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A ROBUST DIRECTION OF ARRIVAL ESTIMATION ALGORITHM FOR PLANAR ARRAYS TO DETECT COHERENT SOURCES IN NOISY ENVIRONMENT

Arun Prakash Singh and Nachiketa Tiwari

Indian Institute of Technology Kanpur, U.P.-208016, India.
email: ntiwari@iitk.ac.in

There are several methods of microphone array processing for estimating the location of a sound source. For localization of sound source, the Direction of Arrival (DOA) of signal is estimated. A typical sound source has two angular co-ordinates; the azimuth angle and the elevation angle. Linear microphone arrays provide information only on the azimuth angle of the sound source. In contrast, a planar array may be used to detect both these angles. Several algorithms used for planar arrays work well only if there are not multiple coherent sources. These methods, like MUSIC, resolve the angular positions of these sources by using only the noise subspace of the overall signal. In this work, we overcome the problem of multiple coherent sources by making use of the signal subspace as well as noise subspace for sound localization purposes. Secondly, MUSIC and similar algorithms do not work well for signals with low values of SNR. The work shown here demonstrates that multiple incoherent noisy signals can also be located satisfactorily. Finally, in-vogue MUSIC and similar algorithms require a large number of sensors, especially to localize multiple coherent sources. The method presented in this work addresses this limitation as well, as it requires lesser number of sensors. Thus, in an overall sense, the method proposed in this work is less expensive as it requires lesser number of sensors, can handle noisy signals effectively, and works for multiple coherent sources present.

Keywords: Music, DOA, Microphone Array Signal Processing, Signal Subspace, Noise Subspace

1. Introduction

It is difficult to determine DOA for multiple sources using correlation based or beamforming based source localization methods. However, subspace based methods like Multiple Signal Classification (MUSIC) are widely used for such cases due to their computational efficiency. These methods use magnitude spectrum using MUSIC algorithm to compute the DOA of multiple sources. Details of this algorithm for source localization using uniform linear array (ULA) of sensors can be found in [1]. However, the limitation of using linear array is that estimated DOA has only information of azimuth angle. For estimating both angular positions (azimuth and elevation angles) we need a planar array. Such an array can localize sources in the azimuthal plane as well as in elevation in the range of 0^o to 90^o. Several type of planar arrays may be used for source localization. These include uniform square array, cross array, triangular array and circular array [2, 3]. The algorithm for MUSIC for source localization using uniform circular array (UCA) has been discussed in [4].For both linear and planar arrays, DOA estimation using MUSIC algorithm deteriorates when multiple incident sources are highly coherent in nature. To overcome this problem a subspace based method named Eigen Space-DOA MUSIC (ES-DOA MUSIC) has been developed in [5, 6]. However, this method works only for a linear array. There is a need to extend this method for two dimensional array as well. In this work a method based on ES-DOA MUSIC spectrum is proposed for two dimensional for DOA estimation in terms of azimuth and elevation angles for a circular array.

2. Problem formulation

Consider L narrowband far-field sound sources incident on a uniform circular array of radius a with M sensors, where M>L. The sensors are uniformly spaced apart on the circumference as shown in Figure 1.The azimuth and elevation angles of l_{th} source are $\varphi_l$and θ_l respectively, and signal from it is$s_l(t)$. It is assumed that the reference of the array is at the centre of the circular array. Since (φ_l, θ_l) represent the direction of arrival of source with respect to reference, the instantaneous pressure amplitude at i_{th} microphone due to l_{th} source is $s_l(t-\tau_i(\varphi_l,\theta_l))$, where $\tau_i(\varphi_l,\theta_l)$ is delay of arrival at i_{th} microphone corresponding to l_{th} source with respect to the reference . Thus, the total pressure at i_{th} microphone at time t is $y_i(\varphi;\theta;t)$ as it depends on DOA of different sources and time. It can be expressed as Equation (1) as based on [8].

$$y_i(\varphi;\theta;t)=\sum_{l=1}^{L}\propto_i(\varphi_l,\theta_l)s_l\big(t-\tau_i(\varphi_l,\theta_l)\big)+w_i(t), \tag{1}$$

where $\propto_i(\varphi_l,\ \theta_l)$, is attenuation factor for i_{th} microphone corresponding to the l_{th} source and w_i is uncorrelated noise for i_{th} sensor. The noise is assumed to be white Gaussian. Here Equation 1 is valid in anechoic environments [8].

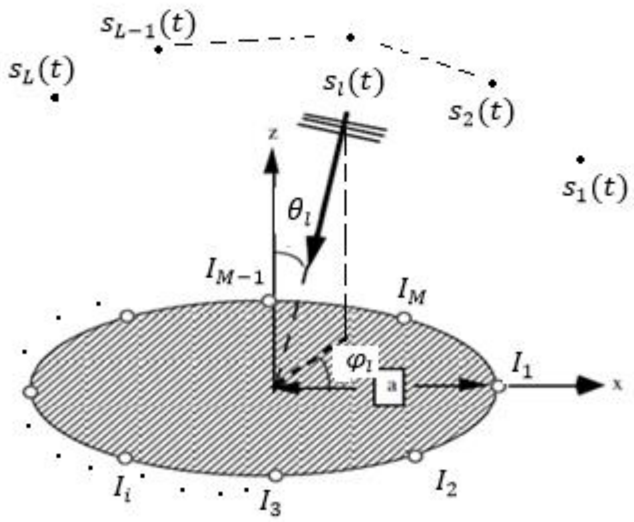


Figure 1: A uniform circular array of radius a. A plane wave is incident at an azimuth and elevation angles φ and θ.

For narrowband approximation, the pressure at i_{th} microphone can be re-written as,

$$y_i(\varphi;\theta;t)=\sum_{l=1}^{L}\propto_i(\varphi_l,\theta_l)s_l(t)e^{-j\omega_l\tau_m(\varphi_l,\theta_l)}+w_i(t). \tag{2}$$

Here ω_l is the angular frequency of signal from l_{th} source.

For $\propto_i(\varphi_l,\theta_l)$=1, the pressure at i_{th} microphone in Equation 2, can be written in terms of wave vector k_l and i_{th} microphone's position vector r_i as Equation 3, as developed in [9].

$$y_i(\varphi;\theta;t) = \sum_{l=1}^{L} s_l(t)e^{-j\mathbf{k}_l^T r_i} + w_i(t). \tag{3}$$

Taking K number of snapshots, Equation 3 can be re-written in matrix form as proposed by [10] as:

$$\mathbf{Y}(t) = \mathbf{A}(\varphi,\theta,k)\mathbf{S}(t) + \mathbf{w}(t),\ t = 1,2,3 \dots\dots\dots.., K\,. \tag{4}$$

Here $\mathbf{Y}(t) = [y_1(t), y_2(t) \dots\dots\dots\dots. y_M(t)]^T$ is a matrix of dimension $M{\times}K$ representing signals read by M sensors, $\mathbf{A}(\varphi,\theta,k)$ is the steering matrix of size $M{\times}L$, $\mathbf{S(t)}$ is a $L{\times}K$ matrix of signals, and $\mathbf{w}(t)$ is $M{\times}K$ matrix representing additive white Gaussian uncorrelated noise corresponding to M sensor locations. Here, the steering matrix is used to compute time series signal $y_i(t)$ for i_{th} microphone in simulations. Also the steering matrix $\mathrm{A}(\varphi,\theta,k)$ can be expressed as

$$\mathbf{A}(\varphi,\theta,k) = \left[\mathrm{a}\left(\varphi_{1,}\theta_1,k\right) \quad \mathrm{a}\left(\varphi_{2,}\theta_2,k\right) \quad \dots\dots\dots. \quad \mathrm{a}\left(\varphi_{L,}\theta_L,k\right)\right]. \tag{5}$$

Hence each column of steering matrix represents a direction vector for particular DOA. Thus if the DOA for l_{th} source is $(\varphi_{l,}\theta_l)$, and k_l is wavevector for l_{th} source, and r_i is positon vector for i^{th} microphone then the corresponding direction (steering) vector can be written as

$$\mathrm{a}\left(\varphi_{l,}\theta_l,k\right) = \left[e^{-j\mathbf{k}_l^T\mathbf{r}_1} e^{-j\mathbf{k}_l^T\mathbf{r}_2} \quad \dots\dots. \quad e^{-j\mathbf{k}_l^T \boldsymbol{r}_M}\right]. \tag{6}$$

The steering vector is also referred as array manifold vector in literature and it can also be written in forms of time delay τ [7], as

$$\mathrm{a}\left(\varphi_{l,}\theta_l,k\right) = [e^{-\boldsymbol{j\omega\tau_1}} e^{-\boldsymbol{j\omega\tau_2}} \quad \dots\dots. \quad e^{-\boldsymbol{j\omega\tau_M}}]^T. \tag{7}$$

This equivalence exists since

$$\omega\tau_i = k_l^T r_i\,. \tag{8}$$

For l_{th} source the expression for wavevector k_l for UCA geometry in direction of arrival with wavenumber k is given as

$$k_l = -[k sin\theta_l cos\varphi_l \quad k sin\theta_l sin\varphi_l \quad k sin\theta_l]\,. \tag{9}$$

Also for UCA as shown in Figure 1 the elevation angle θ is 90° for all the microphones, the position vector for i^{th} microphone is given as

$$r_i = [a cos\varphi_i \quad a sin\varphi_i \quad 0]\,. \tag{10}$$

Now we can calculate propagation delay τ_i at i^{th} microphone for l_{th} source using Equation 8, 9 and 10. Hence, the relation for propagation delay is

$$\tau_i(\varphi_l\,,\theta_l) = \frac{-a\cos(\varphi_l - \varphi_i)\, sin\theta_l}{c}. \tag{11}$$

Here c is the speed of sound.

Using such an approach, we were able to generate signals received by M microphones, which originated at L sources. These signals were subsequently processed to estimate DOA of each source.

3. Method used for source localization

MUSIC and the proposed ES-DOA MUSIC for circular array have been used for source localization. A brief description of these methods is given below.

3.1 MUSIC

Here, first, the covariance matrix of $[Y]$ i.e. $[R]$ was computed. Next it's Eigenvectors and Eigenvalues were calculated. Thus,

$$[R] = [Q][\Lambda][Q^H] \tag{12}$$

Here $[Q]$ is an $M\times M$ *matrix* of Eigenvectors of $[R]$, and Λ is a diagonal matrix made up of M Eigenvalues of $[R]$.

Next $[Q]$ was divided into a noise subspace $[Q_n]$ and a signal subspace $[Q_s]$. This was achieved by separating Eigenvectors associated with small and large Eigenvalues, respectively. Hence, if there are L sources then the size of $[Q_s]$ and $[Q_n]$ would be $M\times L$, and $M\times (M\text{-}L)$, respectively.

Next, for each source the DOA was computed. For this, a revised steering matrix $[A]$ of size $M\times N_2$, was computed. Here N_2 correspons to all possible values of DOA. Hence, $[A]$ was computed for values of φ and θ varying between 0^o to 360^o and 0^o to 90^o, in steps of 1^o.

For computing DOA, MUSIC magnitude spectrum $P_{MUSIC}$was calculated using the following relation.

$$P_{MUSIC}(\varphi,\theta) = \frac{\{A_i\}.\{A_i\}^H}{\|[Q_n]^H.\{A_i\}\|^2}\,. \tag{13}$$

Here, $[Q_n]^H$ is the Hermitian of $[Q_n]$, and $\{A_i\}$ is the steering vector associated with i_{th} angular position. As, Eigenvectors of $[Q_n]$ are orthogonal to signal steering vectors, the denominator reaches a null value when (φ,θ) coincides with direction of signal. Hence the MUSIC magnitude spectrum $P_{MUSIC}(\varphi,\theta)$ shows peaks at the DOA. However, when sound sources are coherent, MUSIC magnitude spectrum fails to resolve them with a computationally reasonable number of sensors. Hence, ES-DOA-MUSIC for circular array was developed in this work.

3.2 ES-DOA MUSIC

Here, both the signal subspace $[Q_s]$ and the noise subspace $[Q_n]$ were used for computation. Detailed description of this algorithm for ULA are provided in [5]. For circular arrays the ES-DOA MUSIC spectrum is given by Equation (14),

$$P_{ES-DOA}(\varphi,\theta) = \{A_i\}^H R_A^{+}\{A_i\} \times P_{MUSIC}(\varphi,\theta). \tag{14}$$

Here R_A^{+} is the generalized inverse of signal subspace and is given by

$$R_A^{+} = Q_s\Lambda_s^{-1}Q_s^H\,. \tag{15}$$

4. Simulation

For simulations, we had 16 microphones placed in uniform circular array of $0.7\lambda_{2000}$ radius. Also the number of far field sources was taken as 3. The power of these signals were same and were equal to unity.

4.1 Simulation conditions

We have studied 23 cases. The details of first sixteen cases are shown in Tables 1 and 2. Here we have compared performances of MUSIC and ES-DOA MUSIC methods. For the first two cases (C1 and C2) all three sources are incoherent; and the frequencies for these sources were taken as 1600 Hz, 800 Hz and 2000 Hz. For the next two cases (C3 and C4) all three sources were assumed to be highly coherent; and the frequencies for these sources were 1600 Hz, 1597 Hz and 1603 Hz. In the first and third cases we fixed elevation positions at 30^0, and varied azimuth positions (0^o, 120^o, 240^o). In the second and fourth cases elevation positions were 30^o, 60^o and 85^o while azimuth position was fixed at 0^0. The SNR for signals arriving at *M* microphones was 20 dB for all the four cases. The next four cases (C5 to C8) studied were identical to cases C1, C2, C3 and C4, respectively except for the fact that for these sources the SNR which was 5 dB.

Table 1: Cases studied for determining DOA of coherent and incoherent sources

Cases	Frequency (Hz)			Angular positions (degree)						SNR
	s_1	s_2	s_3	s_1		s_2		s_3		
				φ	θ	φ	θ	φ	θ	
C1	1600	800	2000	0^o	30^o	120^o	30^o	240^o	30^o	20dB
C2	1600	800	2000	0^o	30^o	0^o	60^o	0^o	85^o	
C3	1600	1597	1603	0^o	30^o	120^o	30^o	240^o	30^o	
C4	1600	1597	1603	0^o	30^o	0^o	60^o	0^o	85^o	
C5	1600	800	2000	0^o	30^o	120^o	30^o	240^o	30^o	5dB
C6	1600	800	2000	0^o	30^o	0^o	60^o	0^o	85^o	
C7	1600	1597	1603	0^o	30^o	120^o	30^o	240^o	30^o	
C8	1600	1597	1603	0^o	30^o	0^o	60^o	0^o	85^o	

In the next eight cases (C9a-C9d, C10a-C10d), the effectiveness of MUSIC and ES-DOA algorithms was assessed for coherent sources as a function of number of sensors used. Details of these cases are shown in Table 2. Finally seven additional cases studies were conducted to determine the effectiveness of ES-DOA algorithm as a function of number of snapshots in the analysis. For this we assumed same angular positions and frequencies corresponding to cases 9a and 9b. However, the number of snapshots was kept as a variable. Here we used 16 sensors and DOA was computed for a varying number of snapshots, i.e. 1024, 2048, 4096, 8192, 16384, 32768 and 102400.

4.2 Results

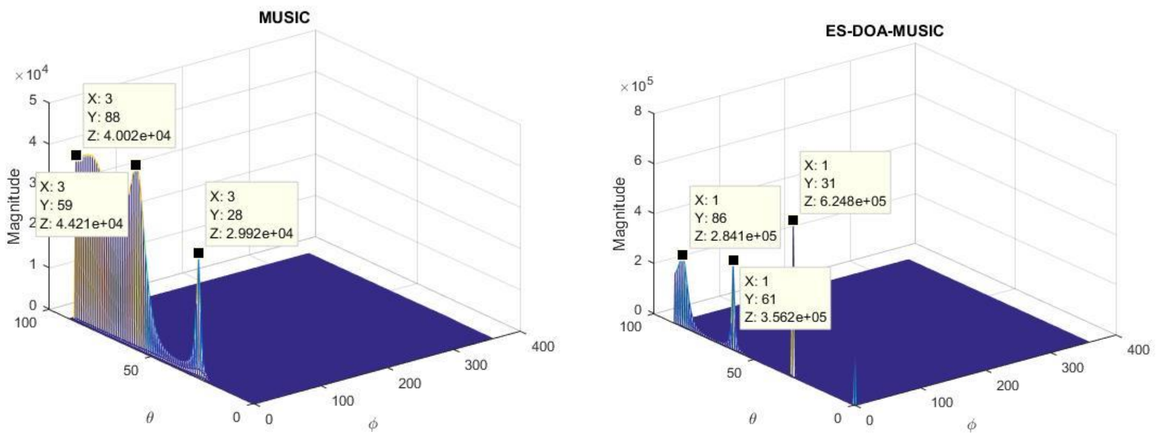
4.3 Results gotten from MUSIC and ES-DOA MUSIC methods for first eight cases are given in Table 3. We make the following observations from this table,

- For incoherent sources, at high SNR, estimated DOA by MUSIC gives 3^o and 2^o deviation in azimuth and elevation positions respectively as compared to actual positions of sources. However ES-DOA method gives only 1^o deviation from actual angular positions of the sources.
- For coherent sources with high SNR, estimated DOA by MUSIC gives similar deviation as for incoherent sources. However the plots for spectral magnitude using MUSIC show that three peaks associated with source frequencies are not sharp. This seen in Figure 2a. In contrast ES-DOA peaks are clearly seen to be sharp as shown in Figure 2b. Also, estimated DOA deviates by no more than 1^o from actual angular positions when ES-DOA method was used.

- For low SNR, estimated DOA deviates as much as by, 6º for MUSIC, but only by 2º for ES-DOA. This is true for both incoherent and coherent sources. Further, spectral magnitude peaks using MUSIC are not clear and sharp for coherent sources.

Table 2: Cases studied for determining the influence of number of sensors on accuracy of estimated DOA

Cases	No. of sensors	Frequency (Hz) and Angular positions (degree)						SNR
		s_1		s_2		s_3		
		1600Hz		1597Hz		1603Hz		
		φ	θ	φ	θ	φ	θ	
C9a	8	0º	30º	120º	30º	240º	30º	20dB
		0º	30º	0º	60º	0º	85º	
C9b	16	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60º	0º	85º	
C9c	24	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60	0º	85º	
C9d	32	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60	0º	85º	
C10a	8	0º	30º	120º	30º	240º	30º	5dB
		0º	30º	0º	60º	0º	85º	
C10b	16	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60º	0º	85º	
C10c	24	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60º	0º	85º	
C10d	32	0º	30º	120º	30º	240º	30º	
		0º	30º	0º	60º	0º	85º	



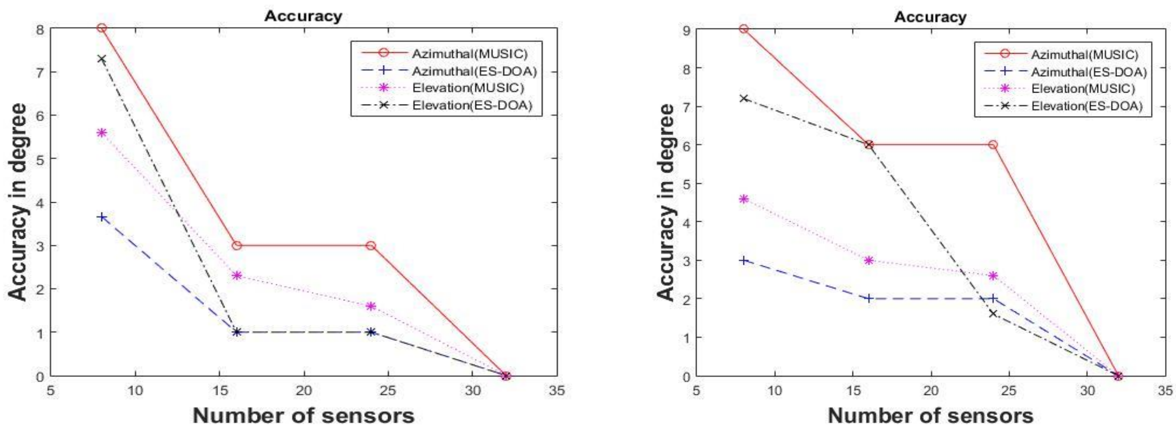
(a) (b)

Figure 2: Spectral magnitude for case C3 (a) MUSIC, (b) ES-DOA.

Table 3: Results for determining DOA of Different sources

Cases	Actual						Estimated (MUSIC)						Estimated (ES-DOA-MUSIC)					
	s_1		s_2		s_3		s_1		s_2		s_3		s_1		s_2		s_3	
	φ	θ	φ	θ	φ	θ	φ	θ	φ	θ	φ	θ	φ	θ	φ	θ	φ	θ
C1	0°	30°	120°	30°	240°	30°	3°	28°	123°	28°	243°	28°	1°	31°	121°	31°	241°	31°
C2	0°	30°	0°	60°	0°	85°	3°	28°	3°	58°	3°	88°	3°	31°	1°	61°	0°	86°
C3	0°	30°	120°	30°	240°	30°	3°	28°	123°	28°	243°	28°	1°	32°	121°	30°	241°	30°
C4	0°	30°	0°	60°	0°	85°	3°	28°	3°	58°	3°	83°	1°	31°	1°	61°	1°	84°
C5	0°	30°	120°	30°	240°	30°	6°	26°	126°	26°	246°	26°	2°	32°	122°	32°	243°	32°
C6	0°	30°	0°	60°	0°	85°	6°	26°	6°	66°	6°	86°	2°	33°	2°	72°	2°	90°
C7	0°	30°	120°	30°	240°	30°	6°	26°	126°	26°	246°	26°	2°	32°	122°	32°	242°	32°
C8	0°	30°	0°	60°	0°	85°	6°	27°	6°	65°	6°	86^0	2°	33°	2°	70°	2°	90°

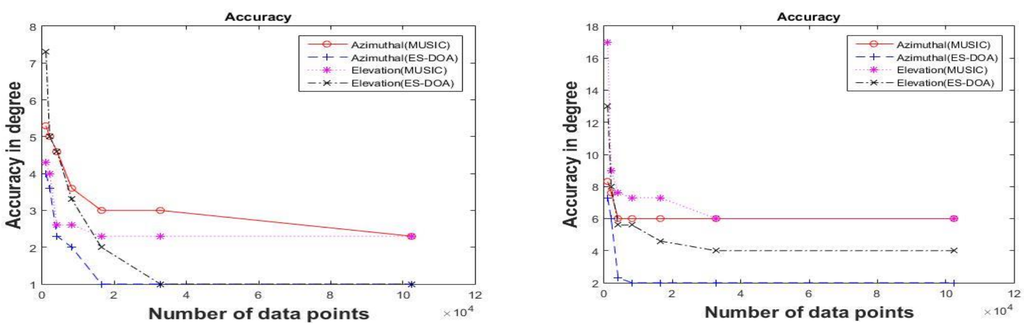
To prove the efficiency of ES-DOA-MUSIC method for UCA, we plotted accuracy of estimated DOA from both methods using varying number of sensors as given in Cases 9 and 10. We see from Figure 3 that ES-DOA is much more accurate even with lesser number of sensors relative to MUSIC.



(a) (b)

Figure 3: Influence of number of sensors on DOA accuracy; (a) SNR=20 dB (b) SNR=5dB.

The results from MUSIC and ES-DOA for highly coherent sources as a function of number of snapshots is shown in Figure 4. It is seen that as number of snapshots increases, DOA becomes more accurate. It is also noted that we do not necessarily need more than 16*k* points to achieve high accuracy.



(a) (b)

Figure 4: Influence of number of data points on DOA accuracy; (a) SNR=20dB (b) SNR =5dB.

5. Conclusions

A high resolution two dimensional DOA estimation algorithm for uniform circular array to resolve coherent sources using a reasonable number of sensors has been proposed in this work. It is shown that ES-DOA MUSIC method when adopted for circular array is able to resolve both azimuth and elevation angles of multiple coherent and incoherent sources with reasonable accuracy compared to MUSIC algorithm. This is shown to be the case even for low SNR signals even when lesser number of snapshots are used. The proposed method can be used in for noise source detection in vehicles and several other field applications.

REFERENCES

1. R. O. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Trans-actions on Antennas and Propagation*, vol. AP-34, pp. 276–280, Mar. 1986.
2. S.Chandran, Advance in Direction-of-arrival estimation. Artech House, 2005.
3. D.Ying and Y.Yan, “Robust and fast localization of single speech source using a planar array”, *Signal Processing Letters, IEEE*, vol.20,no.9,pp. 909-912,2013.
4. MBrandstein and DWard, Eds., Microphone Arrays, Springer Verlag, Berlin, 2001.
5. Zhang Xiaofei; Lv Wen; Shi Ying; Zhao Ruina; Xu Dazhuan, “A novel DOA algorithm based on Eigen space", *International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications,* Pages: 551 - 554, DOI: 10.1109/MAPE.2007.4393677.
6. Arun Prakash Singh; Nachiketa Tiwari, “Source localization algorithm for noisy signal using sparse optimization technique”, *International symposium on Acoustics for Engineering Applications,* Nov, 2016.
7. H. Krim and M. Viberg, “Two decades of array signal processing research: the para-metric approach,” *IEEE Signal Processing Magazine*, vol. 13, pp. 67–94, July 1996.
8. J. Benesty, J. Chen, and Y. Huang, Microphone array signal processing.Springer Verlag, 2008, vol. 1.16.
9. H. L. Van Trees, Optimum Array Processing, Wiley - Interscience, 2002.
10. M. I. Skolnik, *Introduction to Radar Systems*, McGraw-Hill, 2000.
11. Yonghong Xia; Yin Zhang; Wenchao Li, “ An improved MUSIC algorithm for angular super resolu-tion in scanning antenna”, *International Conference on Computational Problem-solving (ICCP),* Pages: 358 - 361, DOI: 10.1109/ICCPS.2013.6893540.
12. Jeffrey Fiutz, Andreas spanias, and Mahesh K. Banavar, Narrowband direction of Arrival estimation for Antenna Arrays. Morgan & Claypool Publishers, 2008.