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PRONY AND LINEAR PREDICTION SPECTRAL ANALYSIS TECHNIQUES APPLIED TO SONIC WELL LOGGING

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1. INTRODUCTION

An application of the Prony and linear prediction spectral analysis techniques to sonic well logging is illustrated in this paper. An array of acoustic receivers on a well logging tool record the response of the formation and borehole to an emitted pulse of sonic energy. Two dimensional (2-D) spectral analysis is used to separate various waveform components according to their sonic velocities versus temporal frequencies. The spatial aperture of the array is limited, requiring high resolution spectral techniques to resolve waveform components travelling at comparable sonic velocities. A hybrid 2-D technique is described to achieve higher resolution in the spatial dimension. Examples of the performance of the method are shown for scale model laboratory data.

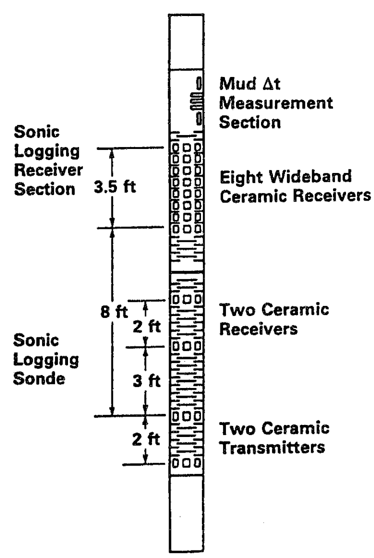

Figure 1-1 Sonic logging tool configuration

2. THE SIGNAL ENVIRONMENT

In sonic well logging, an elongated tool (Figure 2-1) in a well borehole periodically, with depth, emits a burst of sonic energy. After transmission and dispersion through the downhole rock formation, a linear array of eight sonic transducers located some distance away on the same tool records the time-delayed waveform response (Figure 2-2).

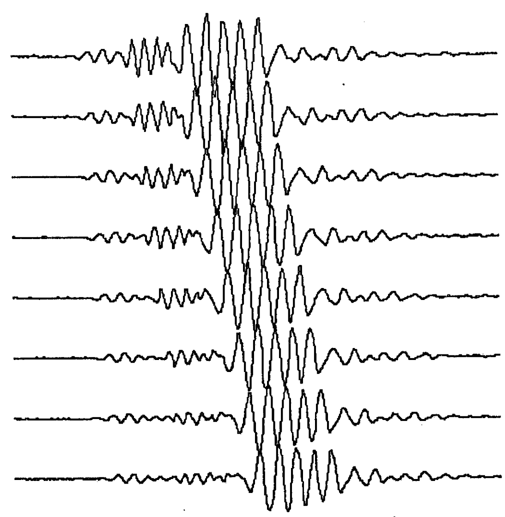

Figure 2-2 Waveforms from a sonic logging tool.

Due to the physics of acoustic wave propagation in the earth, multiple wavelets travelling at various sonic velocities and various time delays may be encountered. The object of digital processing in sonic well logging is to separate the various waveform components and determine their associated wave velocities. The short spatial aperture of the tool receiver array relative to the wavelength yields, at times, inadequate spatial resolution to distinguish arrivals with only slightly different velocities.

3. STANDARD PROCESSING

The magnitude of a two-dimensional Discrete Fourier Transform (DFT) of the linear array data will produce a standard frequency-wavenumber spectrum of the waveform set, as illustrated in Figure 3-3.

Due to the limited spatial aperture of the linear array, waveform components travelling with nearly the same velocity cannot always be distinguished by standard 2-D DFT processing. There are three propagating modes in Figure 3-3, but the spectrum is complicated by sidelobes from the largest event.

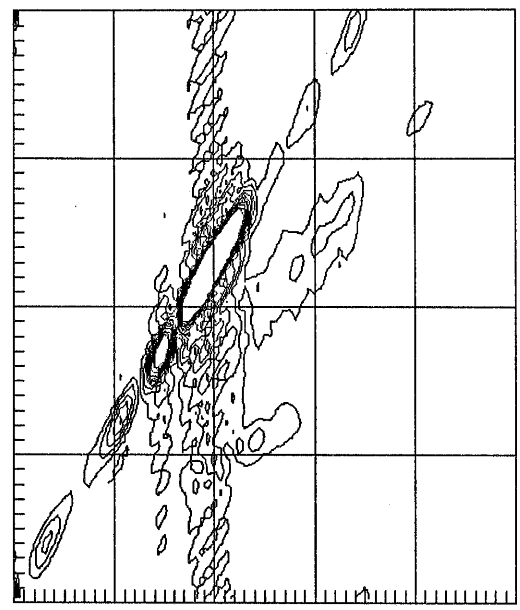

Figure 3-3 Wavenumber / Frequency plot

4. HIGH RESOLUTION PROCESSING

High-resolution spectrum analysis techniques have been applied to improve the spatial resolution. A successful high resolution approach has been the Prony method using the covariance method of linear prediction. This spectral analysis algorithm has been described in detail by Kay and Marple [3]. The concepts described were first developed by Parks, Morris, and McClellan [1,2] for application to this problem.

The high resolution waveform velocity estimation algorithm is a simple modification of the standard 2-D DFT algorithm. Short-time temporal frequency estimates are obtained from the application of 1-D DFTs to each waveform in the array, ie. the time dimension of the array is processed first. Denoting the n-th time sample of the m-th receiver waveform as $s[mT, nZ]$ where Z is the inter-receiver spacing interval and T is the temporal sampling interval, then the temporal transform will produce M frequency estimates for $i = 0$ to $M-1$ of

$$S[iF, nZ] = \sum_{m=0}^{N-1} s[mT, nZ] \exp(-j2\pi im/M) \quad \text{for } n = 1 \text{ to } N \quad ,$$

where $F = 1/NT$, the frequency bin size in Hz. M represents the total number of time samples per waveform. N represents the number of receivers in the array. The Prony signal modelling approach is used to obtain a high resolution spatial frequency estimate from $S[iF, nZ]$. The Prony model assumes the complex envelope signal $S[iF, nZ]$ for a fixed frequency iF, with spatial index n being the variable, is composed of a superposition of P exponential components. Assuming $P = 2$, for example, the model makes the estimate

$$\hat{S}[iF, nZ] = a_1[iF] \exp(-[\sigma_1 + j2\pi iFp_1]nZ) + a_2[iF] \exp(-[\sigma_2 + j2\pi iFp_2]nZ)$$

The Prony method uses least squares procedures to estimate the amplitudes a_1 and a_2, damping factors σ_1 and σ_2, and spatial frequencies p_1 and p_2. This estimation procedure is performed at each temporal frequency index i. Details of the procedure may be found in reference [3]. A key algorithm used in the Prony method is the covariance method of linear prediction. An indicator of the quality of the Prony exponential fit to the spatial complex envelope is to plot the residual squared error

$$\sum_{n=1}^{N} \left| S[iF, nZ] - \hat{S}[iF, nZ] \right|^2$$

relative to the total envelope energy

$$\sum_{n=1}^{N} \left| S[iF, nZ] \right|^2$$

at each temporal frequency iF. The residual squared error will be small relative to the envelope energy if a good exponential fit has been made.

This analysis technique is shown in Figure 4-4 applied to the same waveform set as the preceeding conventional processing. The plot shows the root locations of solutions with a residue which exceeds a certain threshold.

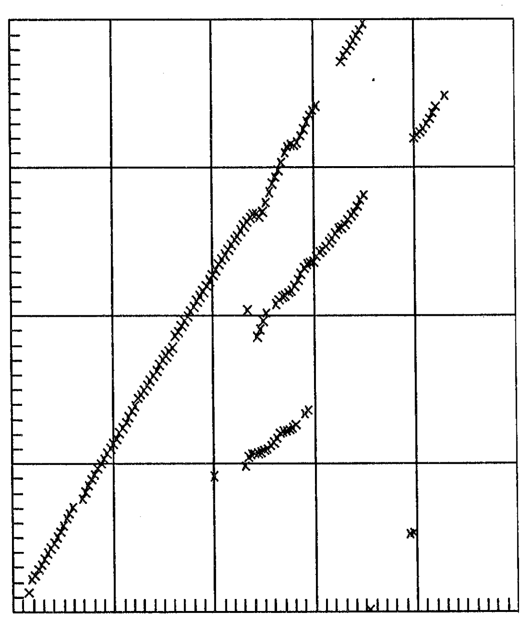

Figure 4-4 Wavenumber / Frequency plot, high resolution processing.

A more useful display is the frequency-inverse velocity plot as shown in Figure 4-5. This spectrum is obtained by transforming the wavenumber / frequency coordinates of Figure 4-4 as shown by the following relation.

$$\frac{1}{v} = \frac{k}{f} \quad .$$

The velocity ridges in the $1/v$ vs f plot permit the simultaneous identification of various wave components in the data. These velocities are then used to infer various formation rock properties.

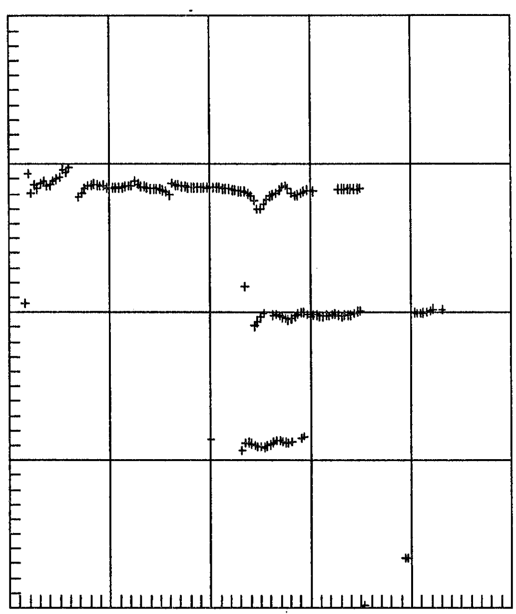

Figure 4-5 Wave slowness / Frequency plot

An indication of the accuracy of the model's fit to the data may be gained from the display of the residual squared error in Figure 4-6.

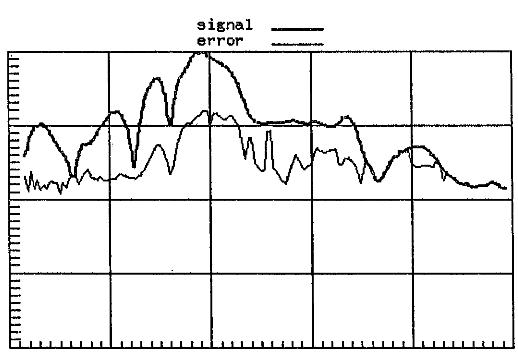

Figure 4-6 Signal and Error power spectra (20 db/vertical div.).

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