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ACTIVE CONTROL OF HARMONIC ENCLOSED SOUND FIELDS OF LOW MODAL DENSITY; A COMPUTER SIMULATION

A.J. Bullmore, P.A. Nelson, S.J. Elliott

Institute of Sound and Vibration Research, The University, Southampton

1. INTRODUCTION

The application of active noise control methods to enclosed sound fields of high modal density has been considered by Nelson *et al* [1,2]. It was shown that to achieve appreciable reductions in the total time averaged acoustic potential energy at the frequency of interest, the primary and secondary sources must be separated by a distance of less than one half wavelength. The objective of this paper is to use the same theoretical basis to investigate the application of active noise control to enclosed sound fields of low modal density; i.e., in the case when only a few modes dominate the acoustic response. The results are found to differ considerably from the high modal density case, although it is shown that the positioning of the secondary sources still critically determines the level of reduction that can be achieved. The practical limitations of this method (as discussed in reference [1]) are reiterated, and an alternative, more practically applicable, method for reducing the total time averaged acoustic potential energy (E_p) is presented. This method involves sampling the sound pressure at a number of discrete locations, and then minimising the sum of the squared pressures at these locations. It is shown that this method can give reductions in E_p close to the optimal levels of reduction achievable using a particular secondary source distribution, even when using surprisingly few sensors. However, the levels of reduction in E_p that can be achieved are heavily dependent on having "reasonable" sensor locations. Some initial guidelines for the location of sensors are suggested.

2. THE SOUND FIELD IN A LIGHTLY DAMPED RECTANGULAR ENCLOSURE

The sound field considered in this work is that produced in a "two dimensional" lightly damped rectangular enclosure (Figure 1) excited at frequencies for which the wavelength of sound is of the same order as the larger dimensions of the enclosure. The solution for the sound field that will be used is that given by Morse [3]. The complex pressure due to a harmonic source (having a time dependence $e^{j\omega t}$) is given by the series solution

$$p(\underline{x}, \omega) = \sum_{n=1}^{N} \psi_n(\underline{x})a_n(\omega) = \underline{\psi}^T\underline{a} \tag{1}$$

where $\underline{x}$ is the position vector in the enclosure, $\psi_n(\underline{x})$ are the *normalised* characteristic functions and $a_n(\omega)$ are the complex mode amplitudes. Thus

$$\psi_n(\underline{x}) = \sqrt{\varepsilon_{n_1}\varepsilon_{n_2}\varepsilon_{n_3}} \cos\frac{n_1\pi x_1}{L_1} \cos\frac{n_2\pi x_2}{L_2} \cos\frac{n_3\pi x_3}{L_3} \tag{2}$$

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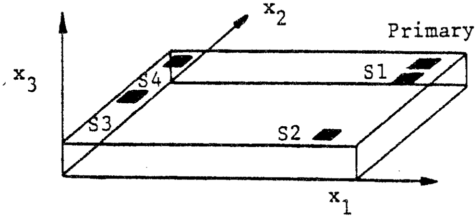

Figure 1. Schematic diagram of the enclosure modelled in all computer simulations, where $L_1 = 2.264$ m, $L_2 = 1.132$ m, $L_3 = 0.186$ m, and the positions of the centres of the sources are: primary = (2.087, 0.993, 0.186), S1 = (2.087, 0.843, 0.186), S2 = (1.892, 0.096, 0.186), S3 = (0.096, 0.566, 0.186), S4 = (0.177, 0.993, 0.186).

$$a_n(\omega) = \frac{\rho_o c_o^2}{V} \cdot \frac{\omega}{2\omega_n k_n - j(\omega_n^2 - \omega^2)} \cdot \int_V \psi_n(\underline{y}) s(\underline{y}, \omega) dV \qquad (3)$$

where ω_n and k_n are the natural frequency and damping of the n'th mode, $V = L_1 L_2 L_3$ is the enclosure volume and $\underline{y}$ is the vector defining the position of the complex source strength distribution $s(\underline{y}, \omega)$. The n'th mode is thus defined by the trio of integers (n_1, n_2, n_3) and the normalisation factors are given by $\varepsilon_\nu = 1$ if $\nu = 0$ and $\varepsilon_\nu = 2$ if $\nu > 0$.

In the work detailed below, computer simulations of the sound field are described in which the total source strength distribution $s(\underline{y}, \omega)$ is split into a single primary component $s_p(\underline{y}, \omega)$ and M secondary components $s_{sm}(\underline{y}, \omega)$. Thus

$$s(\underline{y}, \omega) = s_p(\underline{y}, \omega) + \sum_{m=1}^{M} s_{sm}(\underline{y}, \omega) \qquad (4)$$

Each of these source distributions is modelled as a 0.15 m by 0.15 m rectangular piston mounted in one surface of the enclosure. Each piston moves with a uniform surface velocity. Using this type of piston source ensures that the modal series (1) converges even when calculating the pressure on the surface of the source. This stringent test of the convergence of the series was used to determine the upper bound N on the number of normal modes necessary in numerical simulations. Full details are given in reference [4].

Equation (1) has been used to calculate the acoustic response of the enclosure detailed in Figure 1. A damping ratio k_n/ω_n of 0.01 has been assumed for all

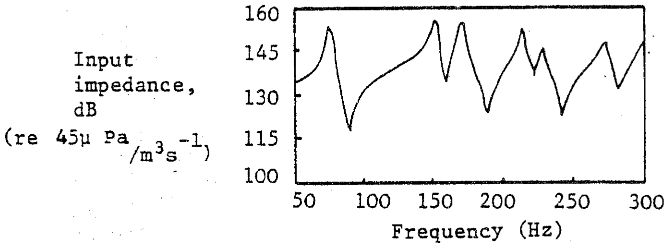

Figure 2. Ratio of sound pressure evaluated on the centre of the surface of the primary source to the velocity of that source. The natural frequencies and associated modal integers are:
1 = 75.3 Hz (1,0,0); 2 = 156.6 Hz (2,0,0)(0,1,0); 3 = 168.4 Hz (1,1,0)
4 = 213.0 Hz (2,1,0); 5 = 225.9 Hz (3,0,0); 6 = 271.5 Hz (3,1,0)

the modes in the series. Figure 2 shows the pressure evaluated on the centre of the primary source for a constant volume velocity of that source for all frequencies from 50 Hz to 300 Hz. Six well-defined acoustic resonances exist in this frequency range.

3. THE MINIMUM VALUE OF THE ACOUSTIC POTENTIAL ENERGY

As shown in Reference [1], the total time averaged acoustic potential energy in the enclosure can be written in the complex quadratic form

$$E_p = \underline{q}_s^H \underline{A}\,\underline{q}_s + \underline{q}_s^H \underline{b} + \underline{b}^H \underline{q}_s + c \tag{5}$$

where the matrix $\underline{A} = (V/4\rho_o c_o^2)\underline{B}^H\underline{B}$, the vector $\underline{b} = (V/4\rho_o c_o^2)\underline{B}^H\underline{a}_p$ and the scalar constant $c = (V/4\rho_o c_o^2)\underline{a}_p^H\underline{a}_p$. Recall that the complex mode amplitude vector $\underline{a} = \underline{a}_p + \underline{B}\,\underline{q}_s$. The model for the sound field described in Section 2 above can be used to define the following components:

$$q_{sm}(\omega) = \int_V s_{sm}(\underline{y}, \omega)\,dV \tag{6}$$

$$a_{pn}(\omega) = \frac{\rho_o c_o^2}{V} \cdot \frac{\omega}{2\omega_n k_n - j(\omega_n^2 - \omega^2)} \cdot \int_V \psi_n(\underline{y}) s_p(\underline{y}, \omega)\,dV \tag{7}$$

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$$B_{nm}(\omega) = \frac{\rho_o c_o^2}{V} \cdot \frac{\omega}{2\omega_n k_n - j(\omega_n^2 - \omega^2)} \cdot \frac{\int_V \psi_n(\underline{y}) S_{sm}(\underline{y}, \omega)\, dV}{\int_V S_{sm}(\underline{y},\omega)\, dV} \tag{8}$$

Thus $q_{sm}(\omega)$ is the m'th component of the secondary source strength vector $\underline{q}_s$, $a_{pn}(\omega)$ is the n'th component of the vector $\underline{a}_p$ of mode amplitudes due to the primary source only, and $B_{nm}(\omega)$ is the n, m'th component of the matrix $\underline{B}$ which defines the extent to which the n'th mode is excited by the m'th secondary source.

The solution for the vector of complex secondary source strengths which minimises E_p is given by [1]

$$\underline{q}_{so} = -\left[\underline{B}^H \underline{B}\right]^{-1} \underline{B}^H \underline{a}_p \tag{9}$$

and the corresponding minimum value of E_p is given by

$$E_{p_o} = E_{pp} - \left(\frac{V}{4\rho_o c_o^2}\right) \underline{a}_p^H \underline{B} \left[\underline{B}^H \underline{B}\right]^{-1} \underline{B}^H \underline{a}_p \tag{10}$$

where E_{pp} is the scalar constant, c, of equation (5) and is equal to E_p due to the primary source operating alone.

4. THE INFLUENCE OF RELATIVE SOURCE LOCATIONS ON THE MINIMUM VALUE OF ACOUSTIC POTENTIAL ENERGY

In this section, equation (10) is used to determine the conditions under which reductions in E_p are possible, and especially to investigate the importance of secondary source locations.

Four different secondary source locations have been used (see Figure 1) and for each of these, and combinations of these, the minimum value of E_p has been calculated at each 3 Hz interval between 50 Hz and 300 Hz. The results are presented as plots of E_{po}, the minimum value of E_p, against frequency.

It has been shown [1] that even for sound fields of high modal densities, substantial reductions in E_p can be achieved provided the primary and secondary sources are spaced within a half wavelength of each other. Figure 3 shows the result of placing the single secondary source S1 adjacent to the primary source and calculating the value of E_{po}. As would be expected, large reductions in E_p are achieved for all the frequencies considered.

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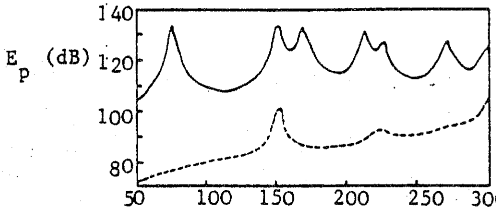

Figure 3. The value of E_p when minimised using the single secondary source S1 adjacent to the primary source; ——— E_{pp} due to the primary source; ------- E_{po} when E_p minimised using S1.

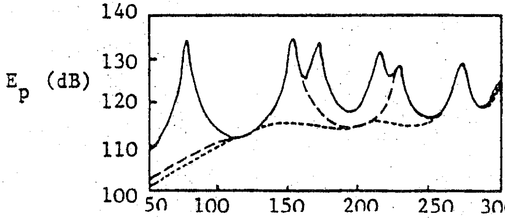

Figure 4. The value of E_p when minimised using secondary sources S2 and S3; ——— E_{pp} due to the primary source; — — E_{po} when E_p minimised using S2 only; ------ E_{po} when E_p minimised using both S2 and S3.

However, when the separation of primary and secondary sources exceeds $\lambda/2$ the situation differs significantly from the high modal density case. If E_p is minimised using source S2 only (see Figure 4), then appreciable reductions in E_p are evident for three of the six resonances, despite the separation of primary and secondary sources being greater than half a wavelength at the frequency corresponding to one of these resonances. The source's inability to substantially reduce E_p at the other three resonances can be explained if the source's position relative to the nodal planes of the dominant mode at these frequencies is considered (Figure 5). For each of these "uncontrolled" resonances, source S2 lies on, or close to, a nodal plane of the dominant mode. When it lies *symmetrically about* a nodal plane it is unable to excite the resonant mode to generate the necessary field to destructively interfere with the sound field dominated by that modal response. When the source lies *close to* a nodal plane, then whilst it can still excite the dominant mode, the large volume velocity required will lead to an increased excitation of any "residual modes" having pressure antinodes near the source location. This then limits the reduction in E_p achievable. Also shown in Figure 4 is E_{po} calculated using both sources S2 and S3. The use of the two sources is clearly advantageous, as this generally allows the contribution of two dominant modes at each frequency to be minimised, depending of course on the location of the sources relative to the nodal planes and the phase relationships of these modes.

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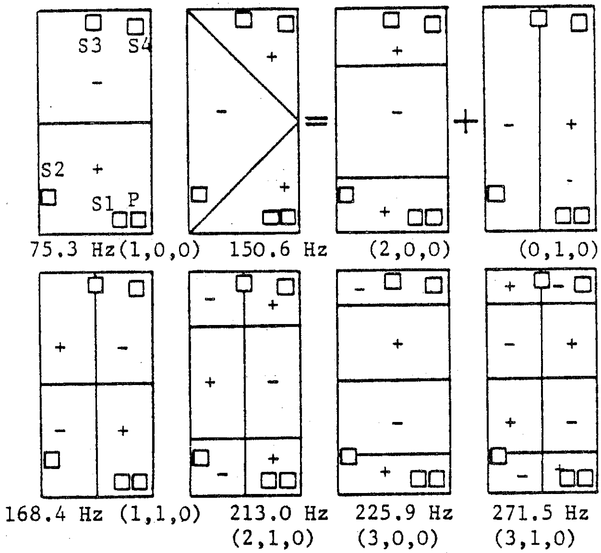

Figure 5. The distribution of nodal planes for the first six resonances of the enclosure of Figure 1 when driven by the primary source only.

The general conclusion of this section is that substantial reductions in E_p are possible using a limited number of sources. However, these reductions are only possible if the system is operating near to an acoustic resonance. Also, the secondary sources must be placed such that they can excite the dominant modes without significantly exciting the residual modes.

5. A PRACTICAL METHOD OF REDUCING THE TOTAL TIME AVERAGED ACOUSTIC POTENTIAL ENERGY

Whilst the method of minimising E_p described in Section 3 is useful in that it provides the "best possible" reductions in E_p for any given primary and secondary source distribution, its application requires exact knowledge of the primary and secondary source distributions, together with the acoustic response of the enclosed space. For many practical situations this will be impossible and some practical method of reducing E_p must be found. One such method is to monitor the sound levels at a number of discrete locations throughout the enclosure, and to minimise the sum of the squared pressures at these sensors. This sum of squared pressures can be written as the "cost function" J_p which is the logical approximation to E_p. This is given by

$$J_p = \frac{V}{4\rho_o c_o^2 L} \underline{p}^H \underline{p} \tag{11}$$

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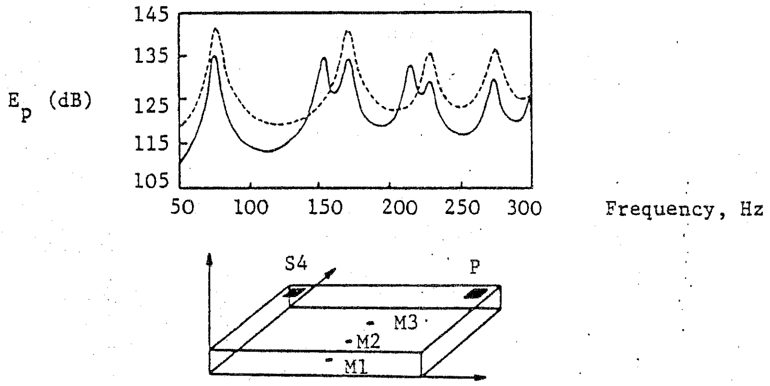

Figure 6. The value of E_p when J_p is minimised using source S4 and three sensors (M1, M2, M3) equispaced along the x_1 = 1.132 m plane of the enclosure; ——— E_{pp} due to the primary source; ----- E_{pJ} when J_p minimised using source S4 and the three sensors.

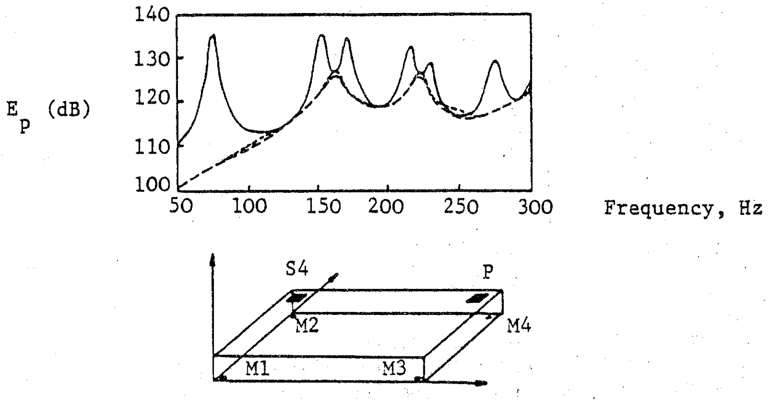

Figure 7. The value of E_p when J_p is minimised using source S4 and a sensor in each of the four corners of the enclosure; ——— E_{pp} due to the primary source; ----- E_{pJ} when J_p minimised using source S4 and the four sensors; — — — E_{Po} when E_p minimised using source S4.

$\underline{p}$ is the vector of pressures at the L sensor locations. Splitting this vector into the pressure due to the primary source only, $\underline{\Psi}_L^T \underline{a}_p$, and the pressure due to the secondary source distribution, $\underline{\Psi}_L^T \underline{B}\,\underline{q}_s$, allows equation (11) to be written in its quadratic form

$$J_p = \underline{q}_s^H \underline{A}\,\underline{q} + \underline{q}_s^H \underline{b} + \underline{b}^H \underline{q}_s + c \qquad (12)$$

where the matrix $\underline{A} = (V/4\rho_o c_o^2 L)\underline{B}^H \underline{\Psi}_L \underline{\Psi}_L^T \underline{B}$, the vector $\underline{b} = (V/4\rho_o c_o^2 L)\underline{B}^H \underline{\Psi}_L \underline{\Psi}_L^T \underline{a}_p$, $c = (V/4\rho_o c_o^2 L)\underline{a}_p^H \underline{\Psi}_L \underline{\Psi}_L^T \underline{a}_p$ and $\underline{B}$ and $\underline{a}_p$ are as defined above and $\underline{\Psi}_L$ is the N × L matrix of the N characteristic functions evaluated at the L sensor locations. E_{pJ}, the value of E_p which results from minimising J_p is found by substitution of the solution for $\underline{q}_{s1}$ (which defines the minimum of the quadratic form (12)) into equation (5), where $\underline{q}_{s1}$ is given by

$$\underline{q}_{s1} = -\left[\underline{B}^H \underline{\Psi}_L \underline{\Psi}_L^T \underline{B}\right]^{-1} \underline{B}^H \underline{\Psi}_L \underline{\Psi}_L^T \underline{a}_p \qquad (13)$$

Comparison of equations (9) and (13) for $\underline{q}_{so}$ and $\underline{q}_{s1}$ respectively shows that the only difference is due to the presence of the matrix $(1/L)(\underline{\Psi}_L \underline{\Psi}_L^T)$. Performing the matrix multiplication $\underline{\Psi}_L \underline{\Psi}_L^T$ yields an N × N matrix whose elements are the products of a pair of characteristic functions summed over the L sensor locations. Thus

$$(1/L)\underline{\Psi}_L \underline{\Psi}_L^T = (1/L)\begin{bmatrix} \sum_{\ell=1}^{L} \psi_{1\ell}^2 & \sum_{\ell=1}^{L} \psi_{1\ell}\psi_{2\ell} & \cdots\cdots & \sum_{\ell=1}^{L} \psi_{1\ell}\psi_{N\ell} \\ \sum_{\ell=1}^{L} \psi_{1\ell}\psi_{2\ell} & \ddots & & \\ \vdots & & \ddots & \\ \sum_{\ell=1}^{L} \psi_{1\ell}\psi_{N\ell} & \cdots\cdots\cdots & & \sum_{\ell=1}^{L} \psi_{N\ell}^2 \end{bmatrix} \qquad (14)$$

If the sound field is monitored at an infinite number of sensor locations spaced evenly throughout the enclosure, then by the orthogonality properties of the normal modes [3] this matrix reduces to the unit matrix, and $\underline{q}_{so}$ will equal $\underline{q}_{s1}$. However, this is clearly impractical, so the effects of using relatively few sensors will be investigated.

Using equation (13), E_{pJ} has been evaluated for two different sensor distributions, in both cases using the single secondary source S4. The first of these is shown in Figure 6, where three sensors have been equispaced along the x_1-plane centre line of the enclosure. Even though J_p has been minimised at the three sensor locations, E_p has at many frequencies increased from its original value. Reference to Figure 5 shows that the frequencies for which this has occurred are those where the dominant mode of the primary field has a nodal plane close to the sensors. In contrast, Figure 7 shows the resulting E_p when evaluated using a sensor in each of the four corners of the "two-dimensional" enclosure. Also shown is the minimum value of E_p achievable using source S4, and close agreement between the two is evident. The reason for this, is that when a single mode dominates the acoustic response, its contribution will necessarily dominate the pressure in all the corners. Thus J_p evaluated at the four corners will be largely due to the contribution of that one mode, and to minimise J_p the secondary source strengths must be adjusted such that they create a sound field to destructively interfere with the primary field dominated by this modal contribution, thus also reducing E_p. Where more than one mode dominates the acoustic response, evaluating J_p in the corners will not guarantee sensing the relative contribution of all the dominant modes, and minimising J_p will not necessarily result in reducing E_p. This is demonstrated in Figure 7 by the difference between E_{po} and E_{pJ} at "antiresonances", where at the low frequencies being considered, two modes generally dominate the acoustic response and thus the pressure is not always at a maximum in the corners of the enclosure.

6. CONCLUSIONS

It has been shown that in a lightly damped, harmonically excited enclosed sound field of low modal density, appreciable reductions in the overall acoustic potential energy, E_p, can be achieved using a small number of secondary sources. This is the case even when these sources are separated by greater than half a wavelength from the primary source. However, to achieve these reductions the system must be excited at, or close to, an acoustic resonance, and the secondary sources must be placed at maxima of the primary sound field.

It has also been shown that minimising the sum of the squared pressures at a number of discrete sensor locations can provide a good practical approximation to minimising the total time averaged acoustic potential energy provided enough sensors are positioned such that the relative contributions to the overall sound field of all dominant modes are detected. For the case of a two-dimensional rectangular enclosure having a low modal density, it has been shown that placing a sensor in each of the four corners and minimising the sum of the squared pressures at these locations will result in near optimal reductions in E_p.

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