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# ASYMPTOTIC ANALYSIS OF AEROENGINE TURBOMACHINERY NOISE

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## ABSTRACT

Existing analyses of aero-engine turbomachinery noise due to rotor stator interaction are based on essentially a strip theory analysis of the flow coupled to either a free field Green function or a modal duct acoustics analysis. The purpose of this paper is to show how methods of asymptotic analysis can be used to both put that theoretical treatment on a sounder footing and to view several aspects of the sound generation and radiation in novel and revealing ways. The approach is based on the existence of a small parameter that is of the same order as the ratio of blade chord or acoustic wavelength to duct diameter. By expansion in this parameter we show how the strip theory approximation arises and how the radiation can be calculated by solving the radiation problem for a given mode by what is essentially ray theory. The advantages of this over the more familiar modal approach are stressed. We show how the radiation arises from sources distributed over the blade span and at the tips and how diffraction and refraction by the mean flow can be allowed for.

## INTRODUCTION

Of the many sources of noise on a modern aero-engine one of the most important is the fan. This fan noise is composed of a number of separate components. These are chiefly broadband noise due to the boundary layer on the rotor blades, buzz-saw noise resulting from the inevitably non-uniform shock waves ahead of a supersonic fan and tone noise at harmonics of the fan blade passing frequency. This latter is due to the interaction of the wakes from the fan with downstream rotor blades and due to the fan passing through distorted inlet flows. In this paper, we shall be concerned in the main with rotor-stator interaction tones, though many aspects of the work are equally applicable to the other sources as well.

The aim of this paper is to present an asymptotic method for the analysis of the rotor-stator interaction tones. It is based on the existence of a small parameter $\epsilon$ in which the solution may be expanded. This parameter is the same order as the inverse of the blade aspect ratio and the ratio of the acoustic wavelength to the fan duct radius. By expanding the solution in terms of $\epsilon$ we show how asymptotically correct solutions to the noise generation problem may be constructed. These may be contrasted with the essentially ad hoc procedures used in existing analysis and prediction methods (see Ventres et al [1] for a typical example). In these analyses, the solution proceeds in the following steps.

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First, the strength of the wakes from the upstream fan rotor is calculated on a quasi-three-dimensional basis and their amplitudes at the downstream row of stators is calculated. Next, the unsteady pressure distribution on the stator blades is worked out. This is done using strip theory. That is, the unsteady wake interaction problem is w were dealing with a locally two-dimensional cascade, which is usually taken to have flat plate blades. The blade pressures are then used in conjnction with an appropriate modal Green function to work out the sound field radiated inside the duct as a sum of duct modes. Thereafter, a correction may be made for sound propagation through the rotor ahead of the stators and the radiation of each mode from the duct to the far field may be calculated using an appropriate theory.

In the work we describe here, we show how by use of a rational expansion of the flow variables in terms of our small parameter we can justify many of the assumptions made above and in addition improve on them. In particular, we stress he importance of not neglecting the radial phase of the wake at different stations of the stator blade, the special treatment that must be employed at the tips of the stator blades and we show how we can usefully think of the in-duct sound field in terms of rays rather than modes. A further feature of the method is that there is no restriction to the use of flat plate blades, the expansion technique will apply equally well to better methods of calculation in which the full geometry of the blade is allowed for.

We begin by showing how the mean flow around the blading may be viewed aymptotically in a manner that is consistent with existing work on steady aerodynamics. We then go on to split up the unsteady flow in the same way, emphasising such frequently ignored flow features as the existence of skew in the wakes and the possibility of significant tip sources. This is followed by a discussion of duct acoustics which aims to show how for a wave of given circumferential mode order, we can view the solution in terms of acoustic rays, rather than the normal radial modes, with a considerable gain in our insight into the properties of the complete solution. Finally, we discuss how the unsteady flow through the blades and the duct acoustics can be matched together and how one could apply the solution to a typical aero-engine installation.

## MEAN FLOW

The purpose of this section is to outline the asymptotic structure of the mean flow through a row of high aspect ratio turbomachine blades. The essentials of this structure are shown on the diagram below.

We assume that the flow has two well defined length scales - a long length scale on which the flow outside the blade varies which is of the same order as the blade span and the radius, and a short length scale of the same order as the blade pitch and chord. We assume that these are in the ratio $\varepsilon$ , with $\varepsilon$ small, and additionally that (though this can be relaxed) the sweep and lean of the blades are of order $\varepsilon$ . It follows from this that if we consider the inviscid flow through these blades, it has the following structure.

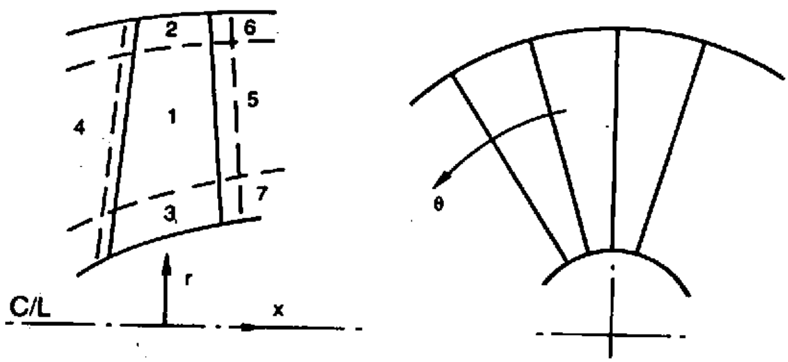


Fig 1 Asymptotic Regions of Flow Through a Blade Row

In region 1, the flow scales as $\phi = \phi_1(x/c, r\theta/c, \varepsilon r/c)$ (with c the chord), for a typical flow variable $\phi$ . To lowest order, when this is substituted into the equations of motion, the radial derivatives vanish and the flow is effectively two-dimensional with the blade row inlet conditions determined by matching to the outer flow. At this lowest order, the blade row appears to the outer flow as an actuator disc. At the next highest order, the flow is still two-dimensional and the inlet and exit conditions are modified slightly. Combining these two lowest orders gives a result that is equivalent to that found by the combined throughflow and blade-to-blade solutions of conventional turbomachinery analysis. The radial flow and blade sweep effects only enter at the lowest order.

In regions 2 and 3, the flow at first order is just as described above, but at the next order, there are three-dimensional tip corrections that depend on secondary flows generated by incoming vorticity and by the blade not meeting the wall at right angles. The second order flow thus scales as $\phi = \phi_1(x/c, r\theta/c, r-R/c)$, where R is the wall radius.

In regions 4 and 5, we have the region outside the blade row where the (inviscid) flow field scales at lowest order on the larger scale, so that $\phi = \phi_4(\varepsilon x/c, \varepsilon r\theta/c, \varepsilon r/c)$. At higher order there will be a flow component that is governed by the trailing vorticity shed from the blades and which will vary on the scale of the distance between the blade wakes. This rapid variation is also present in regions 6 and 7 close to the wall, on account of the rapid variation in the strength of the shed vorticity there.

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From the point of view of blade row interaction, probably the most important feature of the steady flow is that due to the viscous wakes shed from the blade trailing edges. Close to the blade row this will be a flow that has the form $\phi = \phi\,(\varepsilon x/c, r\theta - x\tan\alpha/c, \varepsilon r/c)$. That is, the flow varies rapidly in a direction perpendicular to the wake lines but slowly in a streamwise direction. It has the nearly two-dimensional structure of the blade-to-blade flow that created it. As this wake travels downstream, it is convected according as the amount of swirl behind the blade row.The upshot of this is that the wake sheet is skewed in space when it reaches the downstream stator. In a typical aero-engine fan installation, the distance between the rotor and stator is large so that the skewing is large. When we transfer, further, from the frame of reference rotating with the fan to a stationary one, moving at a relative speed U, say, we find that our wake pattern becomes $\phi = \phi_w(\varepsilon x/c, r\theta - x\tan\alpha - Ut/c, \varepsilon r/c)$. This pattern has clearly got the characteristics of a flow that varies rapidly in the cross-stream direction, has a time scale of order c/U and has a non-negligible phase that varies slowly radially. At the tip, where there are wake components due to secondary flow, the flow varies on a much shorter length scale.

In the above discussion, we have concentrated on the simplest case where the blades are nearly radial. If we have blades that have appreciable sweep and lean, this must be allowed for at the first order and the result is that the blade-to-blade flow will be of the form $\phi_1\,(x - X(\varepsilon r/c)/c, r(\theta - \Theta(\varepsilon r/c)/\varepsilon c, \varepsilon r/c)$, with $X, \Theta$ of order one. There will be a corresponding variation in the phase of the wake that these blades produce. In this case, also, the distortion of the flow due to the blade meeting the annulus wall is also large and occurs at first order. Further complications also arise if we have fully transonic flow when the blade-to-blade and throghflow solutions (regions 1,4 and 5) are more strongly coupled than has been assumed here.

## UNSTEADY BLADE-TO-BLADE FLOW

In this section, we describe the interaction of the wake flow discussed in the previous section with another downstream blade row. For this, we again consider two regions, the main flow region 1 and the tip regions 2 and 3. In region 1, we can write the flow due to the wake as a series of harmonics with typical form

$$u = u_w \exp(inB(\Omega t - \theta) - i\,\Phi(\varepsilon r/c)/\varepsilon) \qquad (1)$$

where we have assumed that the fan has B blades and $\Phi/\varepsilon$ is the phase due to he wake skew discussed above. At large rotor-stator gaps, it is a large quantity and must be allowed for in any unsteady flow calculation. Near any radius r, we can expand u given above as

$$u \sim u_w(r_o)\exp(inB(\Omega t - \theta) - i\Phi(\varepsilon r_o/c)/\varepsilon - (r - r_o/c)\Phi') \qquad (2)$$

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Given the previously discussed structure of the mean flow round the blade row, it follows that we can solve for the main part of the unsteady flow by a two-dimensional strip approach. This is just as in the conventional approach except that we are allowing for the radial phase variation through a radial wavenumber $\Phi'/c$.

Having defined the problem we must now solve it. At the present time, the only well developed methods for doing this as the flat plate cascade methods previously referred to, but in the future, there is considerable hope that we could allow for all the features of the real blade row, such as camber, mean flow turning, using the the various computational fluid dynamics techniques now under development.

When we reach the tip of the blade, its response is much more complicated and the unsteady flow is fully three-dimensional. It is useful to separate the wake into two parts - the sheet like component that we have just discussed, which is presumed to extend to the duct wall and a fully three-dimensional component that is only present at the tip. These generate unsteady pressure on the blades that are fully three-dimensional. These can,though, be split up into several bits. These are a correction to the field due to the sheet-like component of the wake and a component due to the fully three-dimensional wake component. The former is driven by a wake that consists of the difference between the flow that would exist if the wake went on to infinity through the duct wall and the wake plus its image in the duct wall - a sort of hairpin structure with the legs of opposite sign. At present, there is little that appears to be known about the solution of problems of this type.

These three-dimensional tip components while only affecting a small part of the blade, would be expected to have most impact a angles of sound propagation where the sound from the main part of the blade is weak. In these regions, the radiated sound could well be highly dependent on the details of the flow at the tips of the blade.

Finally, we should add an additional note of warning concerning the problem of acoustic resonance. At resonance conditions the strip theory solution given here will break down and the flow will be more highly three-dimensional.

## DUCT ACOUSTICS

In turbomachinery interaction noise problems such as we are concerned with here, it is usual to handle the problem of noise propagation in the duct by a modal approach. That is, the pressures on the flat plate blades are used as acoustic sources to which a modal Green function is applied and the resulting sound field is a series of duct modes. For each value of frequency and circumferential mode number (determined by the blade numbers and the rotational speed), there are many radial modes that can propagate. This is a fundamental consequence of the large ratio of duct height to wavelength and makes the solution unwieldy, particularly when we are looking at propagation over short distances such as between blade rows. Here, we show that we can use the large duct size to wavelength ratio to construct an asymptotic propagation theory that is equivalent to ray theory.

The use of ray theory in its usual form is now well established for the analysis of broadband noise propagation (see e.g. Kempton [2]). Here, we do not solve for rays in the usual three-dimensional sense, but in a two-dimensional context. We shall illustrate this by considering a duct with no mean flow for which the propagation of a disturbance of form $\exp(i\omega t - im\theta)$ satisfies the equation

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} - \frac{m^2}{r^2} \right) \phi = 0 \qquad (3)$$

This can be solved by ray acoustic methods by writing the solution is the usual form i.e. $\phi = A(x)\exp(-i\Phi(x)/\varepsilon)$ to leading order, with the wavenumber $k_0(=\omega/c)$ and m both taken as large quantities (of order $1/\varepsilon$) consistent with the theory of the previous section. Carrying this out, we find that we can characterise the rays by an angle of propagation $\theta$ , such that if $\theta_\infty$ is the value of $\theta$ at infinity, then at a radius r,

$$(k_0^2 - m^2/r^2)^{1/2} \cos\theta = k_0 \cos\theta_\infty \qquad (4)$$

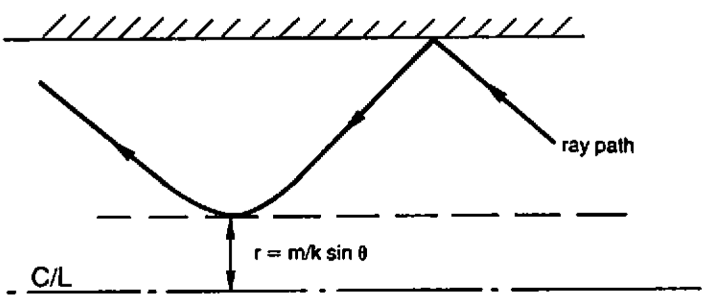


Fig 2 Ray Paths for Given Frequency and Circumferential order m

The rays themselves follow curved paths as shown in Fig 2. For any angle $\theta_\infty$, there is a minimum radius. Any source situated inside that radius cannot radiate (at least not on the ray theory assumption) to the far field. This is the same angle that a conventional approach would tell us is the limit for efficient radiation. If we set $\theta_\infty$ to 90 , we obtain a critical radius of r = k/m, below which a source will not radiate to any angle. Again, this is precisely the same angle that we would have obtained from the modal approach coupling a duct radiation model to the modal approach. The advantages of using this sort of ray approach are the following. First, it concentrates on the parts of the solution that are actually responsible for the radiation, rather than combining them into duct modes that are not necessarily related to the details of the radiation field. Second, by restricting the analysis at the outset to high frequencies, we have made it easy to include all the effects of the mean flow.

In applying the ray theory, it should be remembered that at any point in the duct, the sound level will be composed of rays that have suffered different numbers of reflections off the duct walls. Close to the source, only one of these is likely to be important, except close to the resonances (mode cut-on frequencies) where the rays add in phase, while in the far field, only the rays that propagate to a particular angle need to be studied

## MATCHING

In this section, we discuss how the ray theory of propagation discussed in the previous section can be connected to the blade-to-blade unsteady aerodynamics calculation. Traditionally, this is done by a Green function with the pressures on the blades as sources. This only works, however, when we have flat plate blades, not when the blades have camber and additional quadrupole sources are distributed throughout the flow. Here, we propose that the far field form of the blade-to-blade calculation is matched onto the form of the ray theory solution.

This can be done using the technique described by Tam and Burton [3], in the context of noise generation by jet instability waves. That is, we suppose the blade row to be equivalent to a source of form $q = A(\varepsilon r/c)\exp(-i\Phi(\varepsilon r/c)/\varepsilon)$ and show how in the near field $(x/c = O(1), \varepsilon \to 0)$ the field matches to the local blade-to-blade flow with the local gradient of $\Phi$ matching the local radial wavenumber and in the far field $(\varepsilon x/c = O(1), \varepsilon \to 0)$, we have the ray solution. The initial amplitude of the rays follows from the near field solution and the radius of curvature of their wavefronts from the curvature of the phase. It is found that each radial section of the blade sends out a ray in a particular direction. With typical wake geometries, these rays travel away from the axis. At the tips, the far field of the blade-to-blade tip sources is like that of a directional acoustic source situated at the tip and the rays from this propagate in all the available directions.

For acoustic sources situated inside the critical radius as discussed in the previous section, there is no matching onto the ray-like solutions, just as one would expect for a cut-off source.

## APPLICATION

In this section, we discuss first, the application of the work described above the calculation of tone levels from a particular

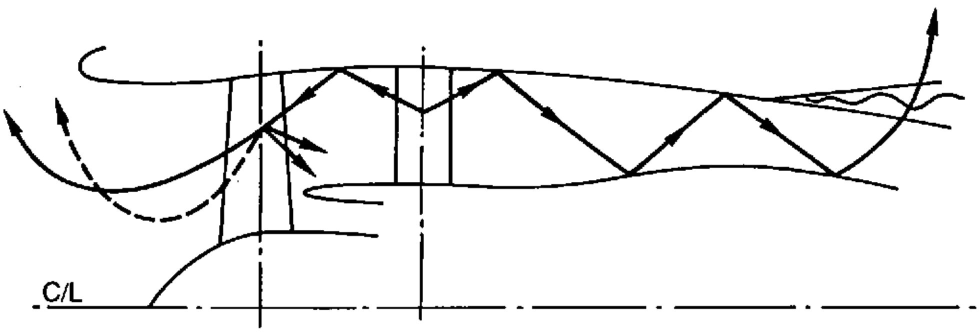


Fig 3 Typical Ray Paths in an Aero-Engine

fan rotor and stator combination. The main steps are as follows. First one works out the strength of the fan rotor wakes as they appear at the leading edge of the stator row. This gives us the amplitude and phase of the incoming disturbances to the stator row. The response of the stator is then worked out in a strip theory fashion, taking due account as we have described, of the radial wavenumber of the wake and also of the tip corrections. Next, the matching procedure is used to give the amplitude and direction of the rays leaving the blades. It is probably best to regard there rays as being driven by sources placed on a plane in the middle of the stator row as shown on Fig 3. These rays then propagate forward to the fan rotor and backwards through the exhaust system. When the rays hit the fan rotor, they are scattered. This process can best be thought of in a frame of reference rotating with the fan. Again, the flow must be solved by strip theory, but taking into account the radial wavenumber of the incoming wakes. The calculation of sound transmission through the fan will cause the energy to be scattered into different circumferential modes, which viewed from a stationary reference frame will correspond to rays propagating in different directions. That is, each ray that hits the rotor will be split as shown on Fig 3 into a number of rays. Their amplitudes are found by matching in essentially the same manner as we have previously described. The subsequent propagation along either the inlet or exit ducts can then be calculated by a ray approach. This has the possible complication that it may be difficult to account properly for the diffraction of sound to angles that direct rays do not reach, but is attractive in that it can handle the effects of changes in the mean flow, which are quite complex for the inlet and exhaust flows, can be handled. Alternatively, we can allow for diffraction by switching to a modal approach at some point.

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In the preceding discussion, we have concentrated on the familiar case of fan rotor-stator interaction. We shall now mention other ways in which the theory can be applied. First, in many engines, the source of noise will not be the bypass section stators, but the smaller engine section (core engine) stators. The theory can be applied to these but it should be noted that they will be too short to be handled by strip theory. The unsteady flow will be fully three-dimensional. There is a possibility of this being simplified by splitting the flow up into radial modes. The theory can easily be applied to complex configurations with counter-rotating blades, the only complication being the large number of different harmonics that can result from the interaction of pairs of rotating blade rows with different blade numbers and speeds. The theory could also be applied to the case of an unducted flow such as is present in a prop-fan. Here, there are no complications due to reflections off the duct walls and diffraction, but there is an extra source due to the tip vortices shed from the tips of the front rotor interacting with the rear rotor.

The theory can also be used to justify the ray treatment of broadband noise [2]. One notes that if the broadband noise is generated by turbulence of a scale of the same order as the blade chord, it will be legitimate to treat each section of the blade as if it is locally of infinite radial extent - the strip theory approach again. Each correlated source region acts as a directional source with respect to the main flow and propagation can be handled by ray theory.

Finally, we point out that there are many aspects of the problem that can be simplified in some circumstances. For example, at high frequencies and Mach numbers, the wavelengths will be smaller than the chord and the cascade effects will be negligible, with the result that the sound generation process can be modelled as the interaction of flow with a single flat plate aerofoil. The fact that we have decoupled the acoustics from the blade response means that approximations made in one part of the calculation do not affect another part.

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