Proceedings of the Institute of Acoustics

GENERATION AND CONTROL OF COMBUSTION NOISE

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1 INTRODUCTION

Unsteady combustion is an effective source of sound and can generate intense pressure fields. In Section 2 we review combustion in free space. It is a strong monopole acoustic source and when the burning region is compact on a wavelength scale, the far-field pressure perturbation is proportional to the time derivative of the total rate of heat release evaluated at retarded time.

Whenever combustion takes place within an acoustic resonator, there is the possibility of self-excited oscillations which can lead to high intensity sound fields. These occur because the sound field within a resonator can be strong enough to alter the instantaneous rate of heat release. Instability is then possible because, while the acoustic waves perturb the combustion, the unsteady combustion generates yet more sound. An investigation of the energetics of acoustic waves within the resonator indicates what measures should be taken to reduce the pressure amplitude. Both passive and active means of eliminating the oscillation are discussed in Section 3 with particular emphasis on the emerging technique of active instability control.

2 UNSTEADY COMBUSTION IN UNBOUNDED SPACE

Unsteady combustion can be an efficient acoustic source and the Lighthill theory [1] provides a convenient description of the sound generation. The Navier-Stokes equation and the equation of mass conservation may be combined to give an inhomogeneous wave equation, which we write in the form

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j - \tau_{ij}) - \frac{\partial^2 \rho_e}{\partial t^2} . \qquad (2.1)$$

p is the pressure, ρ the density, u the particle velocity, c the sound speed and τ_{ij} the viscous stress tensor. ρ_e is the 'excess' density.

$$\rho_e = \rho - \rho_0 - (p - p_0)/c_0^2 , \qquad (2.2)$$

where the suffix o denotes a mean value in the distant acoustic field. ρ_e therefore vanishes in the far-field but is non-zero in regions where the temperature is significantly different from ambient. Jet noise is generated by the quadrupole source, $\rho u_i u_j$, while the term $\partial^2\rho_e/\partial t^2$ describes combustion noise. The strength of this source can be determined from an investigation of the thermodynamics of the source region.

When heat is released at a rate $q(\underline{x},t)$ per unit volume, the entropy equation [2] leads to

$$\rho T \frac{Ds}{Dt} = q + \tau_{ij} \frac{\partial u_i}{\partial x_j} . \tag{2.3}$$

The entropy of a material element is increased both by the addition of heat and by the work done by viscous forces. Now, from the chain rule,

$$\frac{D\rho}{Dt} = \frac{1}{c^2} \frac{Dp}{Dt} + \left.\frac{\partial \rho}{\partial s}\right|_p \frac{Ds}{Dt} , \tag{2.4}$$

where $c^2 = \partial p/\partial \rho|_s$ is the square of the local speed of sound. Differentiation of the perfect gas relation, $s = c_v \ell n p - c_p \ell n \rho$, leads to

$$\left.\frac{\partial \rho}{\partial s}\right|_p = - \frac{\rho}{c_p} . \tag{2.5}$$

Hence, it follows from a combination of equations (2.3) - (2.5) that

$$\frac{D\rho}{Dt} = \frac{1}{c^2} \frac{Dp}{Dt} - \frac{1}{c_p T} \left[q + \tau_{ij} \frac{\partial u_i}{\partial x_j}\right] , \tag{2.6}$$

which can be rewritten as

$$\frac{D\rho}{Dt} = \frac{1}{c^2} \frac{Dp}{Dt} - \frac{\gamma - 1}{c^2} \left[q + \tau_{ij} \frac{\partial u_i}{\partial x_j}\right] , \tag{2.7}$$

since $c^2 = \gamma RT = c_p(\gamma - 1)T$ in a perfect gas, where γ is the ratio of specific heat capacities. Equation (2.7) describes how both pressure fluctuations and heat input change the density of a compressible fluid.

Before the thermodynamic relationship in (2.7) can be used to substitute for $\partial^2 \rho_e / \partial t^2$ in (2.1), one of the partial time derivatives must be replaced by a material derivative. Straightforward algebraic manipulation leads to

$$\frac{\partial \rho_e}{\partial t} = \frac{D\rho_e}{Dt} - \frac{\rho_e}{\rho} \frac{D\rho}{Dt} - \nabla.(\mathbf{u}\, \rho_e) , \tag{2.8}$$

where the continuity equation has been used to replace $\nabla.\mathbf{u}$ by $-\rho^{-1} D\rho/Dt$. Once ρ_e in (2.8) is written explicitly as $\rho - \rho_o - (p - p_o)/c_o{}^2$, equation (2.7) can be used to relate $D\rho/Dt$ to the rate of heat input. This leads to

$$\frac{\partial \rho}{\partial t} = - \frac{\rho_o(\gamma - 1)}{\rho c^2}\left[q + \tau_{ij} \frac{\partial u_i}{\partial x_j}\right] - \nabla.(\mathbf{u}\, \rho_e) - \frac{1}{c_o^2}\left[\left[1 - \frac{\rho_o c_o^2}{\rho c^2}\right]\frac{Dp}{Dt} - \frac{(p - p_o)}{\rho} \frac{D\rho}{Dt}\right] . \tag{2.9}$$

The inhomogeneous wave equation (2.1) is therefore equivalent to

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial}{\partial t}\left[\frac{\rho_0(\gamma-1)}{\rho c^2}\left[q + \tau_{ij}\frac{\partial u_i}{\partial x_j}\right]\right] + \frac{\partial^2}{\partial t \partial x_i}(u_i \rho_e)$$
$$+ \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j - \tau_{ij}) + \frac{1}{c_0^2}\frac{\partial}{\partial t}\left[\left[1 - \frac{\rho_0 c_0^2}{\rho c^2}\right]\frac{Dp}{Dt} - \frac{(p - p_0)}{\rho}\frac{D\rho}{Dt}\right] . \quad (2.10)$$

The first term on the right-hand side of equation (2.10) describes the sound generated by unsteady heat addition and is of monopole type. The second term in (2.10) accounts for the dipole sound generated by a change of momentum of fluid particles with a density different from ambient. The third term involves the familiar quadrupole source of Lighthill's jet noise theory. It is well known that it leads to an acoustic intensity that scales on Mach number to the eighth power. The fourth term is only appreciable if there are regions of unsteady flow with different mean density and sound speed from the ambient flow. This term also leads to an M^8 scaling, but with a different coefficient from the jet noise term [3].

When there is unsteady combustion, the first term on the right-hand side of (2.10) is non-zero. For a low Mach number flow, this monopole generates a far larger sound field than the other source mechanisms in (2.10) and we can write down the pressure perturbation in the distant sound field.

$$(p - p_0)(\underline{x},t) = \frac{1}{4\pi|\underline{x}|}\frac{\partial}{\partial t}\int\left[\frac{\rho_0(\gamma-1)q}{\rho c^2}\right] dV . \quad (2.11)$$

The square brackets denote that the function they enclose is to be evaluated at retarded time [4].

If the combustion takes place at ambient pressure and γ is assumed to be independent of temperature, $\rho c^2 = \gamma p_0 = \rho_0 c_0^2$. The expression in (2.11) then simplifies to

$$(p - p_0)(\underline{x},t) = \frac{\gamma - 1}{4\pi|\underline{x}|c_0^2}\frac{\partial}{\partial t}\int [q] dV . \quad (2.12)$$

This predicted form for combustion generated noise is consistent with that derived by Chiu and Summerfield [5] and Ffowcs Williams [6], but differs by a factor ρ_0/ρ from two earlier theories of combustion noise, [7] and [8]. This discrepancy is resolved in [3], where it is pointed out that the early theories omit significant source terms. Once these are included, they lead to expressions in agreement with (2.12).

When the flame is compact on a wavelength scale, equation (2.12) predicts that the far-field acoustic pressure perturbation should be proportional to the time derivative of the total rate of heat release evaluated at retarded time. Hurle et al [9] confirmed this relationship experimentally by using the light emission from short-lived CH or C_2 free radicals as a measure of the instantaneous rate of combustion. Since their pioneering work the relationship has been verified using more advanced signal processing [10].

Turbulence within furnaces and boilers inevitably leads to fluctuations in the rate of combustion and this can lead to intense sound fields. For example 1 kW of heat modulated at 1 kHz leads to a sound pressure level of about 100 dB 1 m away from the heat source, and generates 0.1 W of acoustic power [6].

Dines [11] has applied the techniques of anti-sound to reduce the radiated noise from an open, turbulent flame. His experimental geometry is illustrated in Figure 1. The light emission from CH free radicals in the flame is used to provide an instantaneous measure of the rate of heat release and hence of the combustion-related acoustic source strength. This signal is processed and used to drive a loudspeaker adjacent to the flame. The aim is essentially to drive the loudspeaker so that it generates a monopole sound field in anti-phase with the combustion generated noise. The superposition of the two fields would then lead to silence. The time delays inherent in the microprocessor and the loudspeaker impose constraints on the performance of

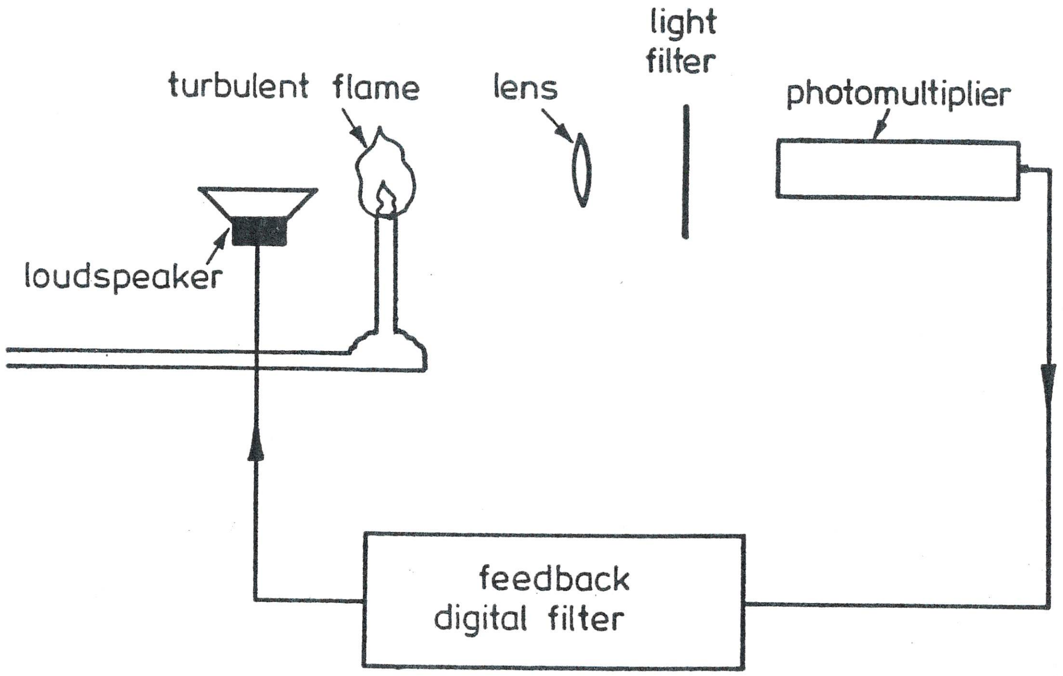

Fig. 1: Anti-sound applied to the noise of a turbulent flame (Dines [11])

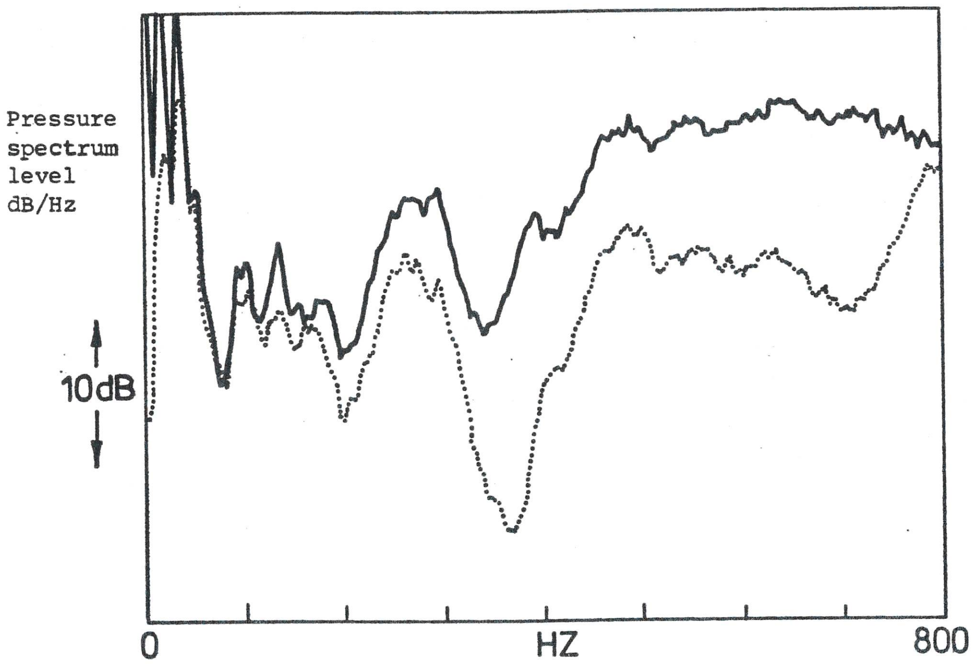

Fig. 2: The noise of a turbulent flame, without --- and with —— anti-sound (Dines [11])

such a system. Nevertheless Dines was able to obtain a 10 dB reduction over a 300-700 Hz bandwidth (see Figure 2.)

3 UNSTEADY COMBUSTION WITHIN A RESONATOR

Whenever combustion takes place within an acoustic resonator, there is the possibility of self-excited oscillations which can lead to high intensity sound fields. These occur because the sound field within a resonator can be strong enough to alter the instantaneous rate of heat release. Instability is then possible because, while the acoustic waves perturb the combustion, the unsteady combustion generates yet more sound!

Combustion instabilities occur in many practical devices. Rockets, ramjets and gas burners are all susceptible. More than one boiler has been unable to attain its design output because of the onset of damaging oscillations. The occurrence of a combustion instability is invariably detrimental. The oscillations can become so intense that they cause structural damage. Alternatively they may enhance the heat transfer leading to overheating, or the perturbations may simply become so violent that the flame is extinguished.

Two excellent reviews describing different types of combustion oscillations have been given recently [12], [13]. In this section we will concentrate on ways of eliminating combustion oscillations, with particular emphasis on the emerging technique of active instability control.

At Cambridge we have been particularly concerned with the combustion instabilities that occur in the afterburners of aeroengines. The fluid emerging from a gas turbine is reheated in an afterburner to provide additional thrust. More fuel is injected and burnt in the wake of a bluff-body flame holder, as illustrated in Figure 3. Increases in the reheat fuel-air ratio increase the thrust but unfortunately the flame becomes susceptible to a low frequency combustion instability called "reheat buzz". This involves interaction between longitudinal pressure waves in the afterburner and the combustion. Similar waves have been detected on laboratory rigs. There schieren photographs [14] have shown that, as these acoustic waves alter the velocity at the flame holder, the flame moves, changes in surface area and alters the instantaneous rate of heat release. If the phase relationship between the pressure perturbation and the rate of heat release is suitable, the acoustic waves gain energy from the combustion. Disturbances grow in magnitude if this energy gain is greater than that lost by radiation at the ends of the duct.

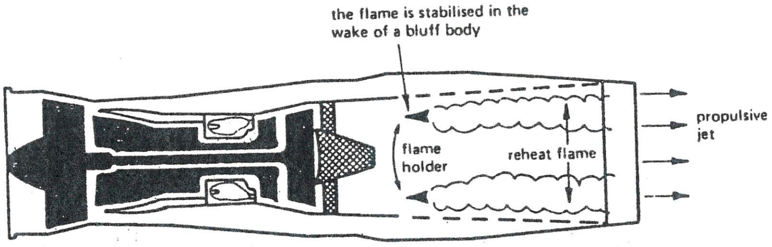

Fig. 3: A gas turbine with an afterburner

Other combustion oscillations may differ in the details of the geometry and the form of the coupling between the acoustic modes and the rate of heat release. However, they can all be explained by investigating the energetics of the acoustic waves. This also indicates what measures should be taken to eliminate the onset of a combustion instability.

Rayleigh [15] gives a clear physical interpretation of the interchange of energy between sound waves and unsteady heating. He states that the amplitude of a sound wave will increase when heat is added in phase with its pressure. The addition of heat out of phase with the pressure reduces the amplitude. Chu [16] adopts a more mathematical approach and his method enables the effects of the boundary conditions to be clearly displayed.

Following Chu let us consider a perfect gas burning within a combustor of volume V, bounded by the surface S, as illustrated in Figure 4. For

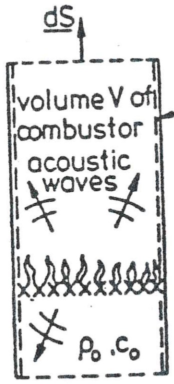

Fig. 4: Combustion within a resonator

simplicity we will just consider the gas to be linearly disturbed from rest with no mean temperature gradients or mean heat release. Then equation (2.7) simplifies to

$$\frac{\partial \rho}{\partial t} = \frac{1}{c_0^2}\frac{\partial p}{\partial t} - \frac{\gamma - 1}{c_0^2} q \ . \tag{3.1}$$

(Extensions can be made to include mean heat release and mean flow [17] and the interaction between liquid and gaseous phases [13].) When equation (3.1) is combined with the linearized equation of mass conservation, $\partial\rho/\partial t + \rho_0 \nabla . u = 0$, it leads to

$$\frac{1}{c_0^2}\frac{\partial p}{\partial t} + \rho_0 \nabla . u = \frac{\gamma - 1}{c_0^2} q \ . \tag{3.2}$$

The linearized momentum equation has the usual form:

$$\rho_0 \frac{\partial u_i}{\partial t} + \frac{\partial p}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j} \ . \tag{3.3}$$

Equations (3.2) and (3.3) may be combined to form an acoustic energy equation. When equation (3.2) is multiplied by $(p - p_0)/\rho_0$ and added to the product of (3.3) with u_i, it yields, after integration over the volume V,

$$\frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\rho_o u^2+\frac{1}{2}\frac{(p-p_o)^2}{\rho_o c_o^2}\right]dV=\frac{\gamma-1}{\rho_o c_o^2}\int_V (p-p_o)q\,dV$$

$$-\int_S((p-p_o)u_j-u_i\tau_{ij})dS_j-\int_V\frac{\partial u_i}{\partial x_j}\tau_{ij}dV \qquad (3.4)$$

The term on the left-hand side of equation (3.4) is the rate of change of the sum of the kinetic and potential energies within the volume V. The first term on the right-hand side describes the exchange of energy between the combustion and the acoustic waves. As noted by Rayleigh the acoustic energy tends to be increased when $p - p_o$ and q are in phase. The surface term describes the rate at which fluid within the volume V does work on the surroundings, and accounts for the loss of energy across the boundary surface S, while the last term is just the rate of viscous dissipation.

Equation (3.4) states that disturbances grow if their net energy gain from the combustion is greater than their energy loss across the boundary. Therefore the acoustic mode grows in amplitude if

$$\frac{\gamma-1}{\rho_o c_o^2}\int_V\overline{(p-p_o)q}\,dV>\int_S\overline{((p-p_o)u_j-u_i\tau_{ij})}\,dS_j+\int_V\overline{\tau_{ij}\frac{\partial u_i}{\partial x_j}}\,dV\ , \qquad (3.5)$$

where the overbar denotes an average over one period of the acoustic oscillation. When it is satisfied, the combustor has a combustion instability. Linear waves increase in amplitude until limited by nonlinear effects.

The inequality in (3.5) demonstrates how to eliminate a combustion instability. Either the driving term, $\int\overline{(p-p_o)q}\,dV$, should be reduced or the damping in the system should be increased. Classical techniques exploit passive ways of producing this change. They involve the insertion of baffles within the combustor or the connection of quarter-wave tubes or Helmholtz resonators to the walls. These suppression devices are described fully in [18]. However, the mean pressure drop associated with acoustically appropriate baffles is too great for their installation to be practical in aeropropulsion applications. Moreover tuned resonators for low frequency instabilities are prohibitively large and could not be used in flight. Passive damping is ineffectual at the low frequencies of reheat buzz. For that reason interest has turned to active ways to reversing the inequality in (3.5). The idea of using active control is not new. Theoretical exercises carried out in the 1950s showed that servo-stabilization should eliminate combustion instabilities [19], [20], [21]. But experimental verification of such a scheme is far more recent. Probably the first demonstration was on a simple Rijke tube.

The Rijke tube is a vertical pipe containing either a heated grid or a flame stabilized on a gauze. The tube has a thermoacoustic instability whenever the gauze is in the lower half of the tube. Then the sound level builds up to a large amplitude with a frequency close to the organ pipe resonance frequency. When the gauze is in the upper half of the tube, the flow is stable. Rayleigh [15] explains this phenomenon. He notes that in the Rijke tube the unsteady rate of heat release lags velocity changes. An investigation of the relationship between pressure and velocity fluctuations at the pipe's fundamental frequency then shows that $\overline{(p-p_0)q}$ is only positive when the heating element is in the lower half tube. It is clear from the inequality (3.5) that it is only for these locations that the fundamental mode can grow in amplitude.

Equation (3.5) also shows that such an unstable Rijke tube can be stabilized by the addition of a controlling heater element to the upper part of the duct, where the unsteady heat release rate is out of phase with the pressure. This has been demonstrated to be effective by several authors [22], [23] and [24]. Interestingly, the power requirement of the controlling heater is considerably less than the power of the primary heat source [24]. This is because the controller does not need to entirely cancel the effect of the primary source. It need only reduce $(\gamma-1)\int\overline{(p-p_0)q}\, dV/\rho_0 c_0^2$ to a level below that of the energy losses. Then, as seen from (3.5), the perturbations slowly decay. Kidin et al. [25] describe an alternative source of unsteady heating. They use a modulated current discharge between high voltage electrodes, and report some success in delaying the onset of combustion oscillations in both laminar and turbulent burners.

Equation (3.5) also demonstrates that combustion oscillations can be stabilized by actively changing the boundary conditions of the tube to increase the acoustic energy lost on reflection. Dines [11] eliminated the instability of a laminar flame burning on gauze in a Rijke tube in this way. He detected the phase of oscillation by measuring the light emitted by CH free radicals in the flame, and altered the boundary condition by driving a loudspeaker near one end of the tube.

Heckl [23, 26] investigated a similar system with a microphone as the detector instead of the photomultiplier used by Dines. Her geometry is illustrated in Figure 5. The signal from the microphone was passed through a narrow-band filter, phase-shifted and amplified and used to drive the loudspeaker. The optimum phase shift was found by inspection. Control reduced the peak in the pressure spectrum to the background noise level. Heckl demonstrated that the imposed phase delay is not crucial to the performance of the controller and that stability can be achieved for a range of phases. This illustrates how different active control is from conventional "anti-sound". There a sound wave is introduced precisely in anti-phase with an existing wave, cancelling it and leading to silence. In contrast the aim of this feedback is just to stabilize the oscillations. All that is required is that $\int\overline{(p-p_0)\mathbf{u}}.d\mathbf{S}$ be made sufficiently large. The phase of the feedback signal may be varied by nearly 180° for large values of gain.

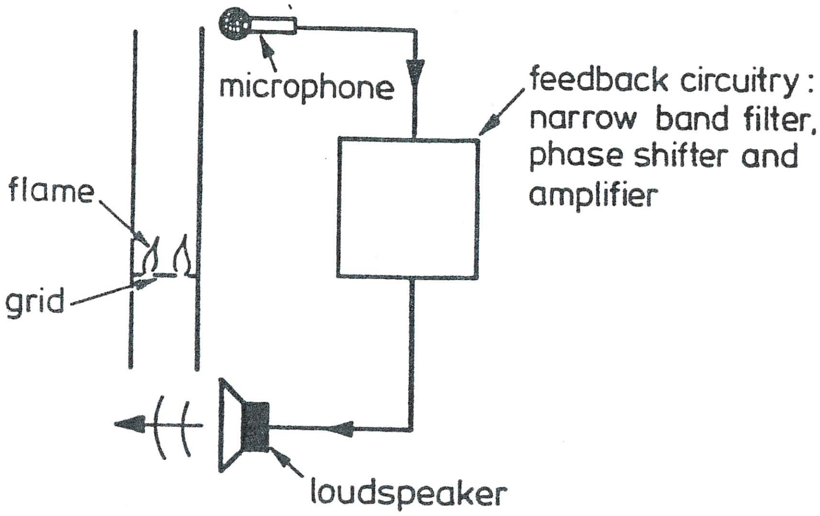

Fig. 5: Active control of a Rijke tube (Heckl [23], [26])

The work of Dines and Heckl has been confirmed in an experiment reported by Lang et al. [27]. Their geometry differed slightly, since their duct had one open and one closed end and the microphone and loudspeaker were mounted on the duct walls. Feedback could again reduce the pressure oscillations to the background noise level. Lang et al. investigated the effect of different loudspeaker positions. They found that, for a fixed microphone and a suitable phase delay, the optimum location for the loudspeaker is near a pressure antinode. This can be readily understood by inspecting the inequality in equation (3.5). With the loudspeaker modelled by a simple monopole source whose phase is chosen appropriately, $\int \overline{(p-p_0)u}.dS$ is maximised when the loudspeaker is placed at a pressure maximum.

The input power to the loudspeaker was measured in Lang et al.'s experiments. When the oscillation was controlled, the power requirement of the loudspeaker was found to be almost negligible. This again emphasizes the difference between active control and anti-sound. Here the control stabilizes the system, reducing the level of all the fluctuations and, in particular, reducing the driving term $\int \overline{(p-p_0)q}\, dV$. We see then, again from equation (3.5), that very little input is required to prevent those low level oscillations from growing.

The control of a combustion oscillation of a turbulent diffusion flame is reported by Poinsot et al. [28]. The flow rate is only 0.024 kg/s but nevertheless the flame is turbulent. They use essentially the same control system as Lang et al. [27], a microphone and a loudspeaker. Control was found to reduce the pressure spectrum by some 24 dB. The residual background noise of the turbulence prevents the complete elimination of any narrow-band peak in the pressure spectrum that was obtained for the laminar flame.

A rig has been set up at Cambridge to model an afterburner and investigate reheat buzz. The geometry is illustrated in Figure 6. Air is supplied at constant pressure and temperature to a choked constriction so that the mass flow rate remains constant irrespective of disturbances downstream. In a similar way, the fuel (ethylene) is supplied through a manifold with ten choked holes. The fuel and air mix well in the constriction, thus supplying a premixed gas of uniform fuel-air ratio and constant mass flow rate to the working section. This premixed gas is burnt in the working section, the flame being stabilized in the wake of a bluff body flame-holder. The whole of the burning length is visible through quartz ducting.

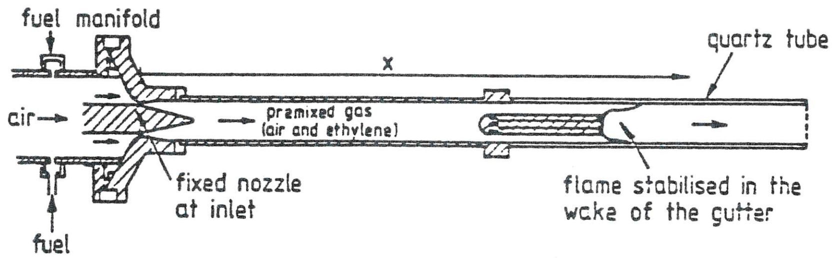

Fig. 6: Geometry of the Cambridge reheat buzz rig

At a typical running condition, the mass flow rate is 0.14 kg/s, the mean flow velocity upstream of the flame is 27 m/s, the turbulence intensity is 10% and about 1/4 MW of heat is released within the duct. Oscillations in the flow are monitored by measuring the pressure perturbation $p'(x,t)$ at various axial positions along the duct. In addition, the light emission from C_2 radicals in the flame is recorded. To a good approximation the instantaneous heat release rate is proportional to this light emission [9, 29].

The upstream end of the working section is choked and its downstream end is open. These boundary conditions mean that the fundamental mode shape is approximately a quarter-wave, and measurements show p' and q to be in phase along most of the duct. It is then evident from the inequality in (3.5) that the driving towards instability is strong, and indeed the observed sound pressure levels can exceed 160 dB. Because the mode is a quarter-wave p' and q are in phase for any position of the flame-holder and the instability cannot be reduced by introducing a second burner as for the Rijke tube. Our first attempts at control therefore concentrated on actively changing the boundary condition [30].

The overall levels of heat release and the intensity of the oscillation mean that the environment is too harsh for loudspeakers to be effective sound sources. Instead for active control the fixed choked nozzle at inlet to the working section is replaced by a choked plate and a moveable centre-body, as illustrated in Figure 7. The choked plate ensures that the mean flow supplied to the rig remains unchanged when the controller is switched on. The centre-body can be moved axially by a vibrator. This alters the blockage to the flow, thus supplying a fluctuating mass flow into the working section and modifying the upstream boundary condition.

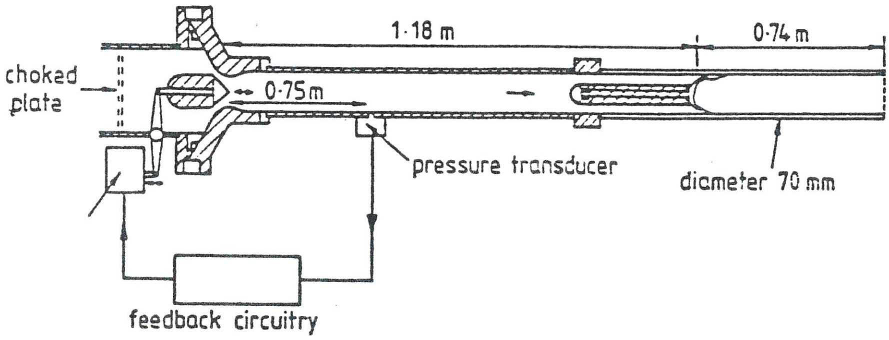

Fig. 7: Reheat buzz rig with active control

The active control experiments were performed for the geometry shown in Figure 7 at an inlet Mach number of 0.08; inlet static temperature of 288 K, and an equivalence ratio of 0.663 (i.e. the fuel-air ratio is 66.3% of the stoichiometric fuel-air ratio). An unsteady pressure signal upstream of the flame was filtered, amplified and phase-shifted and used as input to the vibrator. The optimum phase shift was found by inspection.

In Figure 8 the flow perturbations in the controlled case are compared with those that occur when the centre-body is clamped at the same mean position. In the uncontrolled case the pressure spectrum measured by an upstream

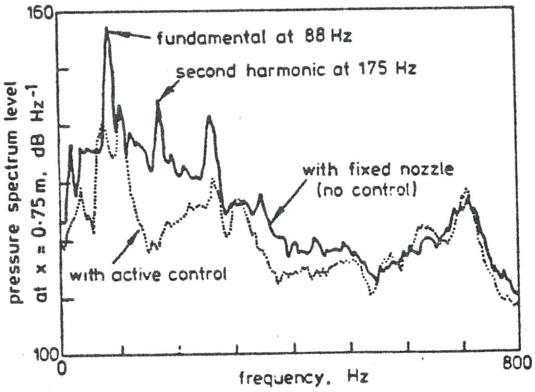

Fig. 8: Effect of active control on the pressure spectrum (0-800 Hz bandwidth)

transducer has an intense peak at 88 Hz. As one might expect, the change in the boundary condition achieved actively alters both the frequency of this peak and its amplitude, producing a reduction of some 20 dB. The second harmonic has been completely eliminated. Indeed the acoustic power in the

0-800 Hz bandwidth is reduced to 11% of the power at the same mean flow conditions but with a fixed upstream nozzle.

We have developed a theory to describe combustion oscillations in the basic uncontrolled rig [31]. Although the sound field is intense, with a sound pressure level of 160 dB, the pressure perturbation is small in comparison with the atmospheric pressure, typically $p'_{rms}/p_0 \sim 0.02$. Similarly the acoustic particle velocity is small in comparison with the speed of sound. The flow perturbations can therefore be described by a linear theory, and our theory is essentially a linear stability analysis of a flame burning within a duct. For linear disturbances each Fourier element may be analysed separately and it is sufficient just to consider disturbances with time dependence $e^{i\omega t}$. We solve the equations of conservation of mass, momentum and energy to find the eigenfrequencies, ω, for which the acoustic boundary conditions at both ends of the duct are satisfied. The real part of ω then specifies, the frequency of this mode, while the sign of the imaginary part of ω determines whether linear disturbances grow or decay. The theory has been tested extensively by comparison with experimental results for a wide range of inlet conditions [32].

The same theory can be applied to the active control geometry. The only difference is that the upstream boundary condition now has a more complicated form. There is a perforated plate with choked holes and a nozzle with either a clamped or oscillating centre-body. Rather than attempt to calculate the reflection of acoustic waves from such an arrangement, we took a pragmatic view and measured it. The results of using this measured boundary condition in the theory are shown in Figures 9 and 10.

With the control switched off and the centre-body clamped, the eigenfrequency is found to have a negative imaginary part. This means that, according to linear theory, disturbances grow exponentially with time and the flow is unstable. In practice, perturbations increase in amplitude until limited by nonlinear effects. Figure 9 compares the calculated mode shape for the pressure fluctuations with the experimental data. There is excellent agreement, although the calculated frequency 95.6 Hz differs by 8% from the measured value of 88 Hz.

With the control switched on, the changed upstream boundary condition alters both the mode shape and frequency. Now ω is found to have positive imaginary part, indicating that the flow has been stabilized. Figure 10 compares the calculated and measured mode shapes. Once again, the agreement is good. The calculated frequency of 78.0 Hz is in excellent agreement with the measured value of 79 Hz.

Although this method of actively changing the boundary condition has been successful on a 1/4 MW burner, it would not be easy to implement on a fullsize afterburner. Instead, in the search for a more practical means of control, we return to equation (3.5). This inequality demonstrates that an alternative form of control could be achieved by adding extra fuel unsteadily. Our aim

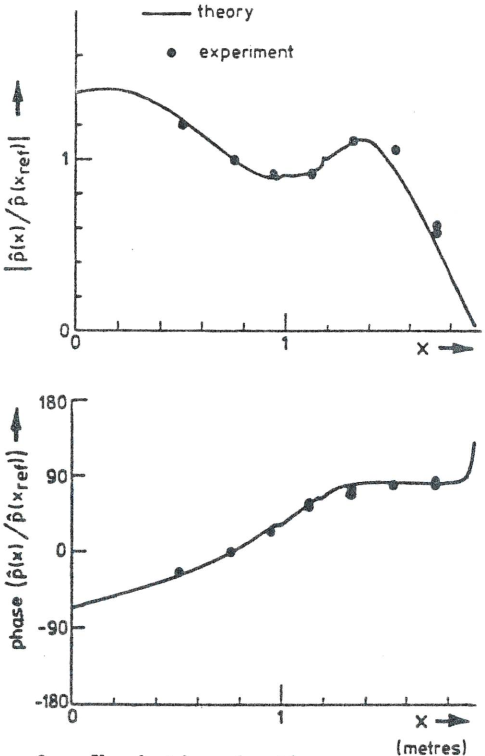

Fig. 9: Variation in the pressure along the duct at the buzz frequency without control; comparison of theory and experiment

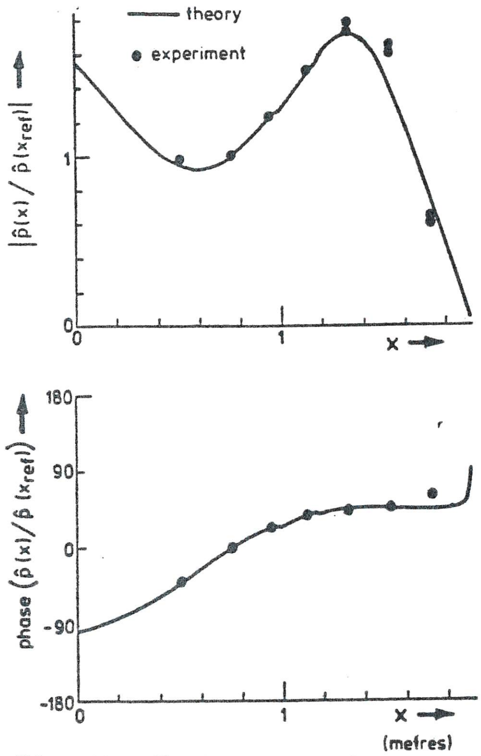

Fig. 10: Variation in the pressure along the duct at the buzz frequency for the optimum control case; comparison of theory and experiment

would be to produce an additional term in the instantaneous heat release rate q, with phase chosen to reduce $\overline{\int(p-p_o)q\,dV}$. The effectiveness of such a controller has been demonstrated on the experimental rig at Cambridge, and is described in detail by Langhorne et al. [33].

The pressure perturbation, $p'(x_{ref},t)$, at a location upstream of the burning region is used as input to the controller. This signal is time-delayed by an amount τ and automotive fuel-injectors are used to pulse additional fuel into the rig whenever $p'(x_{ref}, t-\tau)$ is positive. For negative $p'(x_{ref}, t-\tau)$ the injectors remain closed. The fuel supplied to the working section when the controller is switched on depends on both the length of time the fuel injectors are open and the mean pressure in the supply to the fuel injectors, which we will denote by $\bar{p}_{inj}$. It is not obvious that such feedback will stabilize the system. The transfer function between the input voltage to the fuel injectors and the response $p'(x_{ref}, t)$ was measured and then standard control theory was used to investigate the effectiveness of such a control strategy [33]. It predicted the optimal time delay to be τ = 3.9 ms.

The effect of the time delay across the feedback circuit on the measured pressure band level (PBL) is shown in Figure 11 for two values of the injector back pressure, $\bar{p}_{inj}$. A reduction in the amplitude of the oscillation is obtained for a range of values of τ, while the optimum performance is for τ close to the theoretical optimum of 3.9 ms. Figure 12 shows experimental

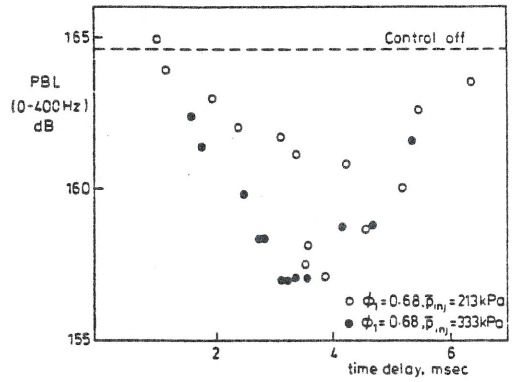

Fig. 11: Effect of time delay across the feedback circuit on the measured pressure band level. The PBL without control is shown by the dotted line.

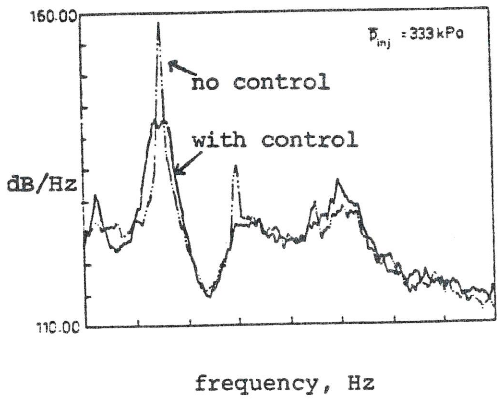

Fig. 12: Effect of active control on the measured pressure spectrum at x_{ref}, for τ = 3.4 ms

results for a time delay of 3.4 ms. "Control on" involves the addition of only 3% more fuel and yet this reduces the peak in the pressure spectrum by more than 12 dB. The higher harmonics are completely eliminated and the pressure band level (0 - 400 Hz) is reduced by 7.5 dB. The control is found to increase the overall rate of heat release within the duct. However, the main advantage is that the controller enables a flame to burn in the rig at a higher fuel-air ratio than is possible without control. This results in an increase in the maximum available thrust.

4 CONCLUSIONS

Combustion noise can become particularly intense when the combustion takes place within an acoustic resonator. Instability is then possible because the flow unsteadiness in the acoustic waves alters the instantaneous rate of combustion, which generates yet more sound. These combustion instabilities can be stabilized by active control. This not only reduces the amplitude of potentially damaging oscillations, but also enables a flame to burn at higher fuel-air ratios within a resonator than is possible without control. The overall heat release rate is therefore increased.

One technique for achieving control is the suitably phased addition of extra fuel. This is easy to implement and we eagerly await its application at full-scale.

Acknowledgement

The Cambridge work described in Section 3 has been carried out in conjunction with Drs P J Langhorne, G J Bloxsidge and M A Macquisten, with the technical assistance of Mr Hooper and the financial support of Rolls-Royce plc.

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