ON GENERAL PROPERTIES OF RAY ARRIVAL SEQUENCES IN OCEANIC ACOUSTIC WAVEGUIDES

A L Virovlyansky

Institute of Applied Physics, Ul'yanov Street, Nizhny Novgorod, Russia

1. INTRODUCTION

A specific feature of sound propagation in the ocean consists of the following. After a single short acoustic pulse emitting a whole series of pulses is registered at an observation point. Each pulse goes through one of acoustic ray paths connecting the source and the receiver. It is important to note that up to distances of a few thousand km travel times of these pulses can be predicted to a high precision using the geometrical optics (see, for example, the papers, Munk & Wunsch [1], Spiesberger & Metzger [2]). Fluctuations of ray arrivals owing to random inhomogeneities (such as those due to internal waves) for many rays are sufficiently lesser than intervals between two consecutive arrivals. This is the main reason to consider ray arrival times as informative parameters in the scheme of the ocean acoustic tomography.

Structures of ray arrival sequences have essential distinctions in waveguides with different sound speed profiles. Nevertheless in spite of all possible diversity there are some general properties of ray arrival sequences taking place in any waveguide. This paper is devoted to discussion of such properties.

Our analysis is based on a simple approximate formula connecting differences in ray travel times and adiabatic invariants of ray trajectories. The derivation of this formula for a plain-layered waveguide as well as for a slowly varying waveguide is given in the paper, Virovlyansky [3]. In the paper, Munk & Wunsch [4], there is a derivation of a simplified version of the formula for a plain-layered waveguide. This formula can be applied to rays with trajectories having many oscillations. So the results obtained here concern only such rays.

2. SIMPLE EXPRESSIONS FOR THE DIFFERENCE IN TRAVEL TIMES

Let us introduce some notions for the main characteristics of ray trajectories that escape a point source in an oceanic waveguide with a sound speed profile $c(z)$, where z is the vertical coordinate. The refractive index profile is defined by the relation $n(z) = c_0/c(z)$, where $c_0 = c(z_0)$, z_0 is the source depth. Each ray trajectory is governed by Snell's law $n(z)\cos(\theta) = \cos(\chi)$, where θ is the grazing angle and χ is the launch angle.

The adiabatic invariant that plays the main role in what follows, is defined by the expression

$$I(\chi) = \frac{1}{c_0} \int_{z_{min}}^{z_{max}} \sqrt{n^2(z) - cos^2(\chi)}\, dz,$$

PROPERTIES OF RAY ARRIVAL SEQUENCES

where z_{max} and z_{min} are the upper and lower turning points of the ray. The cycle length of the ray path (along the horizontal coordinate x) is equal to

$$D(\chi) = 2cos(\chi) \int_{z_{min}}^{z_{max}} \frac{1}{\sqrt{n^2(z) - cos^2(\chi)}} dz.$$

Further we shall consider only the rays connecting the two fixed points: the source and the receiver. Those rays sometimes are called eigenrays. Let us classify them using a "ray identifier" (N, j), where N is the number of cycles of a ray trajectory, j takes on one of four possible values 1,2,3 or 4. Each value defines one of four possible pairs of the ray inclination signs at the beginning and at the end of the trajectory. All rays having the same parameter j we shall call belonging to the jth class.

It can be shown that there is a simple approximate formula connecting the travel times of two adjacent rays belonging to the same class:

$$t_{N+1,j} - t_{N,j} = I(\bar{\chi}), \quad (1)$$

where $\bar{\chi} = \frac{1}{2}(\chi_{N+1,j} + \chi_{N,j})$ is the half-sum of the launch angles. This formula remains valid not only in range-independent but in a slowly range-dependent waveguide as well (under the condition of the adiabatic invariant conservation). It permits one to estimate travel times differences avoiding a rather cumbersome procedure of finding eigenrays and their arrivals. Moreover, in a slowly range-dependent waveguide this formula allows these quantities to be estimated even in the absence of full information about the sound speed profile changes along the acoustic path. For example, knowing only the sound speed profile near the source one can predict the difference in travel times of eigenrays having angles of departure close to some given value χ (of course, if such rays reach the receiver).

The ray identifiers described above do not define each ray unambiguosly. If the dependence of the ray cycle length D on the launch angle χ is not monotonic, then there can be more than one ray with the same identifier. To avoid ambiguity, we shall divide the whole range of launch angles under investigation into smaller intervals where the function $D(\chi)$ is monotonic. We shall call such intervals the intervals of monotony of $D(\chi)$. The relation (1) can be used only when comparing the travel times of two rays with launch angles belonging to the same interval of monotony.

3. ANALYSIS OF RAY ARRIVAL SEQUENCES

First of all, let us introduce the notion of the $t(\chi)$ curve which presents the graphical image of ray arrival sequence in Cartesian coordinates t (travel time) against χ (launch angle). Each arrival in these coordinates is depicted as a point. When the distance between the source and the receiver is large enough and many rays are received, the points representing the arrival sequence are dense and they form a "curve." We shall call this curve the $t(\chi)$ curve. For brevity, we shall consider the $t(\chi)$ curve formed by the rays belonging to one class only. Taking into account contributions of other rays will not change the main results.

PROPERTIES OF RAY ARRIVAL SEQUENCES

Some general properties of arrival sequences discussed below follow immediately from formula (1). For simplicity, we shall suppose that the rays under consideration have nonnegative launch angles. The first property concerns the rays belonging to the same interval of monotony of $D(\chi)$.

(i) As the ray invariant $I > 0$ ($I = 0$ only when the ray trajectory is a straight line) travel times $t_{N,j}$ rises when the index N rises. This means that the more cycles the ray trajectory has, the later the ray arrives at the observation point. This rule holds true for any dependence of the cycle length on the launch angle.

(ii) It follows from (i) that intervals of χ where the $t(\chi)$ curve is monotone (the intervals of monotony of $t(\chi)$) coincide with the intervals of monotony of $D(\chi)$. But the maximums of $t(\chi)$ coincide with minimums of $D(\chi)$ and vice versa. The point is that the number of cycles N (and, according to (i), the ray arrival time) rises when the cycle length diminishes.

(iii) When approaching a maximum the index N of points on both intervals of monotony of $t(\chi)$ joining this maximum rises. When approaching a minimum the index N of points on both intervals diminishes.

(iv) Ray invariant I in any waveguide is a monotonic function of the launch angle. Hence, the difference $t_{N,j} - t_{N-1,j}$ rises monotonically when the launch angles of rays under consideration rise. So, the density of points forming the $t(\chi)$ curve (to be exact, the density of their projections on t-axis) is maximum in the vicinity of $\chi = 0$ and decreases with χ.

(v) Let us discuss the evolution of $t(\chi)$ curve when the distance changes. First of all, it should be noted that the coordinates χ of the extrema of $t(\chi)$ curve do not depend upon distance (according to (ii) they coincide with extrema of the $D(\chi)$ curve). But there exists another conservation law. Its essence is as follows.

When the distance rises (falls) the number of eigenrays with launch angles belonging to the fixed interval $\Delta\chi$ also rises (falls). So, the mean difference in launch angles of "adjacent" rays (having ray identifiers (j,N) and (j,N+1)) changes. This circumstance, nevertheless, does *not* lead to changing of the mean difference in travel times of adjacent rays. According to Eq.(1) the difference in travel times of two rays with launch angles close to some fixed value of χ is approximately equal to $I(\chi)$ and does not depend upon distance. This conclusion remains valid in a slowly range-dependent waveguide as well.

As applied to a range-independent waveguide this consequence of Eq.(1) means also that the mean difference in travel times of rays having turning points in a fixed inteval of depths depends on neither distance, nor depths of the source and receiver.

According to the aforesaid, the evolution of a $t(\chi)$ curve when a distance is rising is going like this. The coordinates of the extrema do not change, but the curve at its intervals of monotony becomes steeper. The received signal duration rises not because of the differences in travel time rising but because the number of eigenrays increases.

4. CONCLUSIONS

Our main conclusion is that important inferences on ray arrival sequences can be made by a very simple analysis of dependences of adiabatic invariant and ray cycle length on launch angle. Sometimes (especially in the range-independent waveguide) it is more convenient to use the turning point depth as an argument. These functions allow one to estimate the mean difference in travel times of rays having launch angles in a fixed interval and find some characteristics of arrival sequences which do not depend on the locations of the source and the receiver but are determined by the sound speed profile only.

It is found that in any waveguide the time of arrival of the ray is the more the more cycles of oscillations are performed by the ray trajectory on the way to the receiver.

The most interesting property among those mentioned above is the independence of the mean differences of ray arrival times, with angles of escape from a fixed narrow angular range, of the path length. This property remains valid for a waveguide that changes adiabatically along the path. As applied to a plain-layered waveguide, this assertion can be augmented as follows: The mean difference of ray arrival times with turning points, lying inside a fixed narrow range of depths, is independent not only of the path length but also of the depths of location of the source and receiver (of course, if the rays with turning points from a given range arrive at the receiver). Thus, in spite of the increase in the number of received rays with increasing path length, there is not tendency towards the changes of the mean differences of ray arrival times.

5. REFERENCES

[1] W MUNK & C WUNSCH,'Ocean acoustic tomography: A scheme for large scale monitoring', Deep-Sea Res., 26 p123 (1979)
[2] J L SPIESBERGER & K METZGER,'Basin-scale tomography: A new tool for studying weather and climate', J Geophys Res, 96 p4869 (1991)
[3] A L VIROVLYANSKY,'On travel times of sound pulses in the ocean,' Akust Zh, 31 p664 (1985)
[4] W MUNK & C WUNSCH,'Ocean acoustic tomography: Rays and modes', Rev Geophys Space Phys, 21 p777, (1983)