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THE PERFORMANCE LIMITS OF DIGITAL LOCAL OSCILLATORS FOR A SECTOR SCANNING SONAR RECEIVER

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1. Introduction

Modulation type sector scanning sonar receivers require the use of a series of local oscillators whose frequencies are arithmetically related. The techniques employed to achieve the necessary performance requirement frequently result in complicated electronic circuits with many, ofter inter-dependent, adjustments. A simpler system using digital techniques was proposed in a recent paper (ref.1) which required only a few adjustments. In this paper, we study the limitations of this scheme particularly with respect to the generation of unwanted local oscillator sidebands.

2. Sector Scanning Preliminaries

First consider the (2n+1) linear array of point acoustic sensors of figure 1 connected in a beam steering arrangement. The use of phase shifting circuits rather than time delays restricts use to narrow-band operation which is a satisfactory mode for most applications. The angular sensitivity of the beam-former is

$$V(\theta) = P \sum_{r=-n}^{n} a_r \exp\left(j\{\omega_o t - r\phi - 2\pi r d(\sin\theta)/\lambda\}\right). \qquad 2.1$$

The angular frequency of the incident acoustic radiation is ω_o, a_r is the sensitivity of sensor r and p is the peak pressure of the acoustic wave. When the hydrophones are of equal sensitivity, the summation of equation 2.1 can be easily performed, giving the envelope sensitivity of the beamformer:

$$|V(\theta)| = pa \frac{\sin(\{2n+1\}\{\pi d(\sin\theta)/\lambda + \phi/2\})}{\sin\{\pi d(\sin\theta)/\lambda + \phi/2\}}. \qquad 2.2.$$

The maximum value of $|V(\theta)|$ is (2n+1)pa and is attained for those values of θ satisfying the equations:

$$\sin(\theta_i) = (2i - \phi/\pi)\ \lambda/2d \quad , \quad |\sin\theta_i| \leq 1 . \qquad 2.3.$$

The angles corresponding to maximum sensitivity may be varied by changing the phase taper parameter, ϕ. A sector-scanning sonar receiver results when ϕ varies cyclically. Modulation-type sector-scanners have ϕ varying as a linear function of time:

$$\phi = \omega_s t = 2\pi f_s t \; . \qquad 2.4$$

The angles of maximum sensitivity also vary cyclically:

$$\sin(\theta_i) = (i - f_s t)\,\lambda/d. \qquad 2.5$$

With this restriction on the time-variant behaviour of ϕ, each phase shifter has the effect of introducing a change in frequency. This may be demonstrated by considering the output of the phase shifter in the r^{th} channel:

$$V_r = pa_r \quad \exp\,(j\{\omega_o t - r\omega_s t - 2\pi rd(\sin\theta)/\lambda\}). \qquad 2.6$$

The change in frequency is easily identified as the $(-r\omega_s t)$ term in equation 2.6. It is thus possible to realise a sector-scanning receiver by implementing the 2n phase-coherent frequency changes using n local oscillators with arithmetically related frequencies. Techniques have been reported (see for example, ref. 2) by which the number of local oscillators can be reduced for high resolution systems having a large number of sensors.

3. Digital Generation of Local Oscillator Signals

A block diagram of the system proposed in reference 1 for the generation of phase coherent sinusoids with arithmetically related frequencies is illustrated in figure 2. Each read-only memory (ROM) contains quantised sample values from an integral number $^{(P)}$ of cycles of a sine wave. Successive time samples of the sine wave are stored in successive address locations in the memory. A counter and clock are arranged to address successively each memory location. The output of the read-only memory is fed to a Digital-to-Analogue converter (DAC) which thus produces a quantised sine wave output. The output waveform is smoothed by passage through a bandpass filter which attenuates all unwanted output signals. A convenient hardware realisation results when the counter is an m-stage binary one and the read-only memory was 2^m locations of k data bits each. The sine wave output from the digital to analogue converter has frequency f_p:

$$f_p = \frac{f_c P}{2^m} . \qquad 3.1$$

A set of sinusoids whose frequencies are in arithmetic progression may thus be generated by incrementing P from one read-only memory to the next. The phases of the output signals may be controlled digitally through presetting a start address into the address counter when a synchronising signal occurs. Figure 2 illustrates the control of the relative phases of two groups of sine wave generators.

4. Calculation of the Relative Intensity of the Output Noise

Output at frequencies other than the wanted one from the system of figure 2 result from the quantisation inherent in using a digital store. The i^{th} sample value of the sinusoid which is stored is derived from

$$S_p(i) = \sin\left(2^{-m+1}\pi p i\right) \qquad 4.1$$

and has P complete cycles contained in 2^m samples. The process of quantisation produces a digital equivalent $\hat{S}_p(i)$ and a quantisation error $\varepsilon(i)$:

$$\varepsilon_p(i) = S_p(i) - \hat{S}_p(i) . \qquad 4.2$$

The quantisation error pattern repeats at least as often as the counter cycles through all memory locations and therefore has a spectrum with non-zero components not closer than $f_c/2^m$. In fact, it can be shown that when P is odd, the only finite spectral components in the output occur at odd harmonics of $f_c/2^m$. This is due to the half wave symmetry of $S_p(i)$ and $\hat{S}_p(i)$:

$$S_p(i) = -S_p(i + 2^{m-1}) ,$$

and

$$\hat{S}_p(i) = -\hat{S}_p(i+2^{m-1}) . \qquad 4.3$$

When P is even, it may be factorised into an odd number and a power of 2:

$$P = a.2^b . \qquad 4.4$$

The repetition interval of $S_a(i)$ and $\hat{S}_a(i)$ for this case is 2^{m-b} and the quantisation error spectrum contains only odd harmonics of $f_c/2^{m-b}$.
By using statistical techniques, reference 1 deduces the ratio between the spectral levels of the wanted and unwanted components (SNR);

$$\mathrm{SNR} = 3 \times 2^{2k+m-b-3} . \qquad 4.5$$

An alternative method of analysis, which we pursue in this paper, is a direct technique using the Discrete Fourier Transform (DFT) to find the amplitude $F_p(k)$ of the various components of $\hat{S}_p(i)$:

$$F_p(k) = \sum_{i=0}^{N-1} W^{ki} \hat{S}_p(i) \quad , \quad N = 2^m, \quad W = \exp\{j\ 2\pi/N\} . \qquad 4.6$$

Computer programs are widely available for the calculation of $F(k)$ in the case (as here) when N is a power of 2.

5. Cyclic relationships in $\hat{S}_p(i)$ and $F_p(k)$

In this section, we study the various cyclic relationships which exist in $F_p(k)$ through those in $\hat{S}_p(i)$. We will find that the various numbers P can be formed into groups depending on their decomposition through equation 4.4. The spectral levels of $F_p(k)$ are invariant for changes of P within a group, only the order of the non-zero components of $F_p(k)$ being changed. It is therefore only necessary to compute the spectrum, $F_p(k)$, for one P value in each group. The rearrangement of the components of this spectrum may be easily determined from the inverses of the two P values modulo 2^m.

First of all, group the values of P so that each group has the same value of b in the factorisation of equation 4.4. For m=4, these groups are as follows:

group 0 :	b=0 :	P = 1, 3, 5, 7, 9, 11, 13, 15 ;
1 :	b=1 :	P = 2, 6, 10,14;
2 :	b=2 :	P = 4 12;
3 :	b=3 :	P = 8. 5.1

The sample values which are stored in the read-only memory may be simply derived from the sequence $\hat{S}_1(i)$ by decimation:

$$\hat{S}_p(i) = \hat{S}_1(r) \ , \quad r = Pi : \text{Mod } N \ . \qquad 5.2$$

For values of P in group O, every value of $\hat{S}_1(i)$ appears in $\hat{S}_p(i)$ since P is relatively prime to N. For values of P in other groups only a subset of the $\hat{S}_1(i)$ appears, of length 2^{m-b} and is repeated 2^b times in $\hat{S}_p(i)$. In these cases, several sequences are possible depending on the starting value, $\hat{S}_p(O)$. These correspond to reconstructed sine waves having different phase relationships to the reconstructed version of $\hat{S}_1(i)$. The in-phase version of $\hat{S}_p(i)$ results if its starting value is $\hat{S}_1(O)$.

Next, we consider the relationship between the various spectral components of $F_p(k)$ for changes in P. Recalling the definition of $F_p(k)$, we have, for values of P in group O:

$$\begin{aligned} F_p(k) &= \sum_{i=O}^{N-1} W^{ki}\, \hat{S}_p(i) \\ &= \sum_{i=O}^{N-1} W^{ki}\, \hat{S}_1(r) \qquad , \quad r = Pi \ , \ \text{modN}, \\ &= \sum_{r=O}^{N-1} W^{k\bar{P}r}\, \hat{S}_1(r) \qquad , \quad \bar{P} = 1 \quad \text{modN}, \\ &= F_1(k\bar{P}) \\ &= F_1(j) \qquad\qquad j = k\bar{P}, \ k=jP, \ \text{modN}. \end{aligned} \qquad 5.3$$

$\bar{P}$ is known as the inverse of P, modN. Values of $\bar{P}$ for N = 16, group O are as follows:

P = 1, 3, 5, 7, 9, 11, 13, 15

$\bar{P}$ = 1, 11, 13, 7, 9, 3, 5, 15.

We conclude from the manipulation of equation 5.3, that the spectral components of $F_p(k)$ are just found by decimating those of $F_1(j)$, P at a time - i.e. $k = jP$, modN. The spectrum of $S_p(k)$ is not therefore unique - it shares its spectral levels with all the other spectra with P in group O.

When P is not relatively prime with respect to 2^m, it does not directly have an inverse and the necessary spectral relationships are a little more complicated. We note that $P.2^{-b}=a$ has an inverse modulo 2^{m-b}. Consider the spectrum of a sequence with P in group b :

$$S_p(k) = \sum_{i=0}^{N-1} W^{ki}\, \hat{S}_p(i)$$

$$= \sum_{i=0}^{2^{m-b}-1} N^{ki}\, \hat{S}_p(i) \sum_{n=0}^{2^b-1} W^{2^{m-b}kn} \qquad 5.4$$

The last step can be made because $\hat{S}_p$ repeats 2^b times. The final summation in equation 5.4 takes the values O or 2^b depending on whether k is not or is a multiple of 2^b respectively:

$$F_p(\ell) = \sum_{i=0}^{2^{m-b}-1} (W^{2^b})^{\ell i}\, \hat{S}_p(i)\; 2^b \quad , \qquad k = 2^b \ell$$

$$= \sum_{i=0}^{2^{m-b}} (W^{2^b})^{\ell \bar{a} r}\, \hat{S}_{2^b}(r)\; 2^b \qquad r = ai \bmod 2^{m-b}$$

$$\bar{a}a = 1 \bmod 2^{m-b}$$

$$= F_{2^b}(\ell\bar{a}) \qquad j = \ell\bar{a} \bmod 2^{m-b}$$

$$= F_{2^b}(j) \qquad \ell = ja \bmod 2^{m-b} \qquad 5.5$$

Equation 5.5. demonstrates that the spectra $F_p(\ell)$ permute within their P group as the sample values $\hat{S}_p(i)$ are permuted by decimation. Next, we relate the spectrum $F_p(k)$ for composite $P = 2^b a$ to the spectrum $F_1(k)$. Consider the sequence of samples:

$$\hat{S}_1(i) \quad \sum_{n=0}^{2^b-1} (W^{2^{m-b}})^{in} . \qquad 5.6$$

The summation is zero except when i is a multiple of 2^b. This is just the sequence of sample values appropriate to a p value in group b. The spectrum of these samples is:

$$\sum_{i=0}^{2^m-1} \hat{S}_1(i)\, W^{ki} \sum_{n=0}^{2^b-1}\left(W^{2^{m-b}}\right)^{in} = \sum_{i=0}^{2^m-1} \hat{S}_1(i) \sum_{n=0}^{2^b-1} W^{i(k+2^{m-b}n)}$$

$$= \sum_{n=0}^{2^b-1} F_1(k+2^{m-b}n). \qquad 5.7$$

The last step was made by a valid reversal of the order of the summations and shows that the spectrum of a subset of $\hat{S}_1(i)$ obtained by decimation is the sum of some of the spectral values of $F_1(k)$. We can relate equations 5.5 and 5.7 by using the fact that the sequence of sample values in 5.6 is non-zero only for i= 2^b r, where r is some integer :

$$\sum_{n=0}^{2^b-1} F_1(k+2^{m-b}n) = \sum_{r=0}^{2^{m-b}-1} \hat{S}_1(2^b r) \sum_{n=1}^{2^b-1} \left(W^{2^{m-b}}\right)^{2^b rn}.\ W^{2^b kr}$$

$$= \sum_{r=0}^{2^{m-b}-1} \hat{S}_{2^b}(r)\ .\ 2^b.\left(W^{2^b}\right)^{kr}$$

$$= F_{2^b}(k) \qquad 5.8$$

Thus, the spectra for composite P may be simply determined by operating on the spectrum of $\hat{S}_1(i)$ with a shift and add routine.

6. Results and Discussion

Computer programs have been written to calculate the spectra of sample values drawn from each of the p-value groups. The Fourier Transform routine used was a standard FFT package. Average levels of unwanted spectral components have been found together with the highest and lowest levels of these components. These results are tabulated below for m=8 and k=4 - that is for a Read-only memory of 256 storage locations of 4 bits capacity each. Tabulated along with these results are calculations resulting from the statistical theory of reference 1.

It should be noted that the statistical theory rests on the assumption that the quantisation errors are independent from sample to sample and that this normally requires at least 6 bits capacity in each memory location.

P - group b	Unwanted Spectral Component Levels, dB		Highest Level	Lowest Level
	Direct Fourier Method	Statistical Method		
0	- 45.8	- 43.9	- 34.4	- 60.1
1	- 41.9	- 40.9	- 34.5	- 53.9
2	- 38.4	- 37.9	- 31.9	- 48.0
3	- 35.7	- 34.9	- 31.7	- 40.8
4	- 31.9	- 29.6	- 27.5	- 33.5
5	- 28.8	- 28.2	- 28.5	- 28.5

The two methods of calculation give results in fairly close agreement. In practice, the differences are likely to be insignificant in comparison to the errors introduced by the inexact switching instances of the address counter and the read-only memory, and by a crude implementation of the Digital to Analogue converter. The main choice for a potential user of this scheme is of the size of the read-only memory and its use. For the table, it may be observed that the best average unwanted levels are available when the harmonic number, P, of the desired output signal is relatively prime to the number of address locations used. One way of ensuring that this condition is met for all P values is to make the number of address locations used N, a prime number. Although such a procedure keeps the average levels of unwanted signals low, the table also indicates that the highest levels only decrease by 6dB in going from a relatively prime P-value to a highly composite one with b=5. On a worst case design basis it is probably a better compromise to use output sine waves of frequencies with highly composite P-values since the unwanted spectral components are much further removed in frequency than those associated with relatively prime P-values. In this case, it is possible to achieve more effective filtering of the unwanted components.

10. References

1. A.R. Pratt., The use of digital techniques to improve modulation-type sector-scanning sonar systems. Radio & Electronic Engineer, Vol. 49 No. 3 1979.

2. Voglis, G.M. A General Treatment of Modulation Scanning as applied to acoustic Linear arrays. Ultrasonics 9. No.3 July 1971, No.4, October 1971.

FIG.1 AN ARRAY PROCESSOR

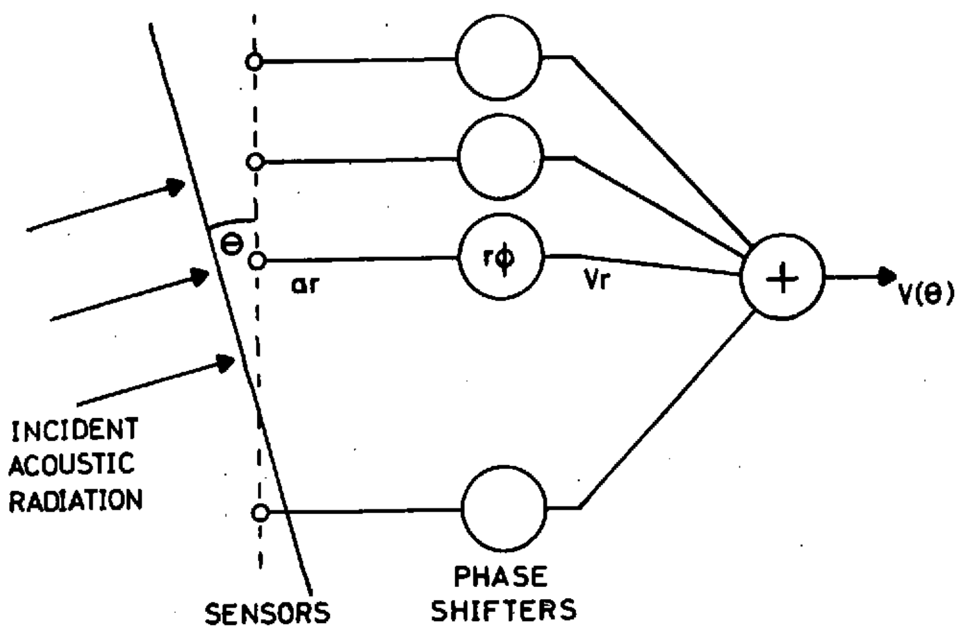

FIG.2