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VIBRATIONAL POWER FLOW FROM MACHINES INTO STRUCTURES

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INTRODUCTION

When installing building services machinery in buildings it is often necessary to resiliently isolate the the vibration source from the structure. Without isolation vibrational energy enters the structure unchecked and can cause excessive response levels at the foundation of the machine or, when transmitted through structural members such as columns, in remote parts of the building. A properly designed isolation setup will attenuate the flow of vibrational energy sufficiently that acceptably low response levels are obtained throughout the structure. There are well established methods by which the attenuation afforded by the resilient mounts can be calculated. For example transmissibility or response ratio are often used to predict isolator effectiveness to an accuracy of a few dBs. In contrast no corresponding theory exists by which the requirements for attenuation can be assessed, taking due account of the strength of the source , the dynamic characteristics of the machine foundation, and the sensitivity of the situation.

Some guidance can be found in the literature, see for example tables 1 and 2 in reference 1 which give recommended isolation efficiencies for a range of machines mounted on concrete slab floors in 'critical' and 'less critical' areas. The recommendations would appear to be based on previous experience of what gives acceptable vibration levels rather than a knowledge of the levels themselves. Such guidelines are invaluable but they do not by themselves provide a good insight into the problem. It would be difficult for example to use this information to choose the optimum position for siting a machine in a building, or to assess the change in isolation requirements for an increase in say floor stiffness. Also if the problem falls outside the range of previous experience then an understanding of the mechanisms involved is essential in order to 'extrapolate' from known cases.

One approach to the problem might be to evaluate the vibration levels throughout the structure and to compare these at each point with 'acceptable' levels such as the 'satisfactory vibration magnitudes' in BS6472. There are obvious difficulties however in predicting the response in such detail and it is often the case that if changes in the structure are made a complete re-evaluation of the response is necessary. It would be of more practical value if a single parameter could be found which would indicate, with acceptable accuracy, the levels of vibration likely to be generated in a given situation without the need for an evaluation of the full response. Such a parameter would provide a feel for the problem rapidly and would enable the effect of structural changes to be quickly assessed.

The force applied to the foundation has sometimes been used in this way but this may be misleading since if the foundation were perfectly rigid then even a large force applied to it would not cause vibration in any part of the structure (see fig 1a). The velocity of the foundation is a better criterion but even then it is possible that the highest response levels may occur elsewhere (see fig 1b). It has been suggested that a more suitable quantity may be the vibrational power flow from the machine into its' foundation which combines force and velocity in a single variable [2]. It can be argued that this will give an indication of overall response levels since vibrational energy which is the cause of excessive response can only enter the structure at the driven point. An understanding of power flow should give valuable insight into the problem. It remains to be seen whether power flow can be simply and accurately predicted.

POWER FLOW AND THE IMPORTANCE OF MOBILITY

The power flow into a structure at a point is given in [3] as

$$P = 1/2\ \mathrm{Re}[Y]\ F^2 \qquad (1)$$

where F is the applied force and Re[Y] is the real part of the point mobility. Power flow is therefore dictated by the strength of the source, but also by the dynamic characteristics of the structure. Both F and Y may be functions of frequency in which case P would be in the form of a spectral density. We will assume that the force spectrum is known so that the only further information required is the mobility of the machine foundation. In practice most machines are mounted on several feet and the assumption of single point contact is strictly not valid. Petersson [4] has suggested a method of dealing with the multiple point case which involves transfer mobilities as well as point mobilities for the contact points.

The problem remains of establishing mobilities for the structures of interest. This would appear at first to be a daunting task in view of the huge variety of possible structures and the number of variables involved. The most reliable means of establishing mobility is by direct measurement, but this cannot be used for prediction. Numerical methods such as the finite element method can be used with some accuracy, but this would hardly be any less effort than a complete response calculation and still the results cannot be generalised so that a structural change like an increase in floor span would demand a complete re-evaluation. We still need to find a means of predicting mobility which can be rapidly applied with acceptable accuracy.

MEAN OR CHARACTERISTIC MOBILITY

An important simplification can be made when the foundation to which the machine is attached is continuous and homogeneous such as a beam or plate. In fig. 2 is shown the exact theoretical solution to the point mobility of a simply supported centre driven beam. At high frequencies the curve converges to a straight line known as the characteristic mobility, or Yc, which happens to be the mobility of an infinitely long beam of the same cross section and material. At such high frequencies the ends of the beam are far from the driven point relative to a wavelength and the driver sees a beam which is infinite. It follows that Yc is independent of the boundary conditions and the position of the driver. The same trends in behaviour are observable for rods and plates except that here Yc is frequency invariant. If either the damping or the length (area for a plate) are increased then the mobility curve converges more rapidly to the characteristic line but the line itself is not affected. At lower frequencies the nearby edges reflect waves propagating out from the driver giving rise to resonant peaks and anti-resonant troughs. The peaks and troughs are the same distance above and below the characteristic line and so the mean mobility in this resonant range is equal to Yc.

Since Yc depends only on the material and cross sectional properties of the foundation it can be very quickly calculated. References 2,3,5 and 6 all give formulae for Yc some of which are reproduced in table 1. It is shown in reference 6 that it is possible to combine the mobilities of homogeneous structures to cover cases such as beam-stiffened plates, and this would enable most situations of practical interest to be dealt with.

UPPER BOUND TO MOBILITY

At low frequencies when the machine foundation is resonant it is not sufficient to know the mean power flow as there is a large variation about the mean (fig. 2). More information is needed about the height of the peaks, and in particular, if the design is to be safe it will be necessary to establish an upper bound. Shown in fig. 3 is the theoretical mobility of two centre-driven beams which are identical except for their end conditions; one is simply supported and the other free. The mean mobility is the same for both as expected. It can also be seen that the line through the resonant peaks is common to both (with the exception of the fundamental of the free beam). In fact it is shown in [6] that the resonant peak mobility is given by

$$Yr = 1/\omega_b M_v \qquad (2)$$

where ω_b is the modal bandwidth (equal to radian frequency ω x loss factor), and M_v is the so called modal mass. This relationship applies generally to continuous structures including rods and plates as well as beams. For centre-driven beams and rods M_v is simply half the total mass M/2, and for plates it is usually M/4. Therefore Yr is independent of the boundary conditions if the driver is at the centre. Yr does depend on the position of the driver but as long as it is not near a free edge the highest peaks occur for the centre-driven case, and this therefore forms an upper bound.

The surprising conclusion is that in addition to the mean we can find an upper bound to the mobility at low frequencies which is independent of the boundary conditions and the position of the driver. The only information required for the upper bound is the mass of the foundation and its' loss factor.

By way of an example the dotted line in fig. 2 was computed from equation 2, the same upper bound being applicable for both beams as they have the same mass and loss factor. The agreement is exact (for all but the first mode) provided that only one mode contributes significantly to each peak [6].

To provide an illustration for plates the point mobility was measured at the centre of an suspended aluminium plate measuring 374x342x4.7 mm. A layer of 'DEDSHETE' damping was added to one side which contributed significant mass but neglible stiffness. The plate (including damping layer) was first weighed and the loss factor, which was assumed frequency invariant, was found by modal analysis of the second resonance. It was then possible to compute the upper bound shown as a dotted line in fig 4. The computed characteristic mobility is also shown for which it was necessary to estimate the bending stiffness and the mass per unit area. An impulse hammer and fast Fourier analyser setup was used for the mobility measurement the results of which are shown in fig 4. Due to the sharpness of the lowest resonances a reliable measure of their peak mobility could not be obtained without 'zooming' and so the single curve shown is actually the result of several measurements.

We have seen that at high frequencies the mobility converges to the characteristic mobility Yc, and at low frequencies the mean mobility is given by Yc and the upper bound by Yr. There is a transition region between high and low frequency behaviour where equation 2 requires correction to account for the contribution to each peak from nearby modes. It can be seen in fig. 5 that this transition effectively begins where the height of the peaks above the mean drops below 10 dB. Further details can be found in [6].

CONCLUSIONS

The study of power flow from machines into structures is worth-while as this will give an indication of the requirements for isolation as well as providing valuable insight into the problem. Power flow is dictated by the force applied to the foundation and the mobility at that point; a means of estimating the foundation mobility is therefore required.

For rod, beam and plate-like foundations the mean mobility and an upper bound can be defined which are independent of the boundary conditions and the position of the driver. Both quantites can be calculated with good accuracy using simple formulae. For calculation of the mean the cross-sectional and material properties of the foundation are required, and for the upper bound the mass and loss factor are required.

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ELEMENT	DIRECTION OF FORCE	CHARACTERISTIC MOBILITY Yc	UPPER BOUND AT AT LOW FREQ. Yr
ROD	LONGITUDINAL	$Yc = \frac{1}{2A\sqrt{E\rho}}$	$Yr = 2/(M\omega\eta)$
BEAM	TRANSVERSE	$Yc = \frac{(1-j)}{4A\rho\omega}\left(\frac{A\rho}{EI}\right)^{1/4}$	$Yr = 2/(M\omega\eta)$
PLATE	TRANSVERSE	$Yc = \frac{1}{8\sqrt{B\rho h}}$	$Yr = 4/(M\omega\eta)$

E = Young's modulus
ρ = density
M = total mass
A = cross-section area
I = second moment of area
h = thickness
η = loss factor

B = plate bending stiffness
= $Eh^3/[12(1-\nu^2)]$
ν = poisson's ratio
ω = radian frequency
j = $\sqrt{-1}$

Table 1 Formulae for Characteristic Mobility and Upper Bound

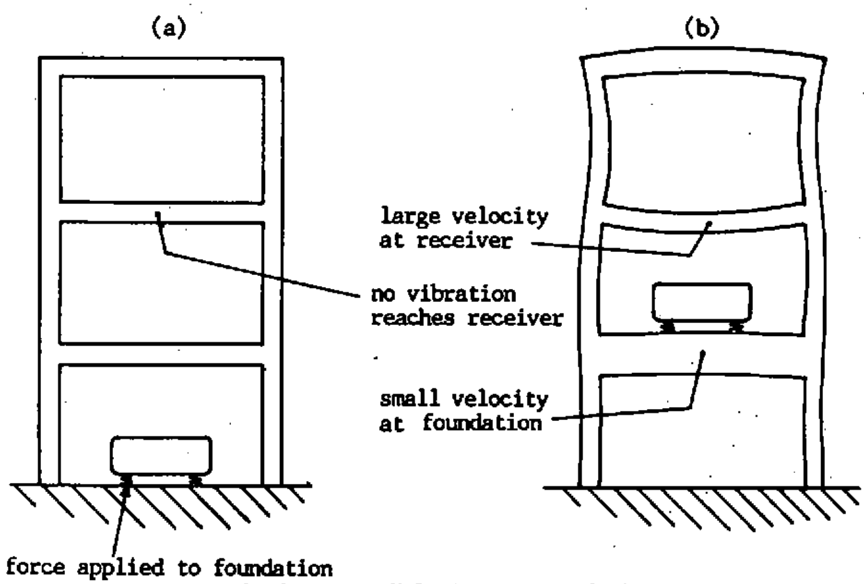

Fig. 1 Assessment by Force or Velocity at Foundation

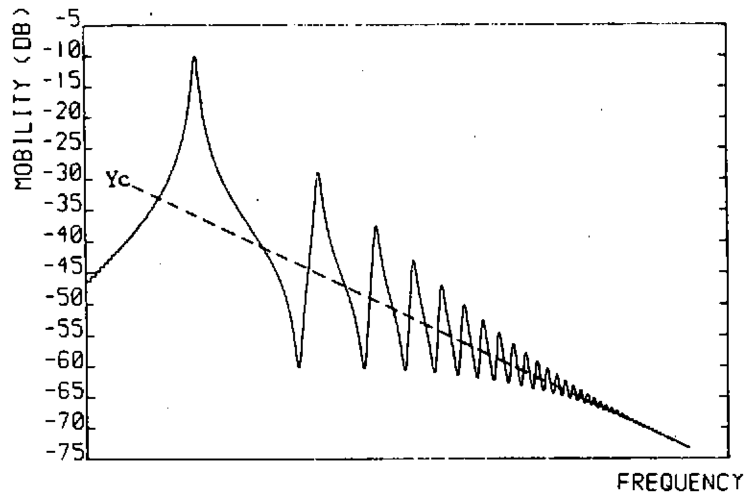

Fig. 2 Mobili:y of Simply-supported Beam Showing Convergence to Characteristic Mobility

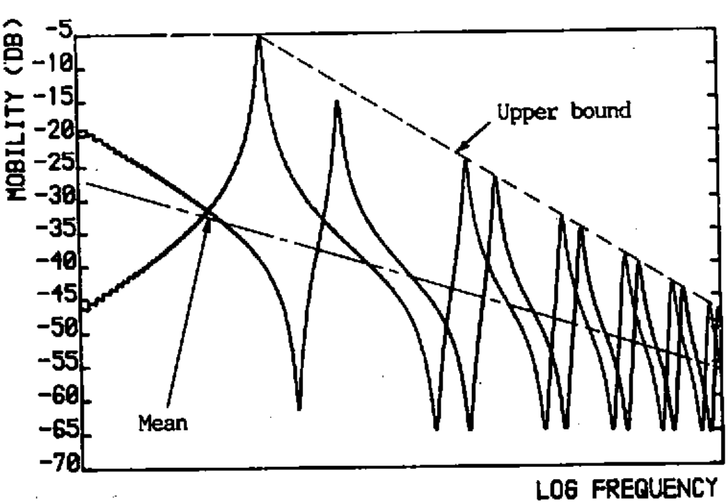

Fig. 3 Mobility of Free and Simply-supported Beams in Resonant Range

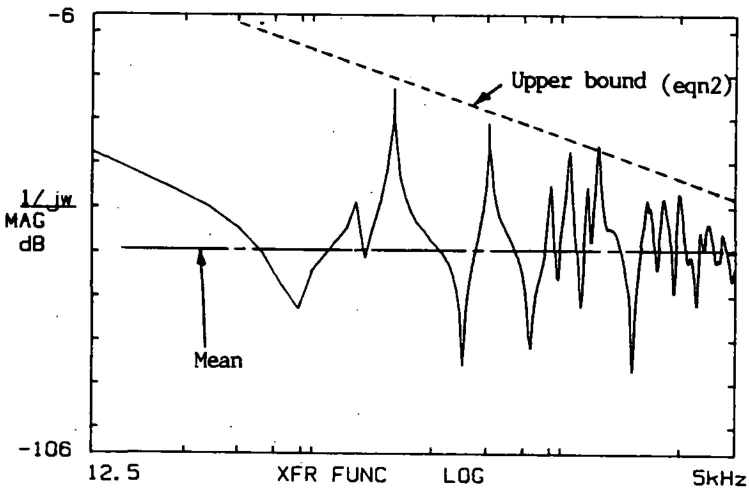

Fig. 4 Measured Mobility of Aluminium Plate Showing Predicted Mean and Upper Bound

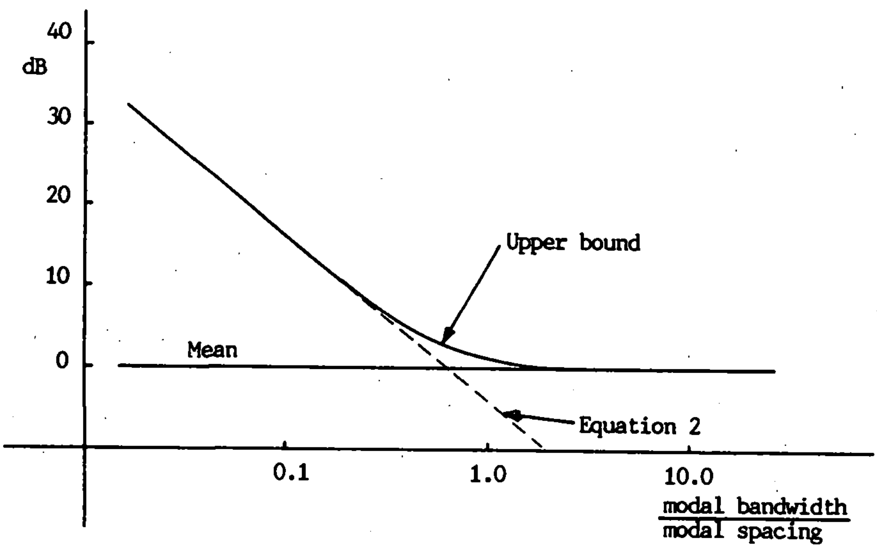

Fig. 5 Height of Resonant Peaks above Mean in Transition Range