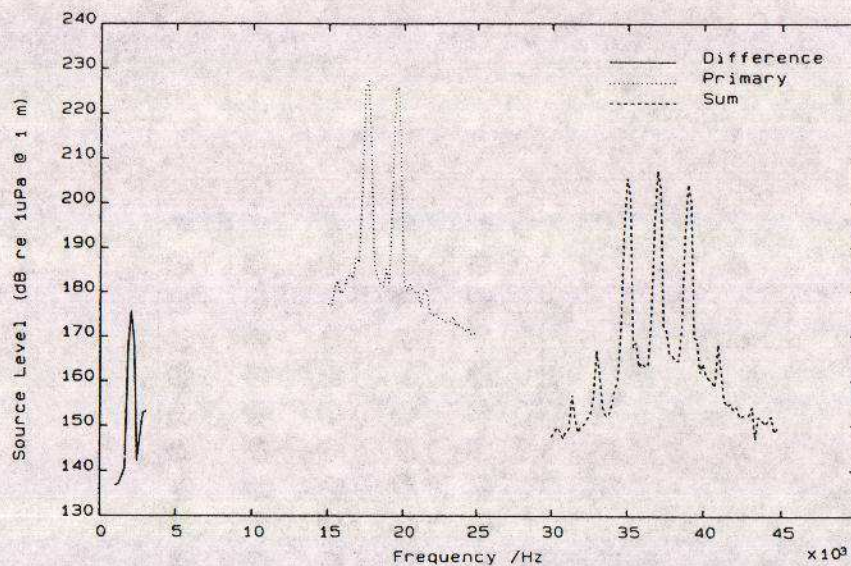
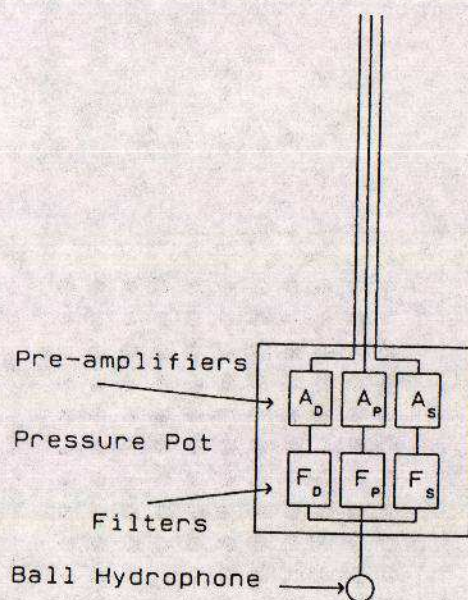


Fig 2



Receive System

COMPARISON OF OUTPUTS FROM THE RECEIVE SYSTEM

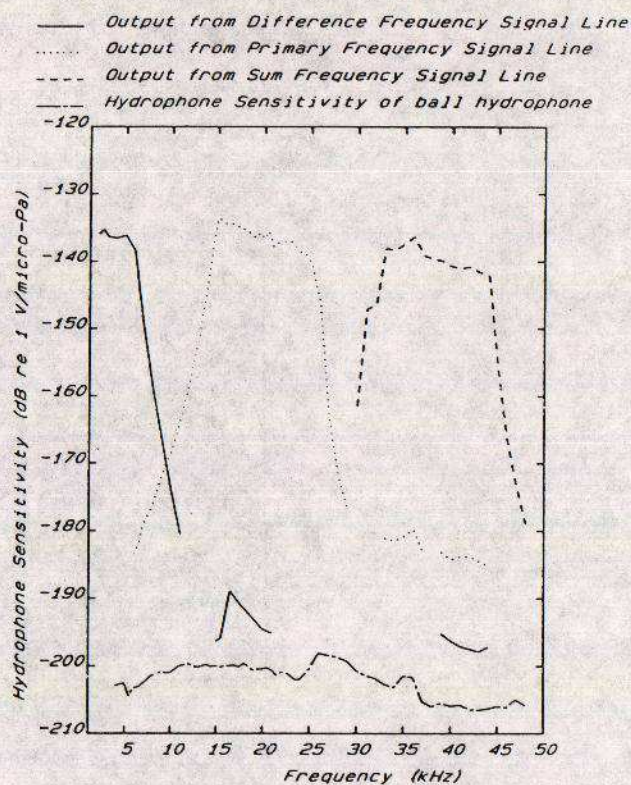


Fig 4

PARAMETRIC ARRAY CONFIGURATIONS

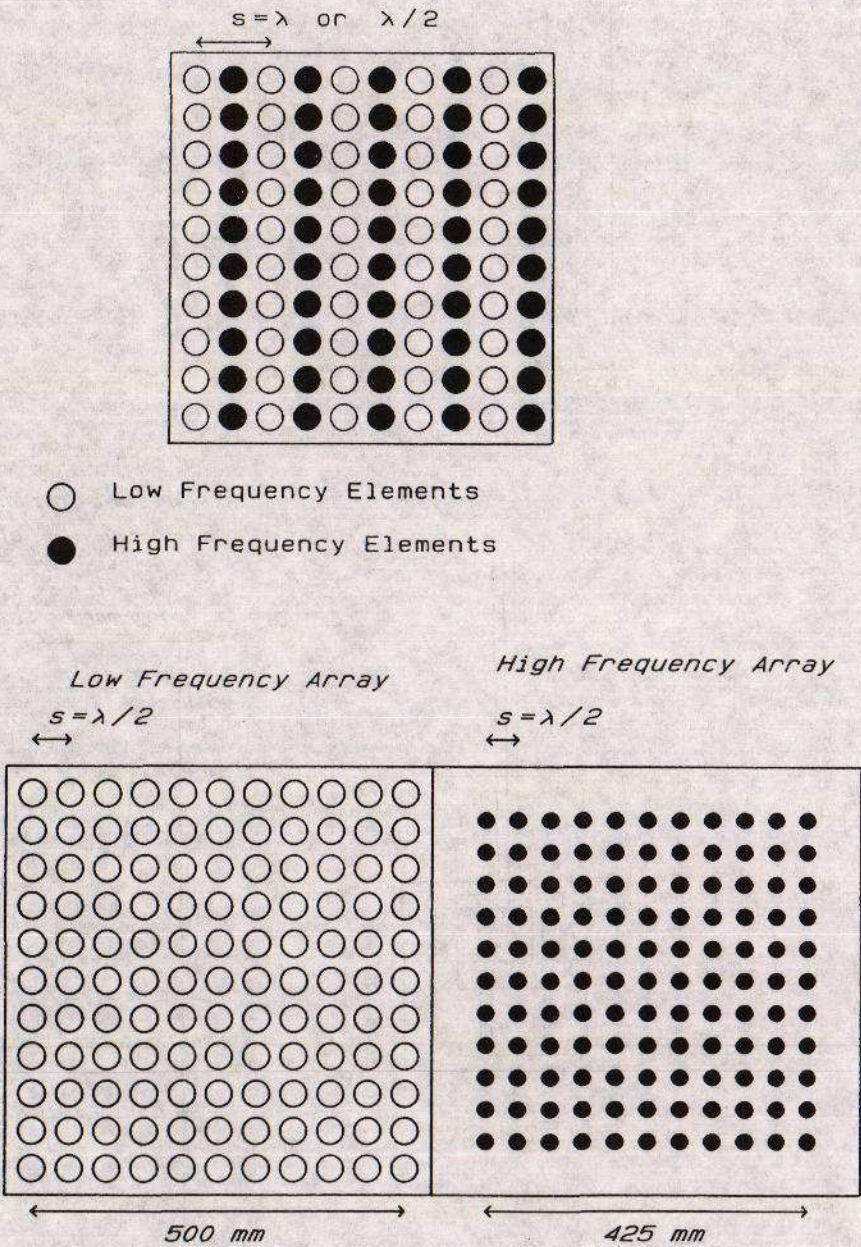
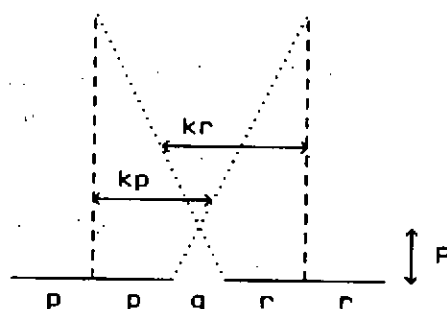


Fig 5

Modification to the Theoretical Models for Spatially-Disjoint Parametric Arrays

Consider two arrays of similar size and operating over a similar range of frequencies. Let the length of the arrays be $2p$ and $2r$ respectively, and separated by a distance q



At any distance from the arrays, the total insonified width from the first array is k_p and from the second array is k_r

When they first overlap: -

$$k_p + k_r = p + q + r$$

$$k = 1 + \frac{q}{p + r}$$

Fig 6
Sketch of Behaviour of Difference
Frequency Source Level

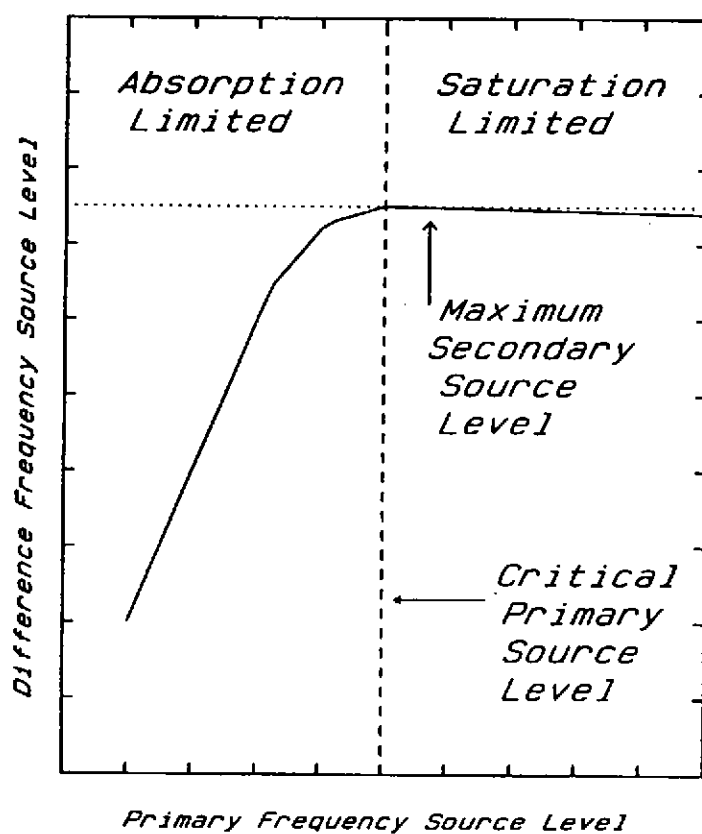


Fig 7

SOURCE LEVELS OF SUM AND DIFFERENCE
FREQUENCIES FOR FIXED PRIMARY LEVELS

DPAS Array Sea Temperature 12 Degrees

—————	Difference Frequency	} Each Primary Level
-----	37 kHz Sum Frequency	
-----	Difference Frequency	} Each Primary Level
.....	37 kHz Sum Frequency	

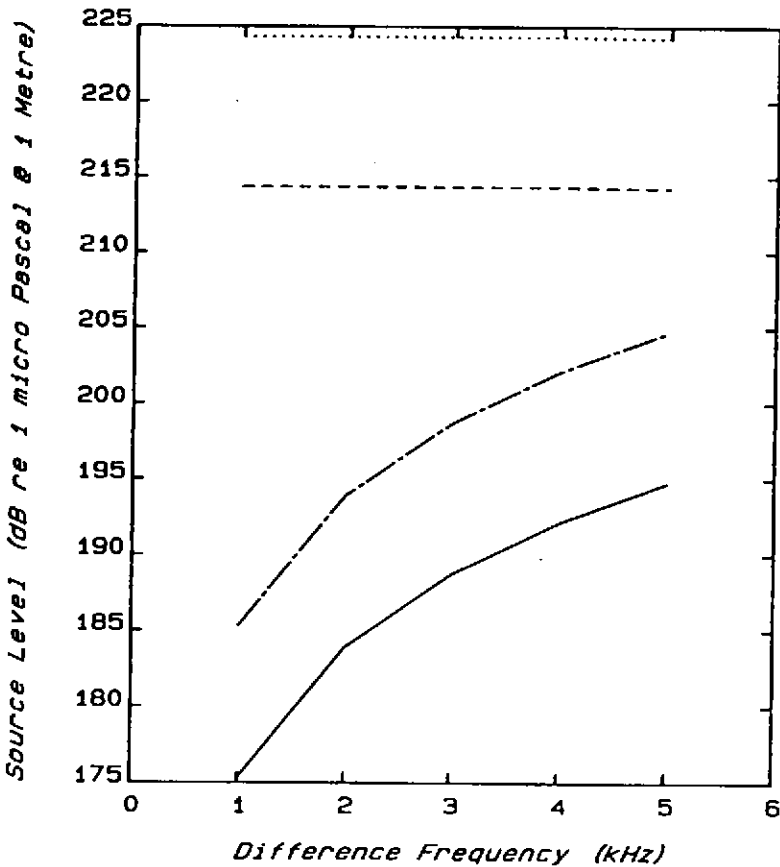


Fig 8

VARIATION OF CENTRAL PRIMARY FREQUENCY
FIXED 3 kHz DIFFERENCE FREQUENCY

DPAS Array Sea Temperature 12 Degrees

Primary Source Level 230 dB re 1 micro-Pascal

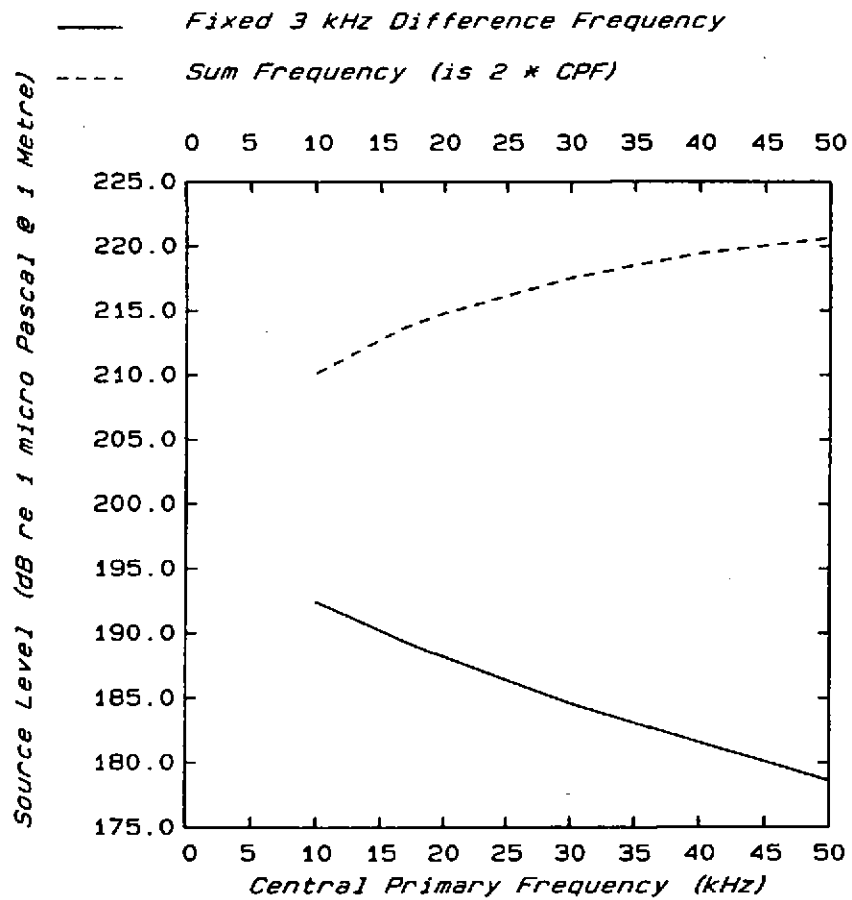


Fig 9

DPAS TRANSMITTING AT 20 kHz and 17 kHz
TO YIELD A 3 kHz DIFFERENCE FREQUENCY
Primary Source Levels 235 dB re 1 micro_Pascal
@ 1 metre

- Theoretical 34 kHz Harmonic Frequency
- Theoretical 40 kHz Harmonic Frequency
- Theoretical 37 kHz Sum Frequency
- Theoretical 3 kHz Difference Frequency
- Theoretical Minimum Ranges for Measurement
- 2.8 metres for 34 kHz Harmonic Frequency
- 2.4 metres for 40 kHz Harmonic Frequency
- 12.5 metres for 37 kHz Sum Frequency
- 154.5 metres for 3 kHz Difference Frequency

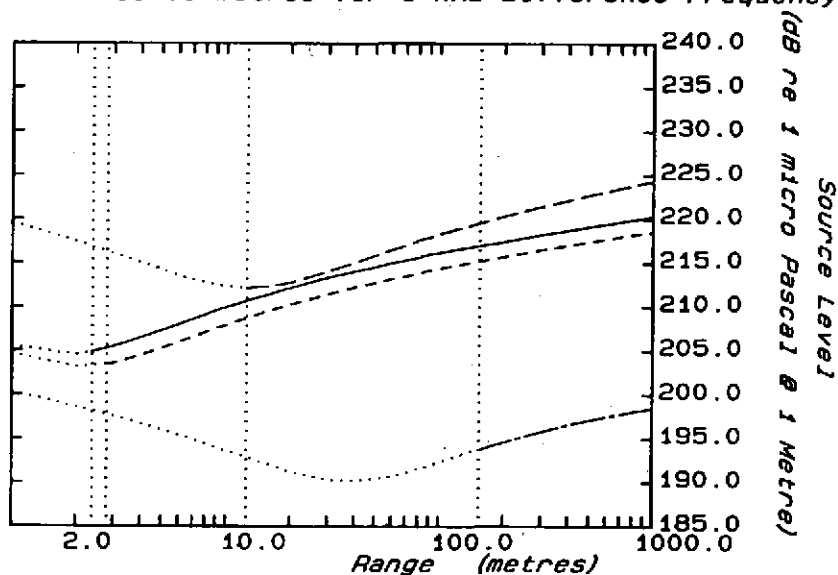


Fig 10

DPAS TRANSMITTING AT 20 kHz and 17 kHz
TO YIELD A 3 kHz DIFFERENCE FREQUENCY

Primary Source Levels 235 dB re 1 micro_Pascal
@ 1 metre

- Theoretical 34 kHz Harmonic Frequency
- Theoretical 40 kHz Harmonic Frequency
- Theoretical 37 kHz Sum Frequency
- Theoretical 3 kHz Difference Frequency

- Minimum Theoretical Range for Measurement of
3 kHz Difference Frequency is 154.5 metres

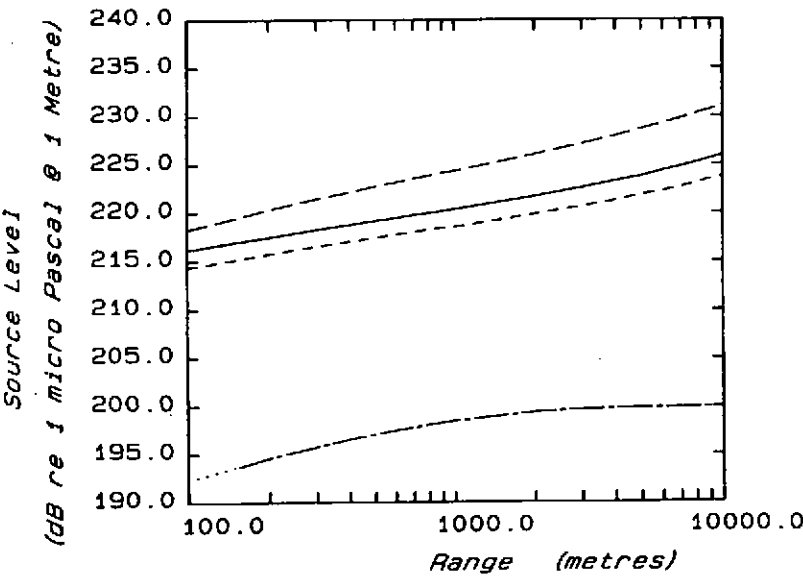


Fig 11

100 kHz Element

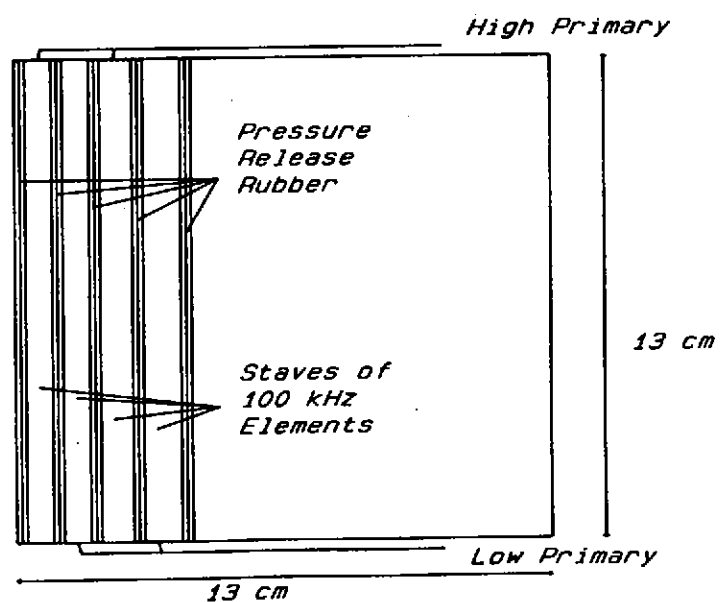
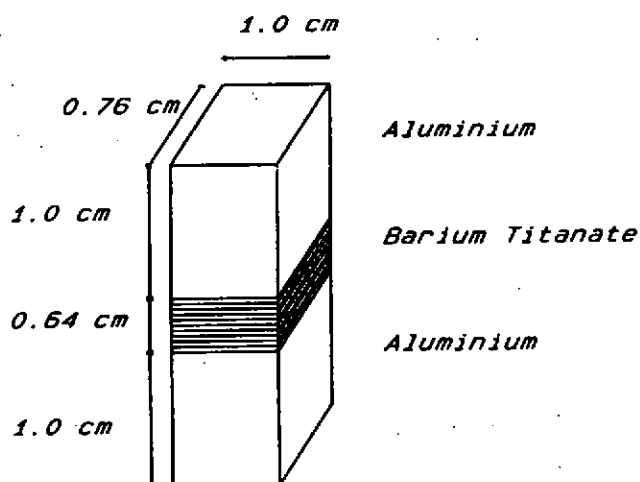


Fig 12

Input Source Level each Primary Frequency

approximately 208 dB re 1 micro-Pascal

Range of measurement 28 metres

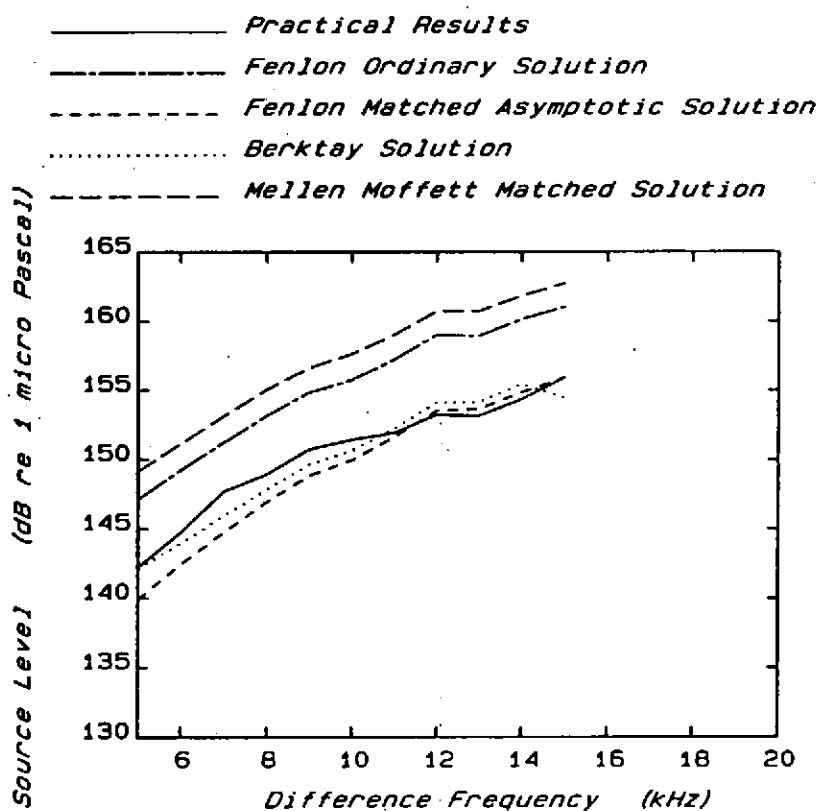


Fig 13

Input Source Level each Primary Frequency
approximately 205 dB re 1 micro-Pascal
Range of measurement 28 metres

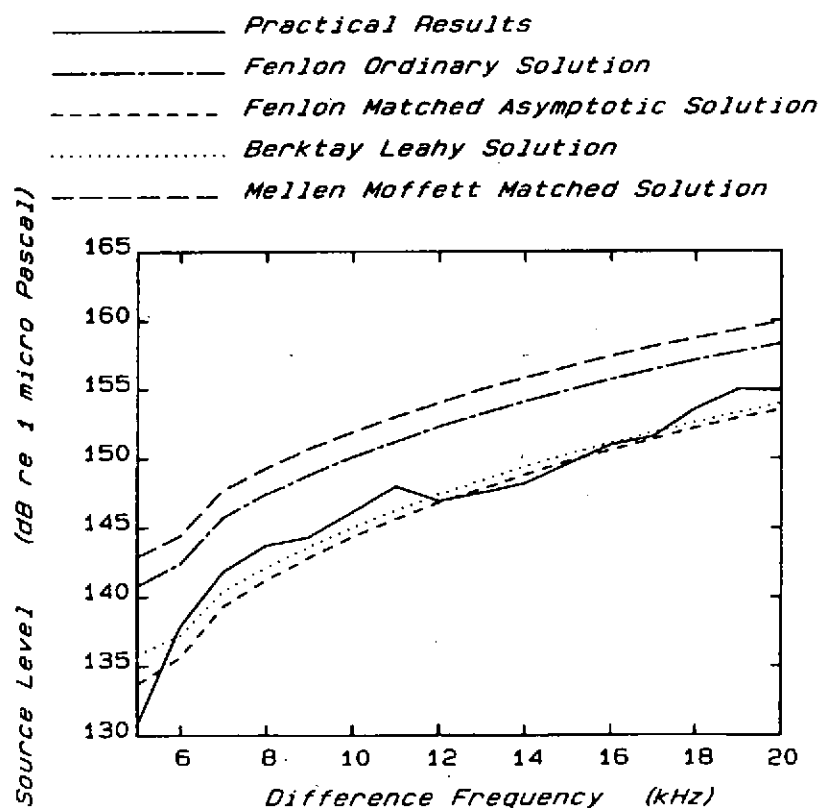
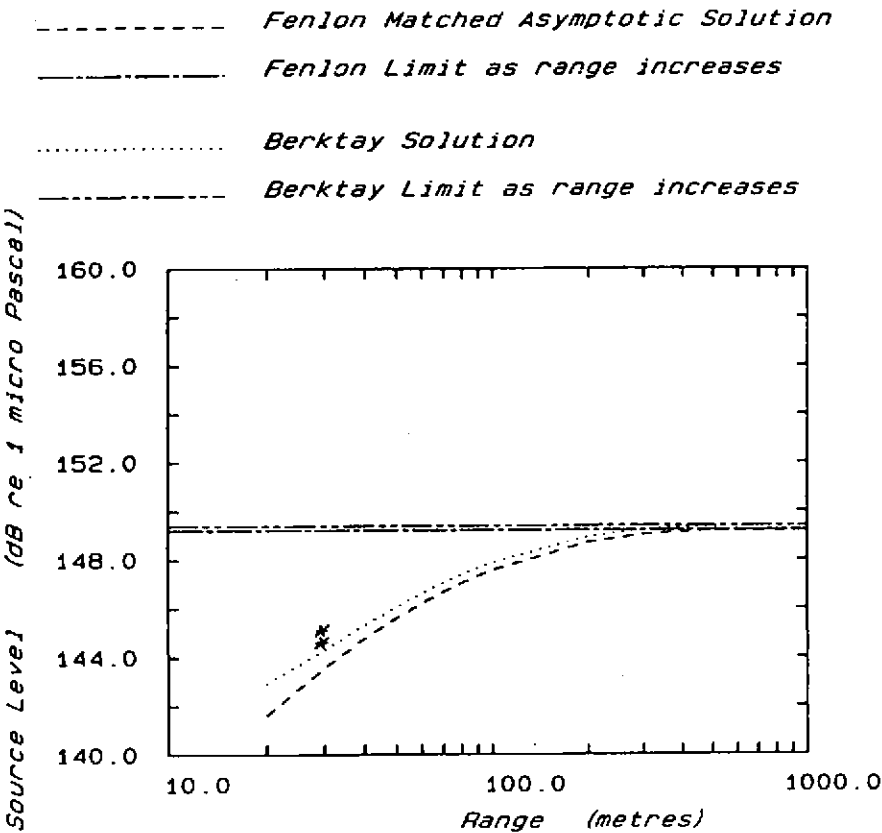
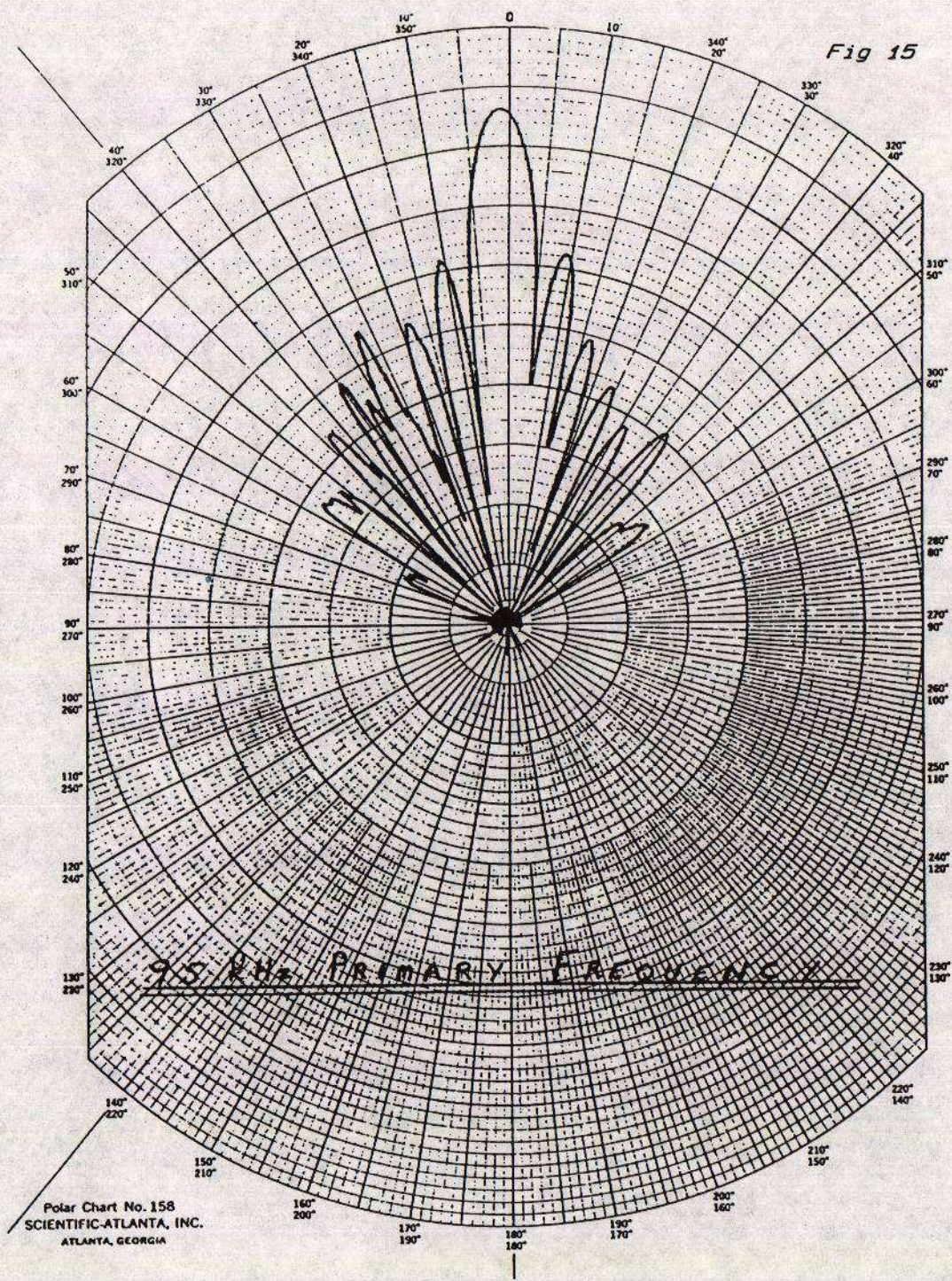


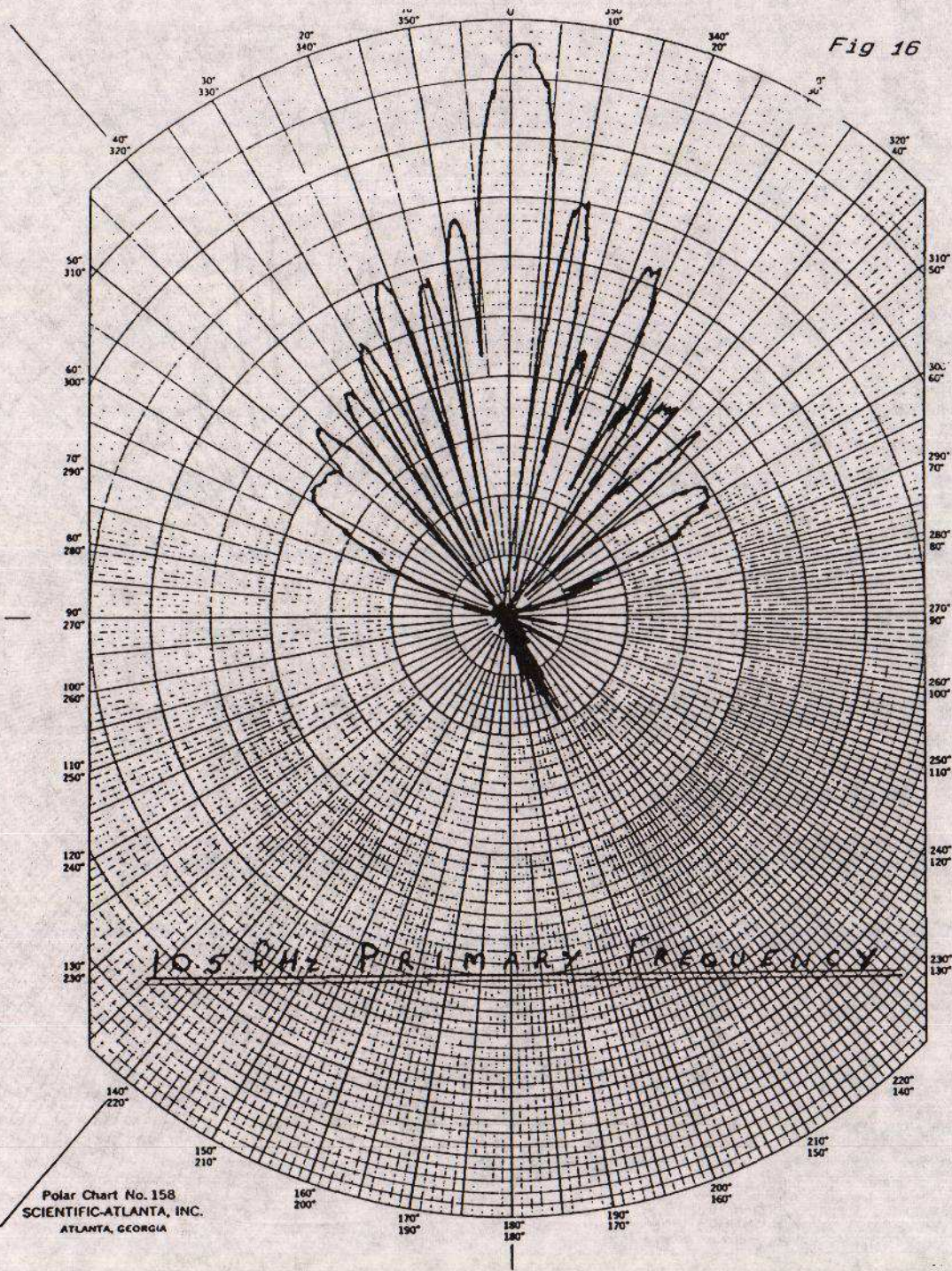
Fig 14

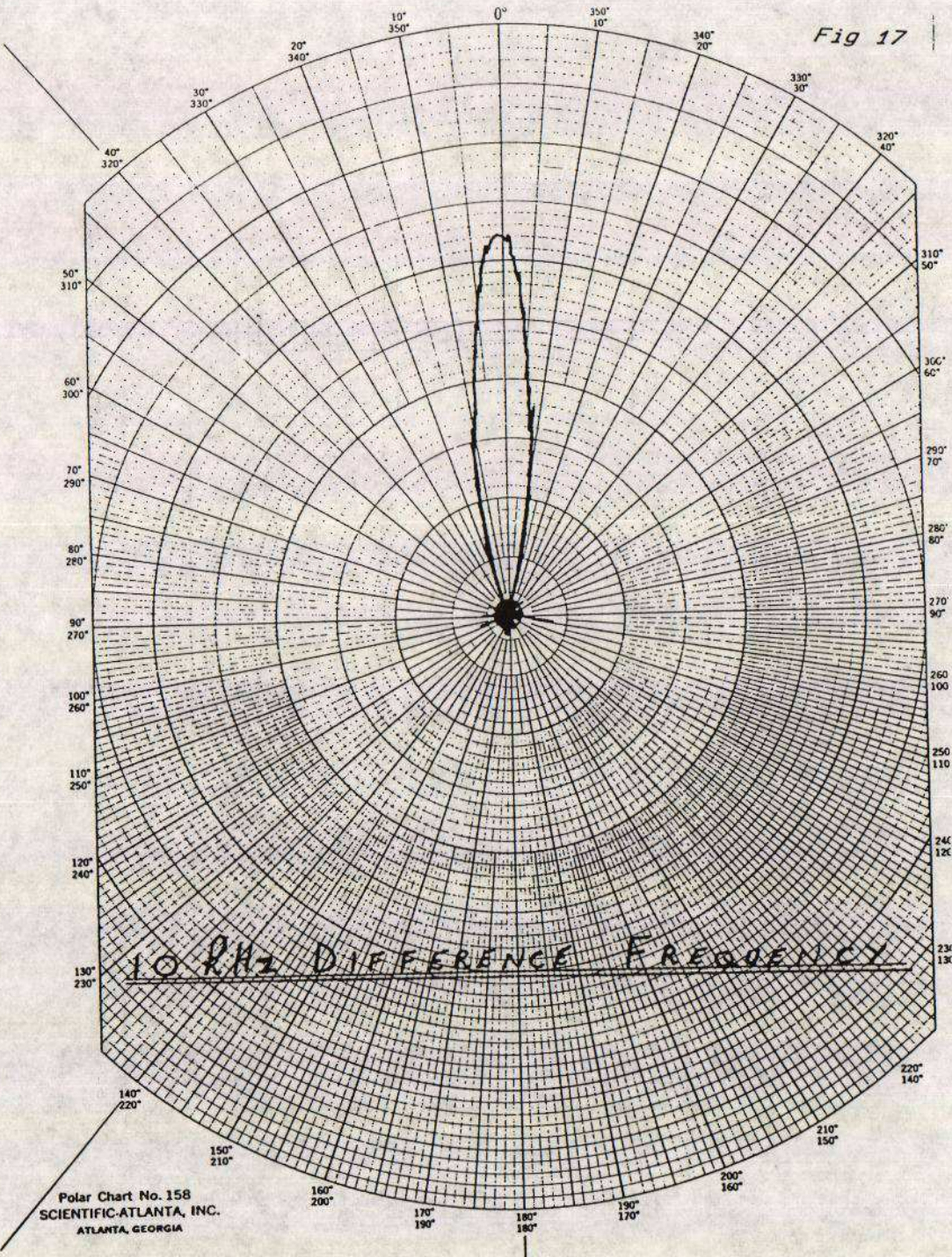
Primary Frequency Source Levels Approximately
205 dB re 1 micro-Pascal for a 9 kHz Signal

* Practical Measurement of 9 kHz
Difference Frequency at a range
of 27.73 metres









DEMONSTRATOR PARAMETRIC ARRAY MEASUREMENT SYSTEM Fig 18

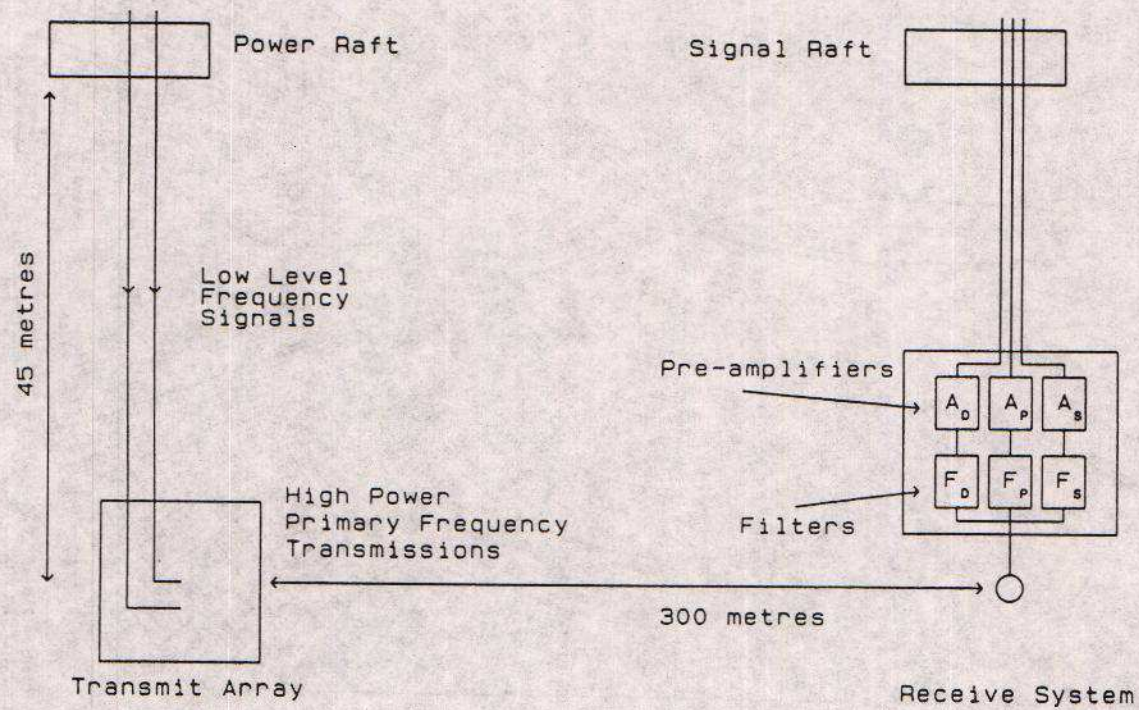


Fig 19

*DPAS SETUP FOR 3 KHz TRANSMISSION
at Loch Goil December 1987*

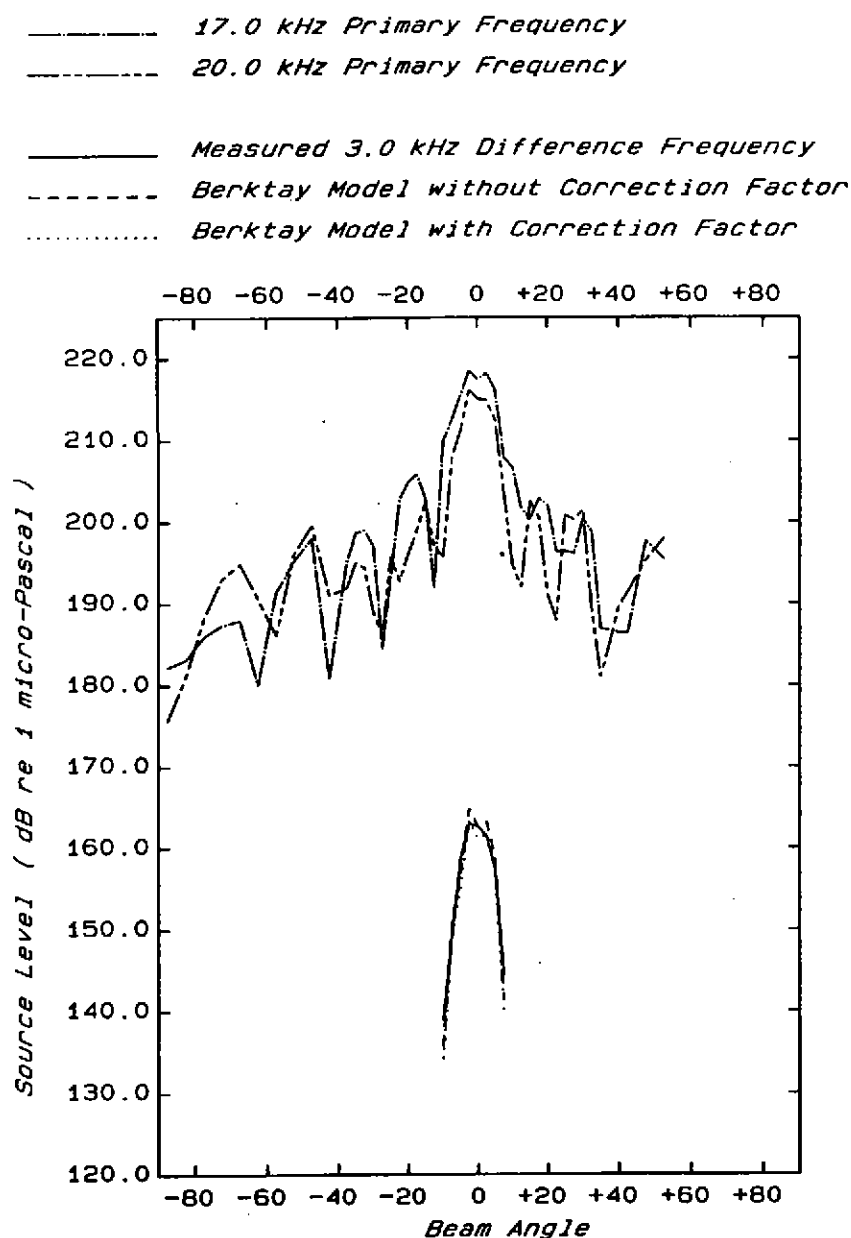


Fig 20

DPAS SETUP FOR 3 kHz TRANSMISSION
at Loch Goil April / May 1988
Electronic Steering 0 Degrees

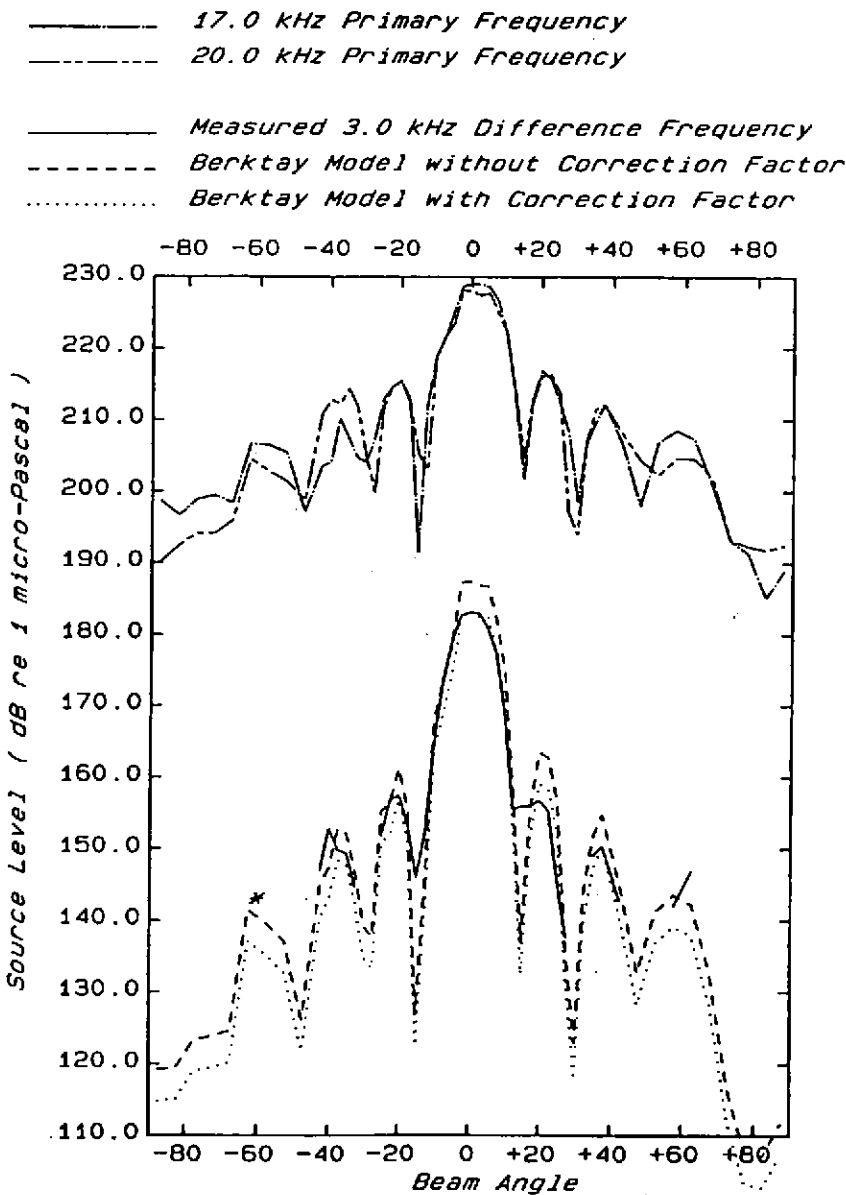


Fig 21

DPAS SETUP FOR 3 kHz TRANSMISSION
at Loch Goil April / May 1988
Electronic Steering 0 Degrees

————	17.0 kHz Primary Frequency	
-----	20.0 kHz Primary Frequency	
Measured		Theoretical
————	37 kHz Sum Frequency

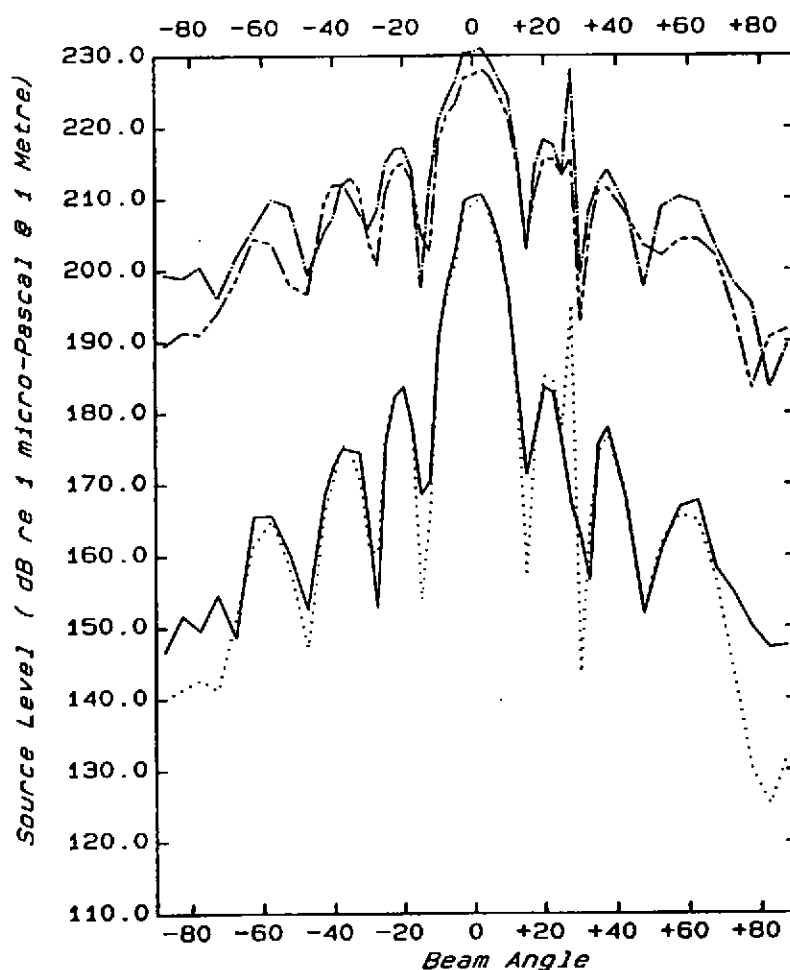


Fig 22

DPAS SETUP FOR 3 KHz TRANSMISSION
at Loch Goil April / May 1988
Electronic Steering 0 Degrees

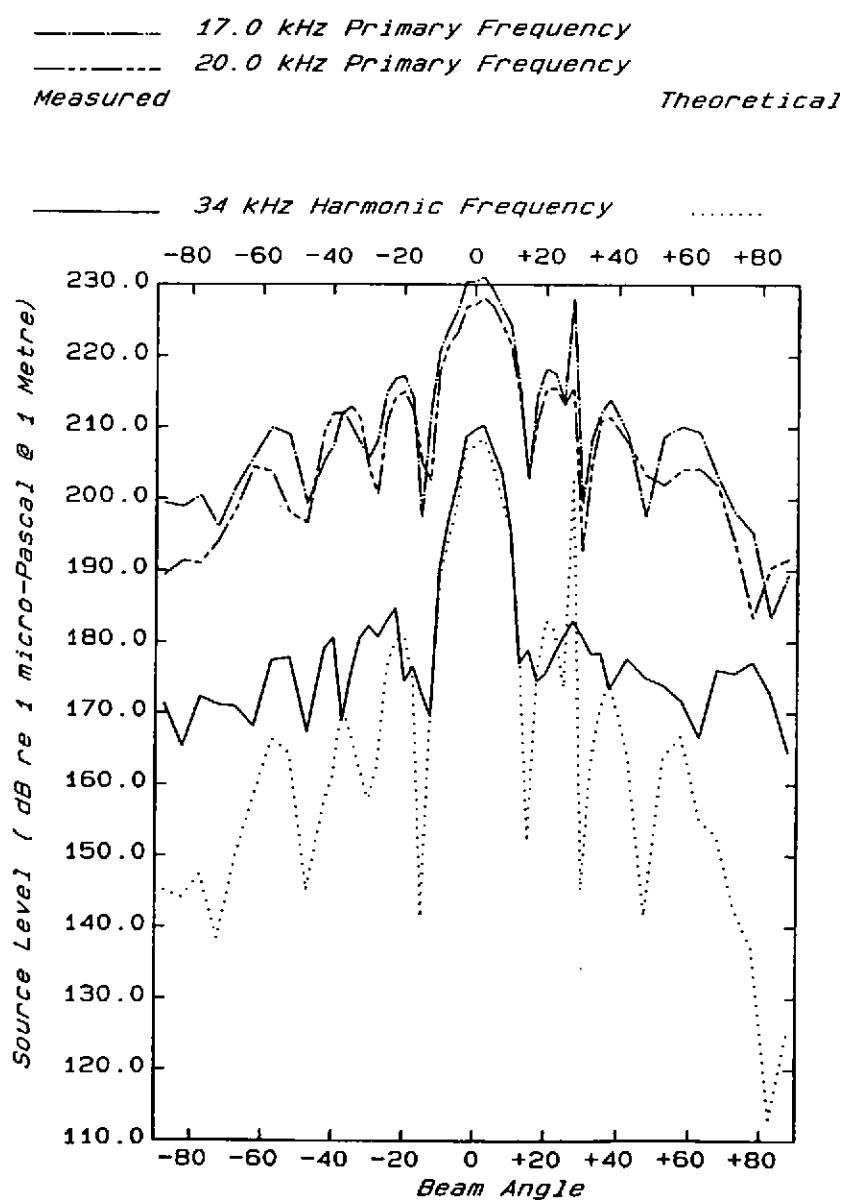


Fig 23

*DPAS SETUP FOR 3 kHz TRANSMISSION
at Loch Goil April / May 1988
Electronic Steering 0 Degrees*

——— 17.0 kHz Primary Frequency
 - - - - - 20.0 kHz Primary Frequency
 Measured Theoretical
 ——— 40 kHz Harmonic Frequency

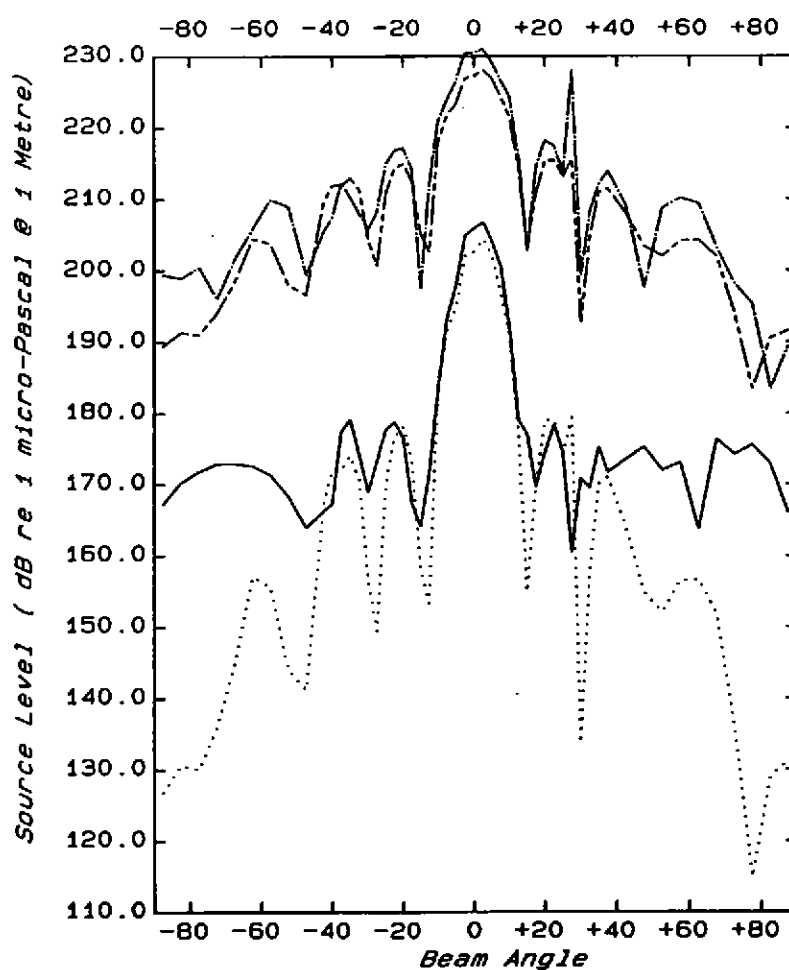


Fig 24
SUMMARY OF RESULTS OF DPAS TRIALS
April / May 1988
Beam Angle 0 Degrees

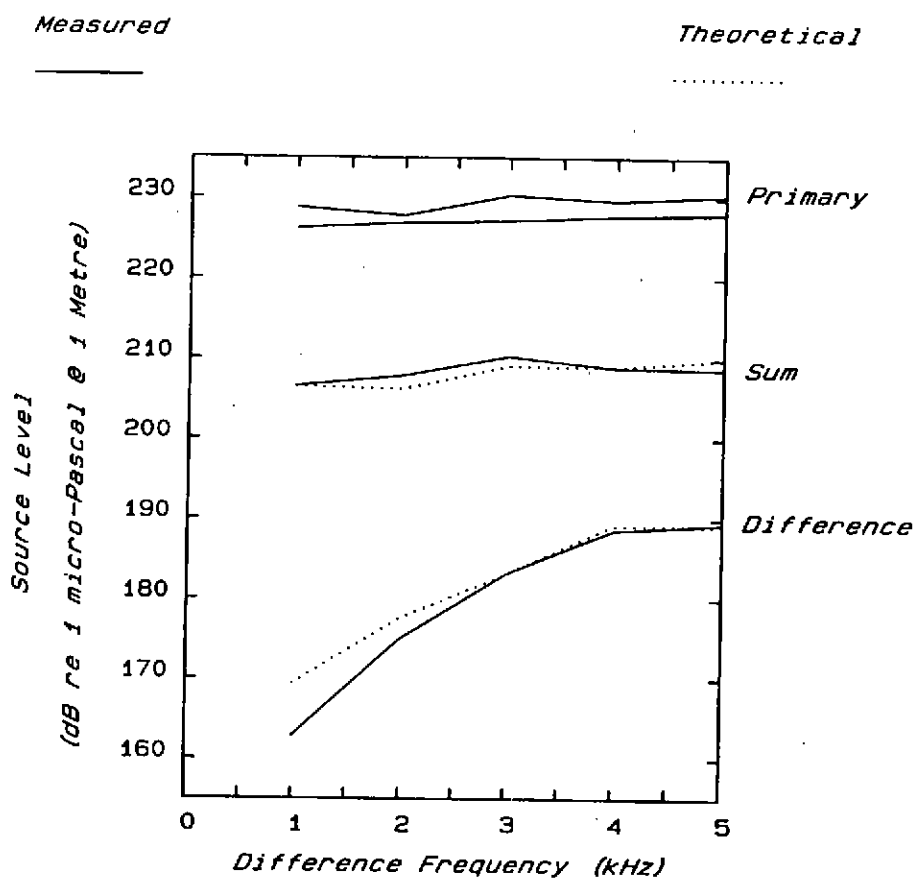


Fig 25

*DPAS SUM FREQUENCY TESTS
on board R.D.V. Crystal January 1990*

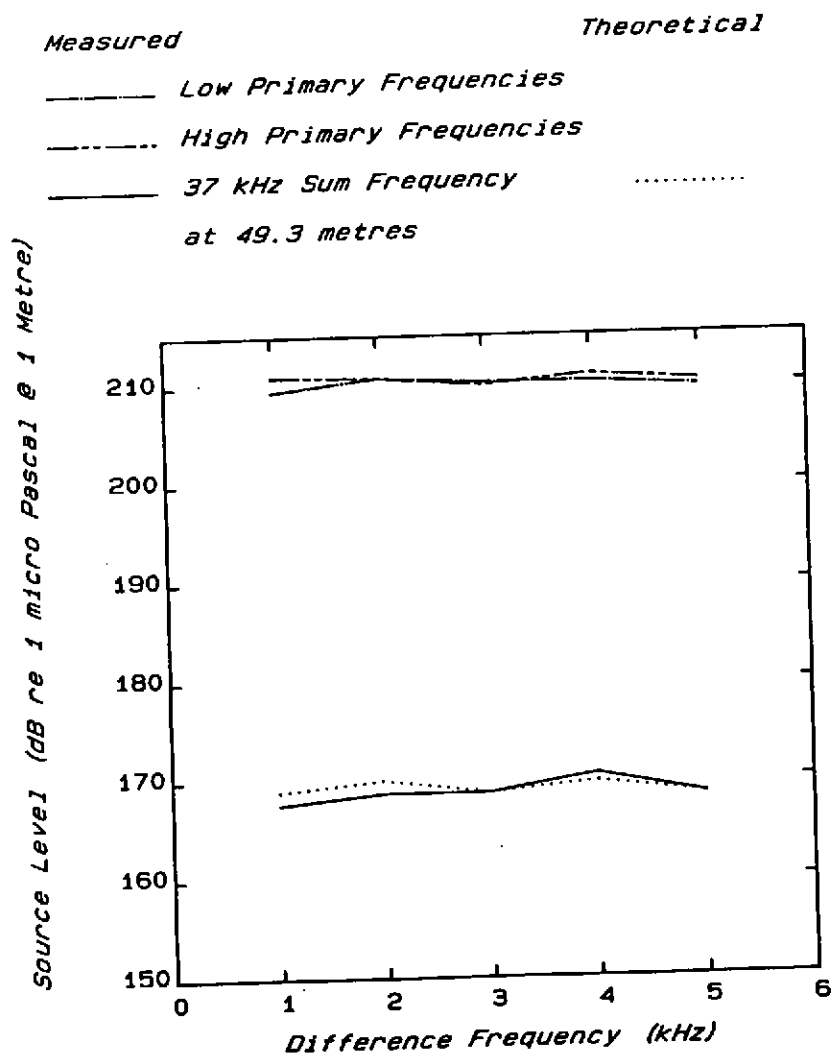


Fig 26

*DPAS SETUP FOR 1 kHz TRANSMISSION
at Loch Goil April / May 1988
Electronic Steering 20 Degrees*

- 18.0 kHz Primary Frequency
- 19.0 kHz Primary Frequency
- 1.0 kHz Difference Frequency
- 37 kHz Sum Frequency
- 38 kHz Harmonic Frequency
- 36 kHz Harmonic Frequency

