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THE USE OF RUBBER NOISE-STOP PADS IN VIBRATION ISOLATION SYSTEMS

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1. INTRODUCTION

Low levels of transmissibility demand soft vibration isolation springs, and yet soft springs are liable to "wave effect" resonances within the range of acoustically significant frequencies. Such resonances result in poor attenuation, or even amplification, of the higher frequency excitations. One solution to the problem is to use a multiple stage isolation system.

Motor cars provide a familiar example of a multiple-stage vibration isolation system [1]. Pneumatic tyres provide the first stage. Tyres have their lowest radial resonance at a frequency of about 80Hz and this frequency is dominant in noise transmission from the road. The suspension springs provide the next stage, and their stiffness is typically a third to a fifth of that of the tyres. The frequency of their first resonance may be less than 300Hz. A final stage in the isolation of vertical vibrations is provided by rubber bushings and pads. These also provide a greater contribution to the attenuation of lateral and longitudinal vibrations than the suspension system itself. About 80% of that sound energy in the interior of a car which originates from tyre-road interactions is structure-borne (the remainder being airborne).

Buildings provide another example of the use of multiple stage vibration isolation systems to limit the problem of structure-borne sound. Sources of vibration are isolated, for example motors by antivibration mounts and footfall by carpets. Transmission of vibrations that are excited in the structure can be reduced by using elastic elements between the structural elements. For example, floors may be resiliently mounted [2] and layers of cork or rubber may be used between relatively rigid structural elements such as brick walls [3].

A particular example of a two-stage isolation system is the use of a rubber "noise-stop" pad in series with a coil spring mount. It is common practice to use such a pad when coil springs are used either as car suspensions or as antivibration mounts for installations in buildings. The object of this paper is to discuss this system in detail. Preliminary work on this topic has already been presented, but it was not possible to draw clear conclusions as there was a puzzling discrepancey between theory and experiment [4]. The origin of this discrepancy has been identified, and the theory can now be applied in confidence to a wider range of systems than those studied experimentally.

2. THEORY

For simplicity, attention will be restricted to translational motion in one direction. A further simplification is that flexural waves will not be discussed. In the previous paper [4] the steady-state response of a mass-spring-mass-spring system (Figure 1) to a sinusoidal excitation was derived, including the effects of the damping and inertia of the springs. The treatment is applicable to either shear or longitudinal motion, and consists of solving the wave equation in a dissipative medium with appropriate boundary conditions. The vibrational displacement $u(x,t)$ may be expressed as a sum of waves travelling in the positive x direction (u_+) and in the negative x direction (u_-):

$$u(x,t) = Au_+(x,t) + Bu_-(x,t) = Ae^{j(\omega t-nx)} + Be^{j(\omega t+nx)} \quad (1)$$

In these expressions, n is the complex wave number given by:

$$n = \omega\sqrt{\rho_L/k_L^*} \quad (2)$$

where ρ_L is the mass per unit length of spring and $k_L^* = k_L' + jk_L''$ is the complex stiffness of unit length of the spring. If the loss angle $\delta = \tan^{-1}(k''_L/k'_L)$ of the material is small we may write:

$$n \simeq \frac{\omega}{c}\,(1-j\delta/2) \quad (3)$$

where c is the wave speed given by $c^2 = k_L'/\rho_L$.

It is apparent from equations (1) and (3) that the amplitude of each wave is reduced exponentially in the direction in which it travels. It follows from equations (1) and (2) that the natural logarithm of the ratio of the amplitudes at positions separated by one wavelength is given by:

$$\Lambda = 2\pi\tan(\delta/2) \approx \pi\delta \quad (4)$$

To the small loss angle approximation, Λ is equal to the logarithmic decrement of the material subjected to free oscillation [5].

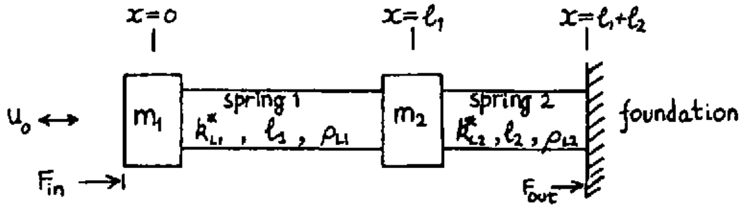

Figure 1 Mass-spring-mass-spring isolation system

The force exerted by the spring on the surface at x is given by:

$$F(x,t) = -k_L^*(\partial u/\partial x) \qquad (5)$$

Using appropriate boundary conditions, equation (1) leads to an expression for the transmissibility T of the system, defined by

$$T = \left|\frac{F_{out}}{F_{in}}\right| \qquad (6)$$

This expression involves trigonometric functions of complex numbers, so to evaluate the expression the following equations are needed:

$$\begin{aligned} \cos(n\ell) &= \cos(p+jq)\ell = \cos(p\ell)\cosh(q\ell) - j\sin(p\ell)\sinh(q\ell) \\ \sin(n\ell) &= \sin(p+jq)\ell = \sin(p\ell)\cosh(q\ell) + j\cos(p\ell)\sinh(q\ell) \end{aligned} \qquad (7)$$

Unfortunately the signs in these expressions were interchanged in equation (14) of [4].

3. COMPARISON OF THEORY AND EXPERIMENT

Experimental results were given in [4] for the longitudinal transmissibility of a 6-turn valve return spring (from a car engine, active length $\simeq$ 32.5mm) in series with pads of rubber (8mm thick) of two different levels of damping. These results are given in Figure 2 where they are compared again with the expression for T given in [4], but using the corrected expansions for the trigonometric functions (equation 9 above).

It is apparent that the predicted frequencies at which the coil spring resonances occur are not precisely in agreement with experiment. However, the shifts in frequency and attenuaton of the height of the peaks are predicted with fair accuracy. The peak values were not attenuated in the theoretical curves given in Figure 4 of [4], this being the puzzling discrepancy which has been resolved on using the correct signs in equation (7).

With this renewed confidence in the theory, its predictions will now be examined in more detail for a wider range of systems.

4. PERFORMANCE OF A SINGLE SPRING

This section deals with a system like that in Figure 1 but with only one spring (parameters k^*_L, ρ_L, ℓ) and one mass (m).

For a single spring excited with amplitude u_o at x=0 and terminated at an immobile base at $x=\ell$ equation (5) becomes:

$$F(x) = k^*_L\, u_o\, n \left\{ \frac{\cos(nx)}{\tan(n\ell)} + \sin(nx) \right\} \qquad (8)$$

Equations (5) and (8) differ from their analogues in [4] because the direction of the x-axis has been reversed.

If ρ_L were zero, equation (8) would reduce to $F(x)=(k_L^*/\ell)u_o$, so that the behaviour is that of a simple (ie massless) spring of complex stiffness $k^*=k_L^*/\ell$. Figure 3 gives plots of the normalized output force $F(x=\ell)/(k_L^* u_o/\ell)$ as a function of frequency with $\tan\delta$ as a parameter. It is assumed in these plots that k_L^* and $\tan\delta$ are independent of frequency. In fact for all rubbers there will be some dependence, and this point will be considered further in the discussion.

It is apparent in Figure 3 that the effect of the inertia of a lightly damped spring is to cause peaks in $F(x=\ell)$ at the resonance frequencies $\omega\ell/c = N\pi$ where N is an integer. In addition, the output force is greater between the resonance peaks than that anticipated for a massless spring. For a more heavily damped spring, the height of the resonance peaks in $F(x=\ell)$ is quickly attenuated as the order of the resonance is increased, and the value of $F(x=\ell)$ between the peaks soon falls below that anticipated for a massless spring. These effects of damping arise from the phenomenon discussed in relation to equation (4).

The input force, F_{in} in equation (6), is dominated at high frequency by the term $m\omega^2 u$ required to accelerate the mass m situated at x=0. The other term in equation (6), F_{out}, is just $F(x=\ell)$. This means that the effect of spring inertia on the transmissibility of a single spring can be appreciated from Figure 3, as will be seen on comparing the plots in Figure 3 with the transmissibility plots in Figure 4.

5. DERIVATION OF DESIGN EQUATIONS FOR NOISE STOP PADS USING FOUR-POLE PARAMETERS

In this section, the effectiveness of a "noise-stop" pad, ie. a rubber pad in series with a lightly damped spring, is discussed in terms of the pad parameters of stiffness and damping. As shown in Figure 2, the expression given for the transmissibility of a two-stage system, including effects of spring inertias in each stage, is satisfactory. However, the expressions are rather complicated (equation 18 in [4]) and a simplification would facilitate design and understanding.

Snowdon [6,7] has presented a method which reduces analysis of the performance of antivibration elements connected in series to the multiplication of 2x2 matrices. An element (or 'subsystem') is characterized by the matrix

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} = \begin{pmatrix} \left.\frac{F_{in}}{F_{out}}\right|_{V_{out}=0} & \left.\frac{F_{in}}{V_{out}}\right|_{F_{out}=0} \\ \left.\frac{V_{in}}{F_{out}}\right|_{V_{out}=0} & \left.\frac{V_{in}}{V_{out}}\right|_{F_{out}=0} \end{pmatrix} \qquad (9)$$

where V stands for velocity and the subscripts represent boundary conditions, for example $V_{out} = 0$ signifies an immobile termination to the element.

This method is applied to the system illustrated in Figure 1, with the simplifications that $m_2=0$ and $\rho_{L2}=0$ so that spring 2 may be characterised just by its stiffness $k_2^* = k^*_{L2}/\ell_2$. Since there is no ambiguity the subscripts 1 and 2 will be dropped. The matrix characterizing this system is then found from the product of the three matrices corresponding to the mass (m), spring with distributed mass (k_L^*, ρ_L, ℓ) and massless spring (k^*):

$$\begin{pmatrix} 1 & j\omega m \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(n\ell) & jZ\sin(n\ell) \\ \frac{j}{Z}\sin(n\ell) & \cos(n\ell) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{j\omega}{k^*} & 1 \end{pmatrix} =$$

$$\begin{pmatrix} \left(1-\frac{\omega^2 m}{k^*}\right)\cos(n\ell) - \frac{\omega m}{Z}\left(1+\frac{k_L \rho_L}{k^* m}\right)\sin(n\ell) & jZ\sin(n\ell) + j\omega m\cos(n\ell) \\ \frac{j}{Z}\sin(n\ell) + \frac{j\omega}{k^*}\cos(n\ell) & \cos(n\ell) \end{pmatrix} \qquad (10)$$

where $Z = \sqrt{k_L^* \rho_L}$, a quantity whose significance will be discussed later. From the definitions (8) and (11) it follows that the transmissibility of the system terminated at an immobile base is given by:

$$T = \left|\frac{1}{\alpha_{11}}\right| \qquad (11)$$

Making the simplification that n is real and the approximations that

$|\omega^2 m/k^*|>>1$ and $|Z^2/mk^*|<<1$ appropriate to high frequency and m much greater than the spring mass, equation (11) becomes

$$T \approx \frac{\frac{1}{\omega^2 m}}{\left| \frac{(k'-jk'')}{|k^*|^2}\cos\left(\frac{\omega \ell}{c}\right) + \frac{1}{\omega Z}\sin\left(\frac{\omega \ell}{c}\right) \right|} \tag{12}$$

It is apparent from equation (12) that resonance peaks occur at ω such that

$$\tan\left(\frac{\omega \ell}{c}\right) \approx -\frac{\omega k' Z}{|k^*|^2} \tag{13}$$

Thus the resonances occur at $\omega\ell/c$ values up to $\pi/2$ less than $N\pi$, the smaller the value of $|k^*|$ and the greater the value of ω the greater being the reduction in frequency. The values of T at the peaks are given by:

$$T_{peak} \approx \frac{|k^*|^2}{\omega^2 m k'' \cos\left(\frac{\omega \ell}{c}\right)} \tag{14}$$

The predictions of equation (12) are compared to the full theory (equation (18) in [4]) in Figure 5. The only difference in the parameters of the systems for the two theories is that the noise-stop pad is treated as having distributed mass in the full theory (in fact $\rho_{L1} = \rho_{L2}$ for the case depicted in Figure 5). This leads to a somewhat greater reduction in the frequencies of the peak values of T than anticipated from equation (12), especially for the higher order resonances (see Figure 5). The failure of equation (12) to predict the primary mass-on-spring resonance is a consequence of the approximation $|\omega^2 m/k^*|>>1$.

6. DISCUSSION

A comparison of Figures 4 and 5 shows that either damping in the main spring, or damping in a noise-stop pad in series with an undamped spring, can attenuate the peaks in T due to wave effects. However, use of a damped noise-stop has negligible benefit to the level of T between the peaks, whereas damping in the main spring is effective in reducing T at all high frequencies. Equation (14) shows that the peak values of T are proportional to $|k^*|^2/k''$. Thus a soft, lightly damped pad would be as effective as a stiff, highly damped pad, an example of which is seen in Figure 2. Figure 5 also shows how use of a highly damped pad and an intermediate mass m_2 reduces T between the resonance peaks, but the peaks themselves are no longer damped, being higher than in the absence of m_2.

With the exception of Figure 2, all the predictions are based on stiffness parameters (k' and k") which are independent of frequency. In fact from the theory of viscoelasticity, based on the assumptions of linearity and the superposition principle, it can be shown that if the dynamic shear modulus G is a function of frequency then it is complex and the real and imaginary parts are interrelated. Schwarzl & Struik [8] have given an approximate form for the relationship:

$$dG'(\omega)/d(\ln\omega) \approx \frac{\pi}{2}\{G''(\omega)-0.8225d^2G''(\omega)/d(\ln\omega)^2\} \qquad (15)$$

From this is follows that a rubber spring with a non-zero value of k" will have a positive frequency dependence of k'. The effect of this will be that the transmissibility of the spring at high frequencies will be worse than anticipated from the spring properties at low frequency. However, k" also rises with frequency for rubbery materials, and the beneficial effect of increasing damping may in part offset the effect of rising $|k^*|$. It would clearly be desirable to apply the theory to a rubber spring with properties fully characterized as a function of frequency. However, the conclusions drawn on the basis of the simplified model will remain qualitatively correct.

Transmissibility of the mass-spring system on an immobile foundation has been used as a measure of the effectiveness of the isolation. However, if the foundation were really immobile there would be no need to isolate the excitation source from it. The impedance Z_T of the foundation (the inverse of the mobility) is defined as the force amplitude required to produce a unit velocity amplitude. The driving point impedance Z_s of the system can be expressed in terms of Z_T and the four pole parameters defined in equation (9)

$$Z_s = \frac{\alpha_{11} Z_T + \alpha_{12}}{\alpha_{21} Z_T + \alpha_{22}} \qquad (16)$$

The object of isolation is to reduce the power flow of the structure borne sound, and the impedance is a useful parameter in this respect. The power passing across a boundary of point impedance Z is just ZV^2, where V is the velocity at the boundary.

An interesting special case of a mobile foundation is a semi-infinite rod. Longitudinal excitation of the termination will cause wave propagation, and the impedance Z of the rod can be derived as in [3]:

$$Z = \sqrt{k_L^* \rho_L} = A\sqrt{E^* \rho} \qquad (17)$$

where A is the cross-sectional area of the rod (considered to be solid),

E^* is the complex modulus and ρ is the density. The "characteristic" impedance Z_* of a rod has already been made use of in equation (10). The quantity $\sqrt{E^*\rho}$ is called the characteristic, or acoustic, impedance of the material.

Cremer & Heckl [3] suggested that a measure of the effectiveness (or rather lack of it) of an elastic layer as a noise barrier between two relatively rigid structural elements is the transmission efficiency τ, defined as the fraction of incident power which is transmitted across the layer. Applying boundary conditions of continuity of force and velocity, it can be deduced that the fraction $1-\tau$ of power (propagating as a wave) reflected at the juncton of a semi-infinite rod with the point of impedance Z_s is:

$$1-\tau = \left|\frac{Z_1 - Z_s}{Z_1 + Z_s}\right|^2 \tag{18}$$

where Z_1 is the characteristic impedance of the long rod. Equations (16), (17) and (18) allow τ to be calculated. Cremer & Heckl [3] give the result for the special case of no damping and equal values of Z_1 and Z_T:

$$\tau = \frac{1}{\cos^2(\frac{\omega\ell}{c}) + \left(\frac{Z_1^2+Z_2^2}{2Z_1Z_2}\right)^2 \sin(\frac{\omega\ell}{c})} \approx \frac{1}{1+(\omega Z_1/2k)^2} \tag{19}$$

where Z_2, ℓ and ω/c are the characteristic impedance, length and wave number respectively of the soft elastic interlayer. The approximate form neglects the effect of distributed mass in the soft layer (the only parameter being its stiffness k) and is appropriate for small values of $\omega\ell/c$.

Four parameters, ρ_L, k^*_L (real and imaginary parts) and ℓ are needed to characterise a spring. From these, other parameters such as characteristic impedance ($\sqrt{k^*_L\rho_L}$), stiffness ($|k^*_L|/\ell$) and damping ($\tan\delta$) may be constructed.

Equation (18) shows that the greater the mismatch in impedances the more energy is reflected. This may appear to indicate that the crucial parameter of the soft layer is its characteristic impedance [9]. However, equation (19) shows that the length of the layer plays a very important role, as described by the four pole parameters, and in fact, at low frequency, the crucial parameter is just the stiffness k of the soft layer. In common with the transmissibility (section 4), the transmission efficiency is undesirably high at the resonance frequencies of the spring ($\omega\ell/c = N\pi$). Material damping will reduce the peak values of τ, just as with the transmissibility. Although an elastic layer between two relatively rigid structural elements may be an effective noise barrier, its function differs somewhat from the noise-stop pad discussed in section 5. While compliance is the crucial parameter for the former element, the latter element must have damping as well as compliance.

7. CONCLUSIONS

If a spring with low damping is used, a "noise-stop pad" can be used in series with it to cut down the peak values of transmissibility (due to wave effect resonances) but the level between the peaks is not reduced. The required properties of the pad are damping and compliance, while density (or acoustic impedance) plays a secondary role. The overall conclusion is that a better solution to the problem of spring resonances is to use a material with inherent damping (eg. rubber) for the spring. This leads to a reduction in both the height of the peaks and the level of transmissibility between them.

8. REFERENCES

[1] POTTINGER, M.G., MARSHALL, K.D., LAWTHER, J.M. & THRASHER, D.B., 'A review of tire pavement interaction induced noise and vibration' p.183 in 'The Tire Pavement Interface' ed. Pottinger & Yager, publ. ASTM, 1986, Baltimore

[2] STEWART, M.A., MacKENZIE, D.J. & FALCONER, D.G., 'The effect of loading on the impact sound insulation of concrete floors with a floating raft on a resilient layer' Proc. I.O.A., 11, p.543 (1989)

[3] CREMER, L. & HECKL, M., 'Structure-borne sound' translated Ungar EE publ. Springer-Verlag, (1973)

[4] MUHR, A.H., 'Transmission of noise through rubber-metal composite springs' Proc. I.O.A., 11, p.627 (1989)

[5] HALL, M.M. & THOMAS, A.G., 'Testing procedure for measurement of dynamic properties of vulcanized rubber' J. Instn. Rubb. Ind. 7, p.65 (1973)

[6] SNOWDON, J.C., 'Mechanical four-pole parameters and their application' J.Sound. Vib 15, p.307 (1971)

[7] SNOWDON, J.C., 'Vibration isolation' US Dept. of Commerce/NBS Handbook No.128 (1979)

[8] SCHWARZL, F.R. & STRUIK, L.C.E., 'Analysis of relaxation experiments' Adv. Mol. Relax. Processes 1, 201-255 (1967/68)

[9] PAYNE, A.R. 'Transmissibility and wave effects in rubber' Rubb. Chem. & Tech., 37, 1190-1244 (1964)

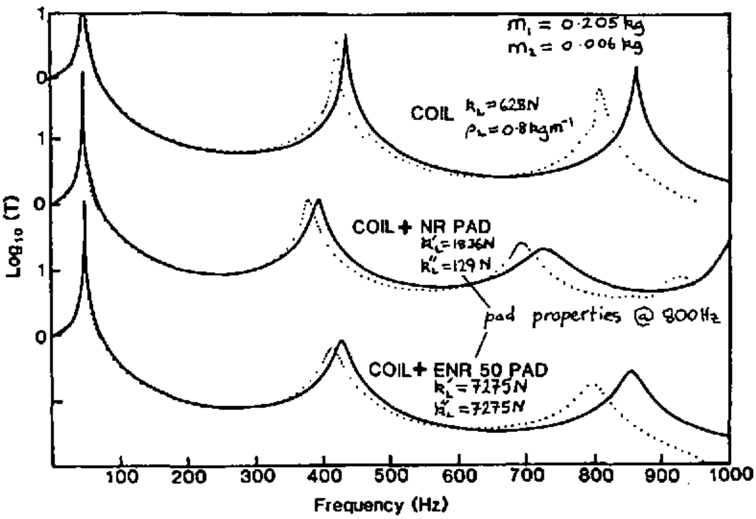

Figure 2 Comparison of theory (——) and experiment (...) for transmissibility of a coil spring with no pad, a soft low damping rubber pad (NR) and a stiffer, high damping rubber pad (ENR 50). Details of the pad properties are given in [4]

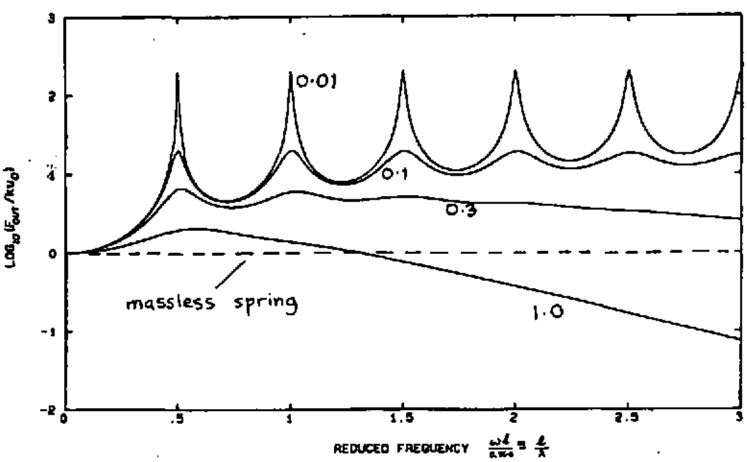

Figure 3 Reduced force (F_{out}/k^*u_o) transmitted by a single spring to an immobile foundation. Parameter is tanδ

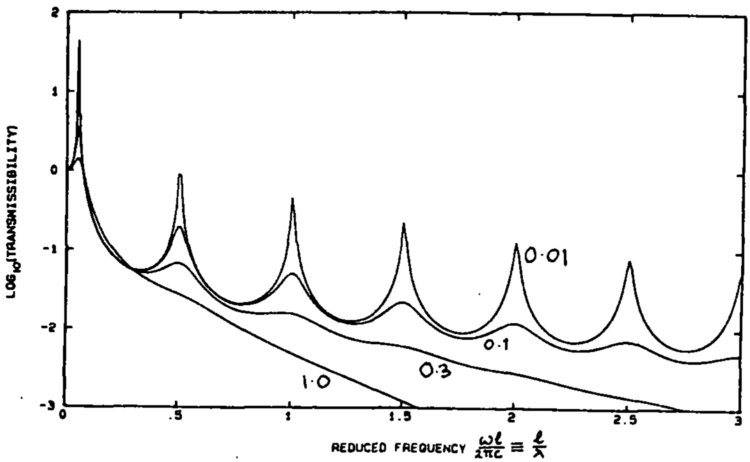

Figure 4 Transmissibility for a mass-spring system ($m/\rho_L \ell = 10$)
Parameter is $\tan\delta$

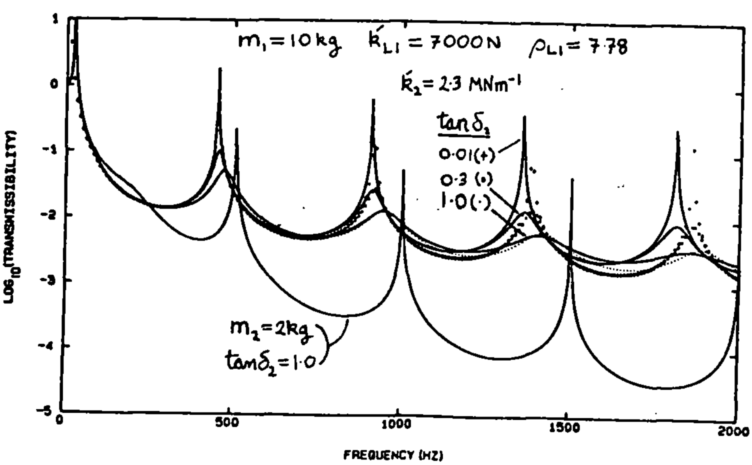

Figure 5 Effect of noise-stop pad (with damping $\tan\delta_2$) on the transmissibility of an undamped spring. —— Equation 18 of [4], +, • and · equation (12). For top three graphs, $m_2 = 0$.