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A MICROELECTRONIC MULTIPLE BEAM FORMING NETWORK

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Introduction

When an array of receiving transducers is illuminated by the echo from a reflecting object in the far field region of the array, the output of each transducer can be viewed as the real part of a phasor rotating at the carrier frequency. A phase difference related to the echo bearing will exist between the outputs of any adjacent pair of transducers, and due to the assumed plane wave nature of the echo, this phase difference will be constant across a linear array of equispaced elements, see figure 1.

Sonar receivers which measure the phase difference between adjacent elements using digital[1] and analogue[2] techniques have been described. Because these receivers reject amplitude information and rely only on the information obtainable from zero crossing positions, their performance is inferior to the more expensive within-pulse sector scanning sonar[3].

This paper describes an analogue and a digital technique for looking at an illuminated sector by creating a number of overlapping beams, the position of each of which is fixed in space, and which together cover the sector of interest. It is hoped that the use of microelectronic and thin film techniques will enable the production of an inexpensive sonar which does not suffer from the disadvantages which a 'phase only' sonar receiver has.

Background

Acoustic energy reflected back to an antenna will give rise to a continuous pressure distribution $P(x)$ across the aperture. The outputs of an array of N transducers constitute a set of samples of $P(x)$, from which the angular distribution $f(\theta)$ of reflectors of acoustic energy in the far field region of the antenna is to be determined.

Only point samples need to be considered since the effect of the finite size of a transducer can be deduced from the pattern multiplication theorem for arrays of identical transducers.

Referring to figure 1(a), if a plane wave is arriving from an angle θ to the normal, the phase difference ϕ between element k and element 0 ($k = 0, 1, 2, \ldots N - 1$) is given by

$$\phi = \frac{k.2\pi d}{\lambda} \sin\theta \quad ; \quad \lambda = \text{wavelength}$$

The contribution from a particular angle can be found by multiplying the array samples x_0, x_1, x_2, etc., by an appropriate phase shifting factor and forming the sum

$$f(\theta) = \sum_{k=0}^{N-1} x_k \exp(-jk \frac{2\pi d \sin\theta}{\lambda}) \qquad \ldots \quad (1)$$

This function, although continuous, can be represented by N independent samples, spaced at intervals in $\sin\theta$ of

$$\Delta(\sin\theta) = \frac{\lambda}{Nd}$$

Thus $f(\theta)$ can be expressed as a series of samples a_r where

$$a_r = \sum_{k=0}^{N-1} x_k \exp(-j2\frac{\pi kr}{N}) \qquad \ldots \quad (2)$$

and $\sin\theta = \frac{r\lambda}{Nd}$ $\quad (r = 0, 1, 2 \ldots N-1)$

Expression (2) indicates the operation of taking the discrete Fourier transform (DFT) of the samples of the pressure distribution across the aperture.

Expanding (2) and putting $w = \exp(-j\frac{2\pi}{N})$ gives

$$\left.\begin{aligned} a_0 &= x_0 + x_1 + x_2 + \ldots\ldots \\ a_1 &= x_0 + wx_1 + w^2x_2 + \ldots\ldots \\ a_2 &= x_0 + w^2x_1 + w^4x_2 + \ldots\ldots \\ &\cdot \\ &\cdot \end{aligned}\right\} \qquad \ldots \quad (3)$$

The process of forming the samples a_r generates N beams whose directions are fixed in space relative to the array.

As an example, if the array consists of 8 elements spaced at intervals of $\lambda/2$, 8 beams are formed at bearings defined by $\sin\theta = 0, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, 1$; this situation is illustrated in figure 2.

The DFT operation (3) is conveniently written in matrix notation

$$A = W.X$$

X is a column matrix having N elements which are the outputs of the array elements.

W is an NxN square matrix, the element in the r th row and k th column is $\exp(-j\frac{2\pi rk}{N})$ $\quad r, k=0, 1, 2 \ldots N-1$

A is a column matrix having N elements which are the outputs of the individual beams.

It has been shown[4,5] that if N is not a prime number the matrix W may be factorised in such a way that the computational effort required to evaluate the DFT is considerably reduced. This procedure leads to the Fast Fourier Transform (FFT).

Only the case where $N = 2^M$ will be considered here because this leads to a convenient result. Specifically, W may be written as the product of M factor matrices, each of order N, but which have only two non-zero elements in each row

$$W = W_1 . W_2 . W_3 . \ldots W_M$$

Rearrangement of the rows and columns of each factor matrix shows that the complete N-element DFT can be obtained using $\frac{N}{2} \log_2 N$ 2-element transforms in conjunction with

$1 - N + \frac{N}{2} \log_2 N$ phase shifts.

The 2-element transform is defined by

$$W = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

This represents a device which takes the sum and difference of two inputs, and is particularly simple to implement in hardware form.

Table 1 illustrates the savings which may be achieved using the FFT factorisation of the DFT for 4, 8, and 16 element arrays.

No. of elements	DFT		FFT	
	Additions and Subtractions	Multiplications	Additions and Subtractions	Multiplications
4	12	4	8	1
8	56	32	24	5
16	240	176	64	17

Table 1

As an example, consider the 4-element DFT

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \\ 1 & w^2 & 1 & w^2 \\ 1 & w^3 & w^2 & w \end{bmatrix} \qquad w = \exp(-j\tfrac{\pi}{2})$$

The individual elements have been written modulo 2π.

The FFT factorisation is

$$W = W_1 \cdot W_2 = \begin{bmatrix} 1 & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & w \\ 1 & -1 & \cdot & \cdot \\ \cdot & \cdot & 1 & -w \end{bmatrix} \begin{bmatrix} 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & 1 \\ 1 & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & -1 \end{bmatrix}$$

Rearranging the rows and columns of W_1 and W_2 gives

$$W = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \left[\begin{array}{cc|cc} 1 & 1 & \cdot & \cdot \\ 1 & -1 & \cdot & \cdot \\ \hline \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 \end{array}\right] \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & w \end{bmatrix} \left[\begin{array}{cc|cc} 1 & 1 & \cdot & \cdot \\ 1 & -1 & \cdot & \cdot \\ \hline \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 \end{array}\right] \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

The operation $A = W.X$ is shown schematically in figure 3. Figure 3 shows how the spatial DFT can be implemented using digital and analogue hardware.

Digital Realisation

It is possible to use a general purpose digital computer to perform the FFT operation in a sonar receiver[6]. However, in many situations, the greater speed of a special purpose FFT processor is required.

Referring to figure 4, the outputs of the array are sampled, amplified, converted to digital form and stored. The FFT can now proceed in a number of ways depending on the processing time available[7].

The Sequential Processor

Each operation depicted by a block in figure 3 is performed sequentially by a processor which has a single arithmetic unit. The time taken is that required to perform $N \log_2 N$ complex additions and subtractions and $1 - N + \frac{N}{2} \log_2 N$ complex multiplications.

The Parallel Processor

The sequential processing time may be reduced by introducing as much parallel processing as is necessary. In the extreme case, one arithmetic unit could be provided for each block in figure 3, and the effective computation time would be that required to perform just one 2-point transform.

The provision of several arithmetic units would be expensive at the present time, but the production of arithmetic units using large scale integration techniques may make such schemes more feasible in the future.

Analogue Realisation

The schematic in figure 3 can be implemented directly using analogue components to build a sum and difference network. It can be shown[8] that any two port network having an output/input relationship expressible in the general form

$$\begin{bmatrix} o_1 \\ o_2 \end{bmatrix} = \begin{bmatrix} \epsilon^{j\alpha_1} & \\ & \epsilon^{j\alpha_2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \\ & \epsilon^{j\alpha_3} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \ldots \quad (4)$$

$$= G \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

i_1, i_2 are inputs

o_1, o_2 are outputs, $j = \sqrt{-1}$

can be used as a basic building block in a multiple beam forming network for an array of $N = 2^M$ elements.

The general expression (4) enables the phase shifts α_1, α_2, and α_3 to be chosen independently to give a network which is convenient to fabricate.

For instance, if $\alpha_1 = 0$, $\alpha_2 = \alpha_3 = -\frac{\pi}{2}$, the network obtained is the hybrid coupler used in the Butler beam forming matrix at microwave frequencies[9].

One network of particular interest in sonar is defined by

$\alpha_1 = \pi$, $\alpha_2 = \alpha_3 = 0$

i.e. $$G = \begin{bmatrix} -1 & \\ & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$$

since this may be fabricated using microelectronic operational amplifiers in conjunction with thin film resistors, see figure 5.

The beam forming network is obtained by substitution into the network shown in figure 3, using the relationship

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & \\ & 1 \end{bmatrix} \cdot G$$

and then simplifying.

The equivalent multiple beam forming network using blocks G is shown in figure 6. If only the magnitudes of the beam outputs are required and their relative phases are unimportant, then the phase shifts of π radians in beam outputs a_1 and a_2 may be ignored.

The phase shifts required within the beam forming networks may be implemented using RC all-pass networks which can be fabricated using microelectronic amplifiers and thin film techniques, fig. 7.

Conclusion

Two schemes for processing the outputs of an array to produce multiple beams covering a sector have been described. Both are being developed at Loughborough University of Technology.

Through the use of low cost analogue and digital microelectronic circuits it is hoped that an inexpensive sector coverage sonar will soon become available.

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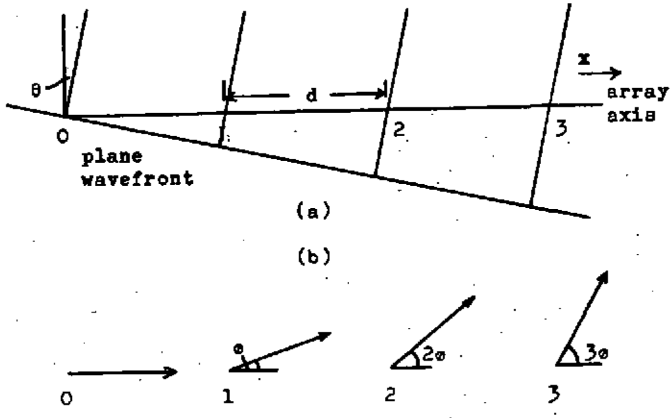


Fig.1
Array geometry and outputs of 4 elements

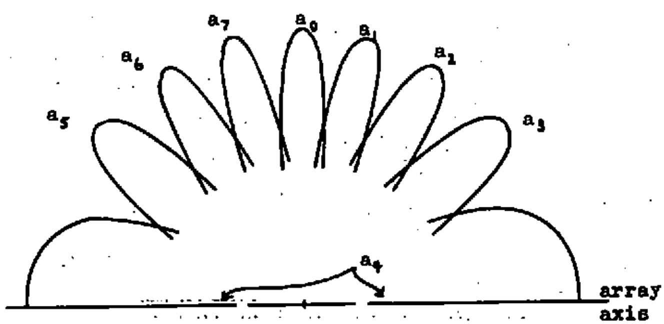


Fig.2
Beams formed from 8 element array ($\frac{\lambda}{2}$ spacing)

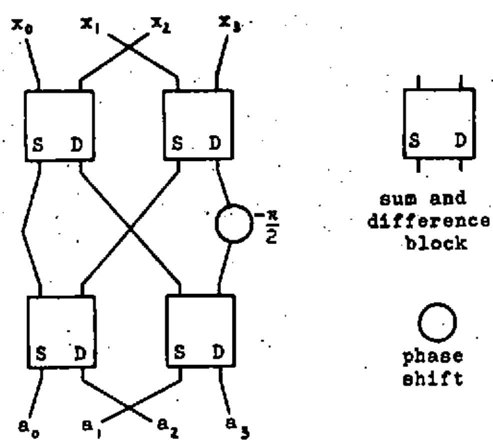


Fig.3
FFT schematic

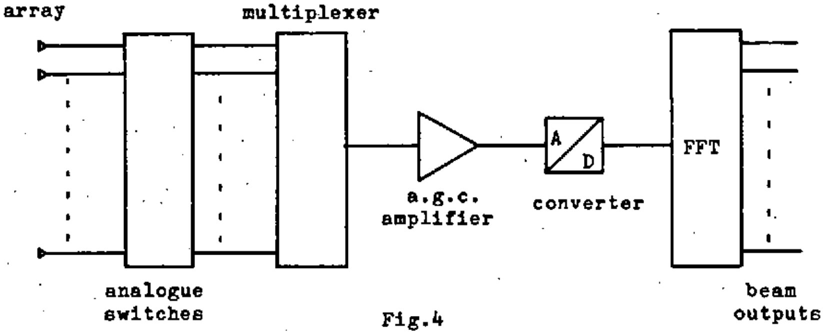


Fig.4
Digital beamformer

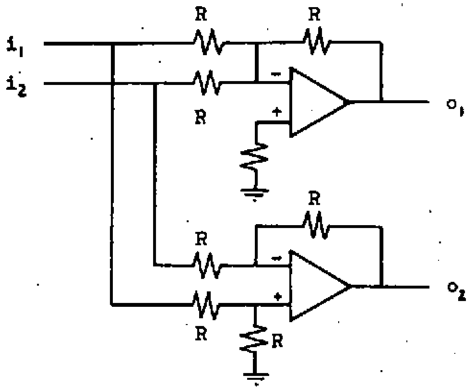


Fig.5
Basic analogue beamforming circuit

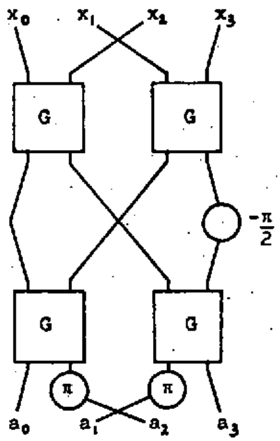
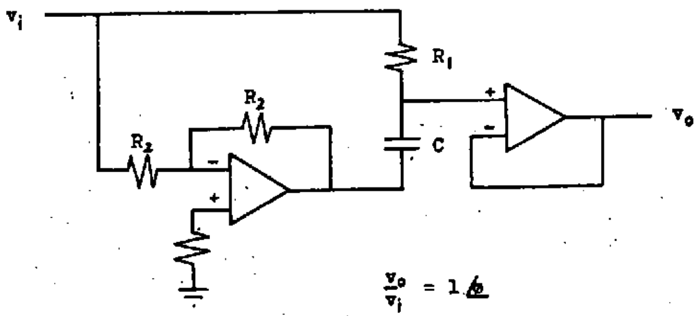


Fig.6
Analogue beamformer



$$\frac{v_o}{v_i} = 1\angle\phi$$

where $\tan\phi = \dfrac{-2\omega CR_1}{1-(\omega CR_1)^2}$

and $\omega = 2\pi.\text{frequency}$

Fig.7
Phase shift circuit