NARROWBAND PULSE PROPAGATION PROBLEM IN A DEEP RANDOM OCEAN

A.G.Sazontov(1) & V.A.Farfel'(1)

(1) Institute of Applied Physics, 46 Ul'yanov Street, 603600 Nizhny Novgorod, Russia

1. INTRODUCTION

A problem of interest in underwater acoustics is the pulse propagation through a randomly inhomogeneous waveguide. From theoretical point of view the analysis of this problem reduces to evaluating the mutual coherence function (MCF) of frequency (see, e.q.[1]). Previous research examing the behaviour of the MCF in a fluctuating ocean has been carried out in a ray-oriented approach using the path-integral formalism [2].

This report investigates the MCF as a function of separations in time, space and frequency in a random ocean in the framework of coupled mode theory which is more suitable for the important case of low-frequency and long-range propagation. We formulated the matrix transport equation for MCF in terms of the modal structure of a sound pressure field and presented useful approximate analytical solution of this equation in unsaturated region. The obtained expression for MCF has been then applied to the derivation of the mean narrowband pulse shape at the output of the matched filter. The accuracy limit of measurements in the arrival time of an acoustic pulse caused by medium fluctuations is estimated as well.

2. PROBLEM FORMULATION

Consider two types of oceanic waveguides: the deep sound channel and the upper-sound channel. Assume that in the first case the sound scattering is caused mainly by volume fluctuations in the index of refraction $\mu(\vec{r}, z, t)$, where $\vec{r} = (x, y)$ is the horizontal two-dimensional position vector, z is the vertical coordinate and t is the time. In the second channel the sound scattering occurs on the statistically rough and acoustically soft boundary $z = \zeta(\vec{r}, t)$. Both μ and ζ are assumed to be random, zero-mean Gaussian fields, and are characterized by the space - time correlation functions $B_\mu(\vec{\rho}, z_1, z_2, \tau)$ and $B_\zeta(\vec{\rho}, \tau)$.

Let there be a nondirectional acoustic source located at coordinates $(0, z_0)$ and emitting a narrowband signal of the form: $g(t) = e^{-i\omega_0 t} S(t)$. Acoustic radiation transmitted through a medium with random fluctuations is registered by correlation receiver (matched filter) located at coordinates $(\vec{r}, z)$. In the subsequent analysis we shall assume that the oceanic inhomogeneities are large-scale, the Rayleigh roughness parameter is small and the characteristic frequencies of the spectra B_μ and B_ζ are small compared with carrier frequency ω_0. Throughout the report all numerical calculations of the primary correlation quantities of a pulsed signal will be performed for summer and winter sound – speed profiles from the North-West Pacific at latitude 45° N (Fig.1) and assuming the Garrett–Munk (GM) spectrum for B_μ and the Pierson–Moskowitz spectrum for B_ζ.

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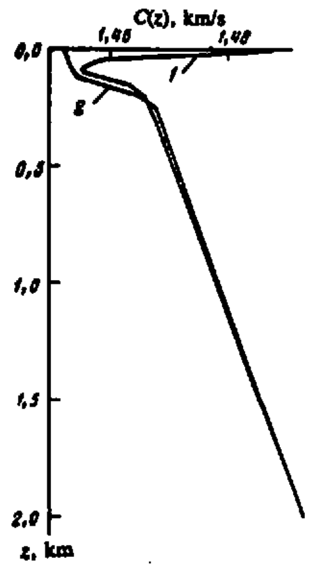

Fig.1 Upper parts of the sound–speed profiles from the North-West Pacific. The profiles are: summer (curve 1) and winter (curve 2).

The acoustic pressure field in a random oceanic waveguide $p(\vec{r},z,t)$ can be formally represented by

$$p(\vec{r},z,t)=\int_{-\infty}^{\infty}d\omega\, e^{-i\omega t}g(\omega)\sum_{n=1}^{M(\omega)}p_n(\vec{r},\omega,t)\,\varphi_n(z,\omega) \tag{1}$$

Here $g(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\, e^{i\omega t}g(t)$ is the frequency spectrum of the transmitted pulse, $\varphi_n(z,\omega)$ denotes the n-th vertical eigenfunction of a regular channel associated with the eigenvalue $\kappa_n^2(\omega)$; M is the number of propagating modes. The modal amplitudes $p_n(\vec{r},\omega,t)$ are governed by the set of coupled wave equations

$$(\Delta_\perp+\kappa_n^2)p_n=\varphi_n(z_0)\,\delta(\vec{r})\quad-\sum_{n=1}^{M(\omega)}V_{nm}(\vec{r},\omega,t)\,p_m \tag{2}$$

where $V_{nm}(\vec{r},\omega,t)$ is the matrix coupling coefficient (depending on t as a parameter) defined according to:

$$V_{nm}(\vec{r},\omega,t)=\begin{cases}2k^2\mu_{nm}(\vec{r},\omega,t) \text{ - for volume inhomogeneities;}\\ \frac{d\varphi_n(0,\omega)}{dz}\frac{d\varphi_m(0,\omega)}{dz}\zeta(\vec{r},\omega,t) \text{ - for irregular surface.}\end{cases}$$

Here μ_{nm} is given by: $\mu_{nm}=\int_0^H dz\; n_0(z)\mu(\vec{r},z,\omega,t)\varphi_n(z,\omega)\varphi_m(z,\omega)$, where H is the ocean depth, $n_0(z)$ is the regular part of the refractive index.

The important correlation properties of a pulsed wave that has transversed a random medium are described by the second moment of the acoustic pressure field

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$$B_p(\vec{r}_1, z_1, t_1;\ \vec{r}_2, z_2, t_2) = < p(\vec{r}_1, z_1, t_1)\ p^*(\vec{r}_2, z_2, t_2) > .$$

The brackets $< . >$ denote averaging over the ensemble of random μ or ζ fields. In multimode waveguide by the use of the representation (1) we have for B_p

$$B_p(\vec{r}_1, z_1, t_1;\ \vec{r}_2, z_2, t_2) = \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\infty} d\omega_2\ g(\omega_1) g^*(\omega_2)\ \Gamma\ exp(-i\omega_1 t_1 + i\omega_2 t_2), \quad (3)$$

where $\Gamma(\vec{r}_1, z_1, t_1, \omega_1;\ \vec{r}_2, z_2, t_2, \omega_2)$ is the two-frequency coherence function defined as

$$\Gamma(\vec{r}_1, z_1, t_1, \omega_1;\ \vec{r}_2, z_2, t_2, \omega_2) = \sum_{n,m} \Gamma_{nm}(1,2) \varphi_n(z_1, \omega_1) \varphi_m(z_2, \omega_2), \quad (4)$$

$$\Gamma_{nm}(1,2) = < p_n(1)\ p_m^*(2) > .$$

The labels 1 and 2 refer to two different horizontal position points, times and frequencies. Thus, the problem of finding a result for B_p in a waveguide channel now reduces to evaluating the two-frequency cross-modal mutual coherence functions Γ_{nm} .

3. BEHAVIOUR OF THE TWO-FREQUENCY COHERENCE FUNCTIONS IN A RANDOM OCEANIC WAVEGUIDE

From the original stochastic Eqs.(2) the conventional transport equation for the cross-modal coherence functions Γ_{nm} taken at two horizontal position points $\vec{r}_1 = (x, y_1)$, $\vec{r}_2 = (x, y_2)$ in the same x plane, two different times and frequencies can be derived with the forward-scattering approximation and has the form [3]:

$$\left[\frac{\partial}{\partial x} + i\big(\kappa_n(\omega_1) - \kappa_m(\omega_2)\big) - i\xi_{nm}^{+} \frac{\partial^2}{\partial\rho\partial R} - \frac{i}{2}\xi_{nm}^{-}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{4}\frac{\partial^2}{\partial R^2}\right)\right]\Gamma_{nm}(1,2) =$$

$$= -\frac{1}{2}\big(\sigma_n(\omega_1) + \sigma_m(\omega_2)\big)\Gamma_{nm}(1,2) + {\sum_{n',m'}}' A_{nn'}^{mm'}(\rho, \tau;\ \omega_1, \omega_2)\Gamma_{n'm'}(1,2). \quad (5).$$

In writing (5) the following notations are used:

$$\rho = y_1 - y_2,\ R = \frac{1}{2}(y_1 + y_2),\ \tau = t_1 - t_2;$$

$$\xi_{nm}^{+} = (\kappa_n^{-1}(\omega_1) + \kappa_m^{-1}(\omega_2)),\ \xi_{nm}^{-} = (\kappa_n^{-1}(\omega_1) - \kappa_m^{-1}(\omega_2));$$

$$A_{nn'}^{mm'}(\rho, \tau;\ \omega_1, \omega_2) = \int_{-\infty}^{\infty} dx < V_{nn'}(0, 0, \omega_1, 0) V_{mm'}(x, \rho, \omega_2, \tau) > e^{i(\kappa_{nm}^{+} - \kappa_{n'm'}^{+})x},$$

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$$\kappa_{nm}^{+} = \frac{1}{2}\left(\kappa_n(\omega_1) + \kappa_m(\omega_2)\right);\ \sigma_n(\omega) = \sum_{n=1}^{M} A_{nm}^{nm}(\rho = 0,\ \tau = 0,\ \omega, \omega).$$

The symbol $\sum'_{n',m'}$ means the summations over all couples of modes satisfying the synchronism condition: $\kappa_{n'}(\omega_1) - \kappa_{m'}(\omega_2) = \kappa_n(\omega_1) - \kappa_m(\omega_2)$.
The system (5) is justified if

$$\kappa_n l_\perp >> 1,\ x >> \rho,\ \sigma_n << max\ \{\Lambda_n^{-1},\ l_\perp^{-1}\},$$

where $l_\perp$ is the characteristic horizontal correlation length, $\Lambda_n = 2\pi/(\kappa_{n+1} - \kappa_n)$ is the cycle length.

In order to describe the behaviour of the second moments the folloing two cases must be distinguished: $n \neq m$ and $n = m$. For waveguides having nonequidistant spectrum of the wavenumbers $\kappa_n(\omega)$, the contribution to the double sum in the right-hand side of (5) in the case $n \neq m$ gives from the terms $n' = n,\ m' = m$, and by using a method similar to the Rytov approximation one can show that

$$\Gamma_{nm}(1,2) = \frac{\varphi_n(z_0,\omega_1)\,\varphi_m(z_0,\omega_2)}{8\pi x \kappa_n(\omega_1)\kappa_m(\omega_2)} exp\Big\{i(\kappa_n(\omega_1) - \kappa_m(\omega_2))x + i\kappa_{nm}^{+}\frac{\rho R}{x} - D_{nm}(1,2)\Big\},$$

where

$$D_{nm}(1,2) = \frac{1}{2}(\sigma_n(\omega_1) + \sigma_m(\omega_2))x -$$

$$- \int_0^x dx' \int_{-\infty}^{\infty} d\kappa_y\ \overline{A}_{nn}^{mm}(\kappa_y, \tau, \omega_1, \omega_2)\ exp\Big\{i\xi_{nm}^{-}\kappa_y^2 \frac{x'(x - x')}{2x} + i\kappa_y \frac{x'}{x}\rho\Big\}$$

and

$$\overline{A}_{nn}^{mm}(\kappa_y, \tau, \omega_1, \omega_2) = \int_{-\infty}^{\infty} d\rho\ e^{-i\kappa_y \rho} A_{nn}^{mm}(\kappa_y, \tau, \omega_1, \omega_2), \qquad n \neq m.$$

Applied to a narrowband tramsmitted pulse, a good approximation for the diagonal elements of the Γ_{nn} playing the dominant role at long ranges is given by [3]:

$$\Gamma_{nn}(1,2) = \frac{1}{8\pi x} \sum_m \kappa_m^{-1} \varphi_m^2(z_0)\ exp\Big\{i\Omega t_n + i\kappa_n \frac{\rho R}{x} - \frac{1}{2}\Omega^2 < \tau_n^2 >\Big\} H_{nm}(1,2).$$

Here $\Omega = \omega_1 - \omega_2,\ t_n = (x/V_n)(1 + R^2/2x)$, where V_n is the group velocity of mode n, $< \tau_n^2 >= \frac{1}{4} d^2\sigma_n(\omega_0)/d\omega_0^2$ is its travel-time variance and the H_{nm}-factor can be computed from

$$H_{nm}(1,2) = \int_0^1 d\alpha\ exp\{-2\pi i(n-m)\alpha - \Psi_{m\alpha}(1,2)\},$$

where $\Psi_{m\alpha}(1,2) =$

$$= \sigma_m x - \sum_{n'} \int_0^x dx' \overline{A}_{mn'}^{mn'}(\kappa_y, \tau) e^{2\pi i(n'-m)\alpha} exp\Big[\frac{i\kappa_y^2 \Omega x'(x-x')}{2\kappa_m^2 x V_m} + iV_{nm}^- x' + i\kappa_y \frac{x'}{x}\rho\Big],$$

and $V_{nm}^- = V_m^{-1} - V_n^{-1}$, $\overline{A}_{nm}^{nm}(\kappa_y,\tau) = \int_{-\infty}^{\infty} \frac{d\rho}{2\pi} A_{nm}^{nm}(\rho,\tau,\omega_0,\omega_0)\ e^{-i\kappa_y\rho}$.
As an example in Fig.2a we present the numerical results for the normalized magnitude of Γ_{nn} for several modes as a function of frequency separation Ω. The calculations have been carried out for winter sound-speed profile of Fig.1 at $f_0 = \omega_0/2\pi = 250\ Hz$, $z_0 = 100\ m$, $z = 300\ m$, $x = 500\ km$ assuming that the main source of signal fluctuations is fully developed wind waves. The wind speed V was taken $13\ m/s$. The coherent bandwidth for separate mode is typically about $2-3\ Hz$.

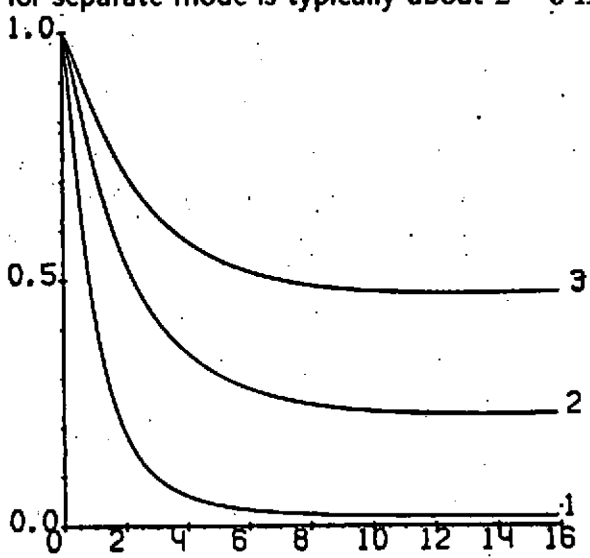

Fig.2a Normalized MCF of frequency for winter sound-speed profile of Fig.1 at different modes: 1 - n=7; 2 - n=12; 3 - n=300.

The total MCF of frequency (4) demonstrates a quite different behaviour (Fig.2b). It is seen that the coherent bandwidth is defined by effects of the deterministic multipath and in the case considered has an order of magnitude of 0.3 Hz.

4. MATCHED FILTERING OF A SCATTERED SIGNAL

The matched filtering is one of the widespread procedures of signal processing. The ambiguity function formed at the output of a receiver system:

$$F(\tau,\Omega_D) = \frac{1}{2\pi} \,|\int_{-\infty}^{\infty} p(\vec{r},z,t)\ S^*(t-\tau)\ e^{-i\Omega_D t} dt\,|, \tag{6}$$

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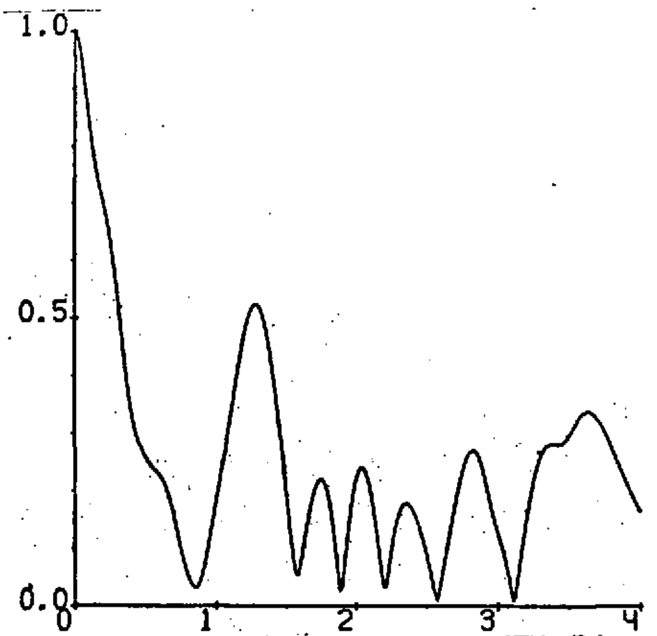

Fig.2b Normalized total MCF of frequency

where τ is the time delay and Ω_D is the Doppler shift, is then employed to determine the source location.

We will first be interested in mean square power output $< F^2(\tau, \Omega_D) >$. It is straightforward to show that for narrowband transmitted pulse the value $< F^2(\tau, \Omega_D) >$ is related to the cross–modal MCF of frequency in the following way:

$$< F^2(\tau, \Omega_D) > = \sum_{n,m} \int_{-\infty}^{\infty} \frac{d\tau'}{2\pi} \int_{-\infty}^{\infty} d\Omega \; e^{-i\Omega(t-t_{nm}^{+})-i\Omega_D \tau'} \overline{\Gamma}_{nm}(x, \rho = 0, R, \tau', \Omega) *$$

$$* \chi_i(\Omega, \tau' - t_{nm}^{-}) \chi_i^*(\Omega, \tau'),$$

where $\chi_i(\Omega, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \; S(t + \tau/2) S^*(t - \tau/2) \; e^{i\Omega t}$ is the ambiguity function of the input signal, $t_{nm}^{+} = \frac{1}{2}(t_n + t_m)$, $t_{nm}^{-} = t_n - t_m$, and $\Gamma_{nm}(1,2)$ is connected with $\overline{\Gamma}_{nm}(1,2)$ according to: $\Gamma_{nm}(1,2) = \overline{\Gamma}_{nm}(1,2) \; e^{i\omega_1 t_n - i\omega_2 t_m}$.

In many acoustic experiments dealing with a fixed source and a fixed receiver the main subject of interest is the function $E^2(\tau) =< F^2(\tau, 0) >$ which describes the behaviour of the ensemble – averaged pulse. Figure 3a shows the distribution of the amplitude $E(\tau)$ (normalized to the value E_0, defined by $E_0^2 = (2\pi)^{-1} \int \int_{-\infty}^{\infty} d\tau d\Omega \; F_0^2(\tau, \Omega)$, where $F_0^2(\tau, \Omega)$ is the processing output in a regular waveguide) versus time delay for the rectangular transmitted pulse $S(t) = T^{-1/2} rect(t/T)$ having the pulse width $T = 50 \; ms$. The

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theoretical curve in Fig.3a has used calculations of the two–frequency coherence function for the example given in Fig.2. For comparison in Fig.3b we plot $F_0(\tau,0)/E_0$ for the same sound–speed profile in absense of random surface irregularities.

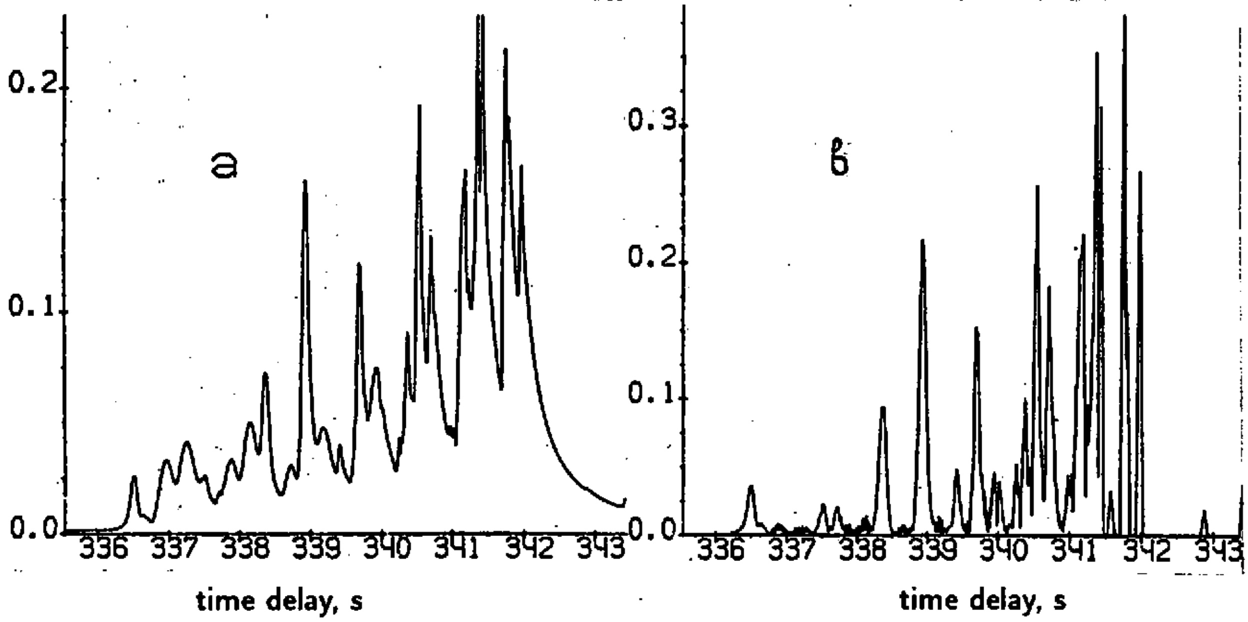

Fig.3 Normalized signal amplitude at the matched filter output versus time delay in the presence (a) and absence (b) of sea–surface scattering.

Consider now the energy loss of matched filtering arising from time and frequency decorrelation of the registered signal. The corresponding power gain reduction is defined by:

$$\delta = -10\ lg \left\{ \frac{max \langle F^2(\tau,\Omega_D)\rangle}{max F_0^2(\tau,\Omega_D)} \right\},\ dB.$$

Figure 4 illustrates the gain reduction as a function of pulse duration T. Numerical calculations were performed for summer and winter sound–speed profiles from Fig.1 at $f_0 = 250\ Hz$, $z_0 = 100\ m$, $z = 300\ m$, $x = 500\ km$ under the assumption that the internal–wave effects dominate in summer conditions (curve 1) and the surface interactions play predominant role for winter profile (curve 2). Wind speed was taken $V = 10\ m/s$ (a) and $V = 13\ m/s$ (b).

5. RANDOM MEASUREMENT ERRORS IN THE ARRIVAL TIME OF AN ACOUSTIC PULSE

In this section we consider the measurement errors in the arrival time of a pulse arising from sound scattering effects assuming that the multipath peaks in the correlation integral (6) are nonresolvable. The corresponding pulse arrival time may be defined to be the centroid:

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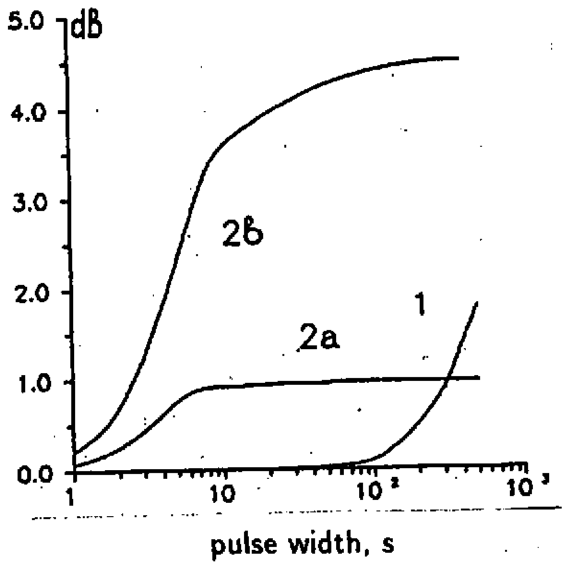

Fig.4 Energy loss of matched filtering versus pulse width. The calculations are for summer profile (curve 1) and for winter profile (curve 2). Wind speed was taken $V = 10\ m/s$ (a) and $V = 13\ m/s$ (b).

$$\tau_{mes} = \frac{\int\int_{-\infty}^{\infty} \tau\, F^2(\tau,\Omega_D)\, d\tau d\Omega_D}{\int\int_{-\infty}^{\infty} F^2(\tau,\Omega_D)\, d\tau d\Omega_D}.$$

This estimate fluctuates due to ocean processes and the statistic of interest is the expected value of the travel - time variance defined as $\sigma_\tau^2 = <\tau_{mes}^2> - <\tau_{mes}>^2$. The useful form for the rms travel time can be obtained in the single–scattering region. In the case of the rectangular pulse having the width much greater than the time delay differences (i.e. $T >> max|t_n - t_m|$) the result is [4]:

$$\sigma_\tau^2 = \frac{1}{2} x \sum_{n,m} \rho_{nm} \left[B^{nm}_{\omega_0\omega_0}(0) + B^{nm}_{\omega_0\omega_0}(T) \right],$$

where $\rho_{nm} = (\kappa_n \kappa_m)^{-1} \varphi_n^2(z) \varphi_m^2(z_0) / \sum_n \kappa_n^{-2} \varphi_n^2(z) \varphi_n^2(z_0)$, and

$$B^{nm}_{\omega_0\omega_0}(\tau) = \frac{\pi}{2\kappa_n(\omega_0)\kappa_m(\omega_0)} \left\langle \frac{\partial V_{nm}(\vec{r},\omega_0,t)}{\partial \omega_0} \frac{\partial V^*_{nm}(\vec{r},\omega_0,t+\tau)}{\partial \omega_0} \right\rangle.$$

For the summer profile from the North–West Pacific and GM–spectrum the numerical estimation of σ_τ at $f_0 = 250\ Hz$, $z_0 = 100\ m$, $z = 300\ m$, $T = 100\ s$ has an order of magnitude of $0.2\sqrt{x}\ ms$ (x is the range in km). For the winter profile and the Pierson–Moskowitz spectrum for wind speed of $V = 10\ m/s$ the value of σ_τ is of order $10^{-2}\sqrt{x}\ ms$ where x is the range in km.

6. REFERENCES

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