Proceedings of the Institute of Acoustics

A BOUNDARY ELEMENT MODEL OF TRAFFIC NOISE PROPAGATION FROM A CUTTING

A.T.Peplow and S.N.Chandler-Wilde

Department of Civil Engineering, University of Bradford, Bradford, U.K.

1. INTRODUCTION

Recently the boundary integral equation (BIE) method has been applied to predict the performance of outdoor noise barriers on a homogeneous rigid or impedance plane [1-6]. In this method an integral equation is solved numerically, by a boundary element method, to determine the acoustic pressure on the barrier surface. Once the integral equation has been solved, the wave-field at any other point in the domain can be determined by a numerical integration over the barrier cross-section. The advantages of the method are that the Helmholtz equation governing the acoustic pressure field is solved accurately, provided the boundary elements used are small enough, and that an arbitrary cross-section and surface treatment for the barrier are allowed. This makes the method very powerful for the investigation of novel barrier designs [4-6]. The disadvantages of the method are that the computational cost increases with frequency since the cross-section must be divided into elements no longer than 1/5 wavelength for accurate results and a system of N simultaneous equations solved, where N is the number of elements at the particular frequency.

In this paper we extend the BIE method to calculate sound propagation out of a cutting of arbitrary cross-section and surface impedance onto surrounding flat rigid or absorbing homogeneous ground. The model developed is two-dimensional; it is assumed that the cutting is straight and infinitely long and that its cross-section and surface treatment do not vary along its length. The source is assumed to be a monofrequency coherent line source. This latter assumption is unrealistic but, on the basis of previous comparisons of boundary element calculations for noise barriers with experimental measurements [3,4], the model is expected to give accurate predictions of attenuation in excess of free-field propagation for the more realistic case of a point source of sound. Alternative BIE formulations of this problem but assuming an entirely rigid boundary are given in Willers [7] and, in the context of predicting water-wave climates in harbours, in Shaw [8].

2. THE BOUNDARY VALUE PROBLEM

The geometry is shown in Figure 1. The region of propagation D, in which the medium is assumed homogeneous and at rest, consists of the cutting labelled S, with boundary ∂S, situated below the half-space U$:=\{(x,y):y>0\}$. To define the other notation in Figure 1, the boundary ∂D of D is composed of γ_1, the boundary of the cutting, and $\gamma_3 := \partial D - \gamma_2$. γ_2 is the interface between S and U and $\partial U := \{(x,y) : y = 0\}$. It is intended to determine the complex acoustic pressure $p(\mathbf{r},\mathbf{r}_0)$ at points $\mathbf{r} = (x,y) \in \overline{D} := D \cup \partial D$ given a monofrequency point source $\mathbf{r}_0 = (x_0, y_0) \in D$. The pressure $p(\mathbf{r}, \mathbf{r}_0)$ is assumed to satisfy the following boundary value problem in which, for brevity, we write $p(\mathbf{r})$ for $p(\mathbf{r}, \mathbf{r}_0)$:

$$(\nabla^2 + k^2)p(\mathbf{r}) \;=\; \delta(\mathbf{r} - \mathbf{r}_0), \;\; \mathbf{r} \in D, \quad (1)$$

the impedance boundary condition

$$\frac{\partial p(\mathbf{r})}{\partial n} \;=\; ik\beta(\mathbf{r})p(\mathbf{r}), \quad \mathbf{r} \in \partial D, \quad (2)$$

and Sommerfeld's radiation condition in D. We assume that β, the normalised surface admittance, satisfies $\beta(\mathbf{r}) = 0$ or $Re\beta(\mathbf{r}) > 0$ at each point $\mathbf{r} \in D$, and that $\beta(\mathbf{r}) = \beta_c$, a constant, for $\mathbf{r} \in \gamma_3$. Throughout the normal, n, on ∂D is directed out of D. The normal on γ_2 is directed out of U and into S.

3. REFORMULATION AS AN INTEGRAL EQUATION

To formulate the integral equation, the solutions $G_f(\mathbf{r}, \mathbf{r}_0)$ and $G_{\beta_c}(\mathbf{r}, \mathbf{r}_0)$ to the following simpler problems are required. The free-field Green's function $G_f(\mathbf{r}, \mathbf{r}_0) := -i/4H_0^{(1)}(k|\mathbf{r} - \mathbf{r}_0|)$ satisfies equation (1) for all $\mathbf{r}$ and $\mathbf{r}_0$. We denote by $G_{\beta_c}(\mathbf{r}, \mathbf{r}_0)$ the solution to the above boundary value problem in the absence of the cutting; i.e. $G_{\beta_c}(\mathbf{r}, \mathbf{r}_0)$ denotes the pressure at $\mathbf{r}$ when propagation is above a plane of homogeneous admittance β_c. Where $\mathbf{r}_0' = (x_0, -y_0)$ is the image in the boundary line ∂U of $\mathbf{r}_0$,

$$G_{\beta_c}(\mathbf{r}, \mathbf{r}_0) \;=\; G_f(\mathbf{r}, \mathbf{r}_0) \;+\; G_f(\mathbf{r}, \mathbf{r}_0') \;+\; P_{\beta_c}(\mathbf{r}, \mathbf{r}_0) \quad (3)$$

where

$$P_{\beta_c}(\mathbf{r}, \mathbf{r}_0) := \begin{cases} 0, & \text{if} \quad \beta_c = 0, \\ \frac{i\beta_c}{2\pi} \int_{-\infty}^{+\infty} \frac{exp(i(y+y_0)(1-s^2)^{1/2} - (x-x_0)s)}{(1-s^2)^{1/2}((1-s^2)^{1/2} + \beta_c)} ds, & Re\beta_c > 0, \end{cases} \quad (4)$$

and $0 \leq arg\{(1 - s^2)^{1/2}\} \leq \pi/2$. Efficient and accurate methods for calculating $P_{\beta_c}(\mathbf{r}, \mathbf{r}_0)$ are described in [9].

To obtain an integral equation, the region V_1 consisting of that part of U contained in a large circle of radius R, excluding small circles of radius η about points $\mathbf{r}_0 \in D$, $\mathbf{r} \in U$ is considered. Applying Green's second theorem to the functions $p(.)$ and $G_{\beta_c}(., \mathbf{r})$ in the region V_1 and then letting $R \to \infty$ and $\eta \to 0$, and making use of the boundary condition (2), the following integral equation is obtained:

$$p(\mathbf{r}) = \int_{\gamma_2} G_{\beta_c}(\mathbf{r}, \mathbf{r}_s)(ik\beta_c p(\mathbf{r}_s) - \frac{\partial p(\mathbf{r}_s)}{\partial n})ds(\mathbf{r}_s) \;+\; \eta(\mathbf{r}_0)G_{\beta_c}(\mathbf{r}, \mathbf{r}_0), \quad \mathbf{r} \in \overline{U}, \quad (5)$$

where $\eta(\mathbf{r}_0) = 1$ for $\mathbf{r}_0 \in U$, 0 for $\mathbf{r}_0 \in$ S.

To obtain an integral equation in S, we apply Green's second theorem to the functions $p(.)$ and $G_f(.,\mathbf{r}_0)$ in S obtaining the boundary integral equation on γ_1 and γ_2,

$$\begin{aligned}\epsilon(\mathbf{r})p(\mathbf{r}) &= \int_{\gamma_2} G_f(\mathbf{r},\mathbf{r}_s)\frac{\partial p(\mathbf{r}_s)}{\partial n} - \frac{\partial G_f(\mathbf{r},\mathbf{r}_s)}{\partial n(\mathbf{r}_s)}p(\mathbf{r}_s)ds(\mathbf{r}_s) \\ &+ \int_{\gamma_1} p(\mathbf{r}_s)(\frac{\partial G_f(\mathbf{r},\mathbf{r}_s)}{\partial n(\mathbf{r}_s)} - ik\beta(\mathbf{r}_s)G_f(\mathbf{r},\mathbf{r}_s))ds(\mathbf{r}_s) \\ &+ (1-\eta(\mathbf{r}_0))G_f(\mathbf{r},\mathbf{r}_0), \quad \mathbf{r} \in \overline{S} \qquad (6)\end{aligned}$$

where $\epsilon(\mathbf{r}) = 1$ for $\mathbf{r} \in S$ and $1/2$ for $\mathbf{r} \in \gamma_1 \cup \gamma_2$. The equations (5) and (6) are a coupled pair of BIE's in which the unknowns are p on γ_1 and γ_2 and $\partial p/\partial n$ on γ_2. It can be shown that the BIE's (5) and (6) have a unique solution, so that this formulation of the problem is equivalent to the boundary value problem. Once this BIE has been solved, by a numerical method, to give the pressure distribution on the boundary, the pressure at a point $\mathbf{r}$ in D can be found by numerical integration of (5) and (6).

4. SOLUTION OF THE INTEGRAL EQUATIONS

In this section we describe how the equations (5) and (6) can be solved for the complex acoustic pressure by a simple boundary element method. Firstly p and $\partial p/\partial n$ on γ_2 and p on γ_1 are determined at regularly spaced points on γ_1 and γ_2 by solving a set of linear simultaneous equations. Once these three quantities are calculated, the value of $p(\mathbf{r})$ at any other point in D can be determined by applying numerical integration to equation (5) if $\mathbf{r} \in U$ or to equation (6) if $\mathbf{r} \in S$.

Let $p_1 := p|_{\gamma_1}$, $p_2 := p|_{\gamma_2}$ and $q := ik\beta_c p_2 - \partial p/\partial n|_{\gamma_2}$ and define g_{β_c} and G_2 on γ_2, G_1 on γ_1 by $g_{\beta_c}(\mathbf{r}) = G_{\beta_c}(\mathbf{r},\mathbf{r}_0)$, $G_2(\mathbf{r}) = G_f(\mathbf{r},\mathbf{r}_0)$, $\mathbf{r} \in \gamma_2$, $G_1(\mathbf{r}) = G_f(\mathbf{r},\mathbf{r}_0)$, $\mathbf{r} \in \gamma_1$. Then we may write (5) and (6) on γ_1 and γ_2 as

$$\begin{aligned} p_2(\mathbf{r}) &= S_{22}^{\beta_c}q(\mathbf{r}) + \eta(\mathbf{r}_0)g_{\beta_c}(\mathbf{r}), \quad \mathbf{r} \in \gamma_2, \quad (7) \\ \frac{1}{2}p_2(\mathbf{r}) &= K_{12}p_1(\mathbf{r}) - ikS_{12}(\beta p_1)(\mathbf{r}) + ik\beta_c S_{22}p_2(\mathbf{r}) - S_{22}q(\mathbf{r}) \\ &+ (1-\eta(\mathbf{r}_0))G_2(\mathbf{r}), \quad \mathbf{r} \in \gamma_2, \qquad (8) \\ \frac{1}{2}p_1(\mathbf{r}) &= K_{11}p_1(\mathbf{r}) - ikS_{11}(\beta p_1)(\mathbf{r}) + ik\beta_c S_{21}p_2(\mathbf{r}) - S_{21}q(\mathbf{r}) - K_{21}p_2(\mathbf{r}) \\ &+ (1-\eta(\mathbf{r}_0))G_1(\mathbf{r}), \quad \mathbf{r} \in \gamma_1. \qquad (9)\end{aligned}$$

The integral operators in (7), (8) and (9) are defined by

$$\begin{aligned} S_{ij}\phi(\mathbf{r}) &= \int_{\gamma_i} G_f(\mathbf{r},\mathbf{r}_s)\phi(\mathbf{r}_s)ds(\mathbf{r}_s), \quad \mathbf{r} \in \gamma_i, \ i = 1,2, \ j = 1,2, \quad (10) \\ K_{ij}\phi(\mathbf{r}) &= \int_{\gamma_i} \frac{\partial G_f(\mathbf{r},\mathbf{r}_s)}{\partial n(\mathbf{r}_s)}\phi(\mathbf{r}_s)ds(\mathbf{r}_s), \quad \mathbf{r} \in \gamma_i, \ i = 1,2, \ j = 1,2, \quad (11)\end{aligned}$$

and

$$S_{22}^{\beta_c}\phi(\mathbf{r}) = \int_{\gamma_2} G_{\beta_c}(\mathbf{r},\mathbf{r}_s)\phi(\mathbf{r}_s)ds(\mathbf{r}_s), \quad \mathbf{r}\in\gamma_2. \quad (12)$$

To solve the above system of integral eqautions numerically we first divide γ_1 and γ_2 into boundary elements. We suppose that γ_1 is polygonal and divide γ_1 into N_1 straight-line elements $\gamma_1^1, \gamma_1^2, \ldots, \gamma_1^{N_1}$. For $n = 1, 2, \ldots, N_1$ let $\mathbf{r}_1^n$ denote the mid-point and h_1^n the length of γ_1^n, and let $h_1 := max h_1^n$. Similarly divide the straight line interface γ_2 into N_2 elements $\gamma_2^1, \gamma_2^2, \ldots, \gamma_2^{N_2}$ and let $\mathbf{r}_2^n$ and h_2^n denote the mid-point and length of γ_2^n and $h_2 := max h_2^n$. Let $h := max\,(h_1, h_2)$.

Having made the above subdivision we approximate the integral operators S_{ij}, K_{ij} and $S_{22}^{\beta_c}$ as follows:

$$S_{ij}\phi(\mathbf{r}) \approx \sum_{m=1}^{N_i}\int_{\gamma_i^m} G_f(\mathbf{r},\mathbf{r}_s)ds(\mathbf{r}_s)\phi(\mathbf{r}_i^m), \quad (13)$$

$$K_{ij}\phi(\mathbf{r}) \approx \sum_{m=1}^{N_i}\int_{\gamma_i^m} \frac{\partial G_f(\mathbf{r},\mathbf{r}_s)}{\partial n(\mathbf{r}_s)}ds(\mathbf{r}_s)\phi(\mathbf{r}_i^m), \quad (14)$$

$$S_{22}^{\beta_c}\phi(\mathbf{r}) \approx \sum_{m=1}^{N_2}\int_{\gamma_2^m} G_{\beta_c}(\mathbf{r},\mathbf{r}_s)ds(\mathbf{r}_s)\phi(\mathbf{r}_2^m). \quad (15)$$

These approximations are accurate if ϕ is approximately constant within each boundary element. Making these approximations in equations (7)-(9) and then collocating at the mid-point of each boundary element we obtain a system of $N = 2N_2 + N_1$ simultaneous equations for the unknown values of p_2 and q at the mid-point of each element of γ_2 and p_1 at the mid-point of each element of γ_1. These equations can be written in matrix form as

$$\mathbf{A}\,\mathbf{p} = \mathbf{g} \quad (16)$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{I} & \mathbf{S}_{22}^{\beta_c} \\ ik\mathbf{S}_{12}\mathbf{B} - \mathbf{K}_{12} & \frac{1}{2}\mathbf{I} - ik\beta_c\mathbf{S}_{22} & \mathbf{S}_{22} \\ \frac{1}{2}\mathbf{I} + ik\mathbf{S}_{11}\mathbf{B} - \mathbf{K}_{11} & \mathbf{K}_{21} - ik\beta_c\mathbf{S}_{21} & \mathbf{S}_{21} \end{pmatrix} \quad (17)$$

$$\mathbf{p} = (p_1(\mathbf{r}_1^1), \ldots, p_1(\mathbf{r}_1^{N_1}), p_2(\mathbf{r}_2^1), \ldots, p_2(\mathbf{r}_2^{N_2}), q(\mathbf{r}_2^1), \ldots, q(\mathbf{r}_2^{N_2}))^T,$$

$$\mathbf{B} = diag(\beta(\mathbf{r}_1^1), \ldots, \beta(\mathbf{r}_1^{N_1})),$$

$$\mathbf{g} = (\eta(\mathbf{r}_0)g_{\beta_c}(\mathbf{r}_2^1), \ldots, \eta(\mathbf{r}_0)g_{\beta_c}(\mathbf{r}_2^{N_2}), (1-\eta(\mathbf{r}_0))G_2(\mathbf{r}_2^1), \ldots, (1-\eta(\mathbf{r}_0))G_2(\mathbf{r}_2^{N_2}),$$
$$(1-\eta(\mathbf{r}_0))G_1(\mathbf{r}_1^1), \ldots, (1-\eta(\mathbf{r}_0))G_2(\mathbf{r}^{N_1}))^T$$

where the elements of the sub-matrices $\mathbf{S}_{ij}, \mathbf{K}_{ij}, \mathbf{S}_{22}^{\beta_c}$ are given by

$$[\mathbf{S}_{ij}]_{lm} = \int_{\gamma_i^m} G_f(\mathbf{r}_j^l, \mathbf{r}_s)ds(\mathbf{r}_s),$$

$$[\mathbf{K}_{ij}]_{lm} = \int_{\gamma_i^m} \frac{\partial G_f(\mathbf{r}_j^l, \mathbf{r}_s)}{\partial n(\mathbf{r}_s)} ds(\mathbf{r}_s).$$

$$[\mathbf{S}_{22}^{\beta_c}]_{lm} = \int_{\gamma_2^m} G_{\beta_c}(\mathbf{r}_2^l, \mathbf{r}_s) ds(\mathbf{r}_s),$$

In the results shown below the elements of $\mathbf{S}_{ij}, \mathbf{K}_{ij}, \mathbf{S}_{22}^{\beta_c}$ are evaluated approximately using the product mid-point rule [10].

5. RESULTS

Two graphs of results are shown. Figure 3 compares the program (CUTIE) implementing the boundary element method of the previous section with the boundary element program for noise barriers on flat ground (BARIE) described in [3,4]. The geometry is as indicated in Figure 2 and for both programs the frequency is 100 Hz, the step-length h=0.1 wavelengths. In Figure 3 the curves denoted A are for receiver positions 10 cm above ground level, those labelled B for receiver positions 10 cm below ground level in the cutting, as shown in Figure 2. The two boundary element programs agree at each receiver position to within 1dB.

Boundary element predictions of propagation from a cutting across rigid ground or grassland are shown in Figure 5. The geometry as shown in Figure 4 and the frequency is 100Hz.

REFERENCES

1. A.DAUMAS 1978 *Acustica* 40,213-222. Etude de la diffraction par un ecran mince dispose sur le sol

2. R.SEZNEC 1980 *Journal of Sound and Vibration* 73, 195-209.Diffraction of sound around barriers: use of the boundary element technique.

3. S.N.CHANDLER-WILDE AND D.C.HOTHERSALL 1987 *Proceedings of the Institute of Acoustics* 9, 37-44. The boundary integral equation method in outdoor sound propagation.

4. D.C.HOTHERSALL, S.N.CHANDLER-WILDE AND N.M.HAJMIRZAE to appear in Journal of Sound and Vibration. The performance of single noise barriers.

5. D.C.HOTHERSALL, D.H.CROMBIE AND S.N.CHANDLER-WILDE to appear in Applied Acoustics. The performance of T-profile and associated noise barriers.

6. D.H.CROMBIE, A.T.PEPLOW, S.N.CHANDLER-WILDE AND D.C.HOTHERSALL to appear in Proceedings of the Institute of Acoustics. Multiple road traffic noise barriers.

7. A.WILLERS 1987*Mathematical Methods in the Appled Sciences*,312-323. The Helmholtz equation in disturbed half-spaces.

8. R.P.SHAW in *Topics in Ocean Engineering* (Bretschneider, editor). Gulf publishing company. Long period forced harbor oscillations.

9. S.N.CHANDLER-WILDE AND D.C.HOTHERSALL Submitted for publication. On the Green function for two-dimensional acoustic propagation above a homogeneous impedance plane.

SOUND PROPAGATION FROM A CUTTING

10. K.E.ATKINSON 1967 *SIAM Journal of Numerical Analysis* 4, 337-348. The numerical solution of Fredholm integral equations of the second kind.

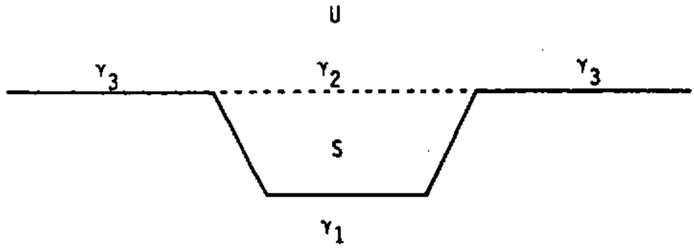

Figure 1. The geometry of the cutting.

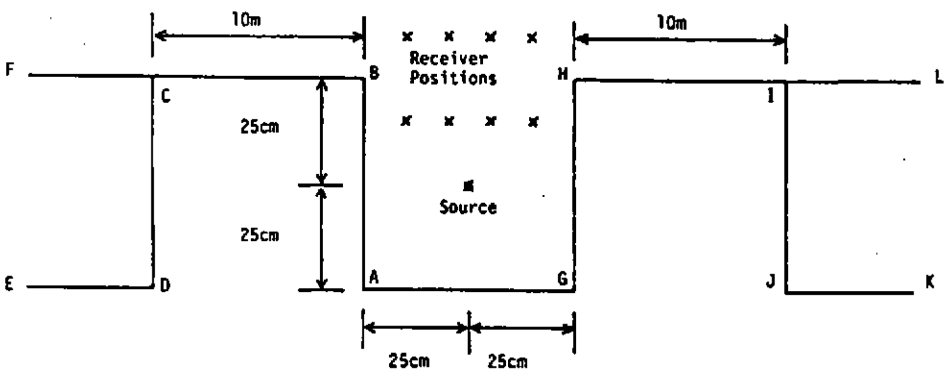

Figure 2. The geometry for comparison of the BARIE and CUTIE programs.

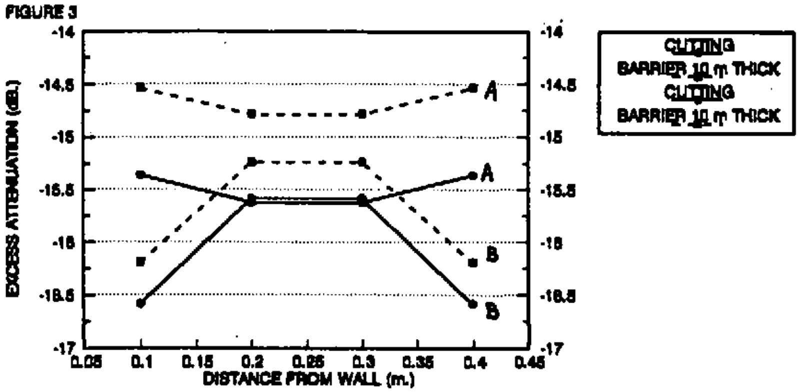

Figure 3. Comparison of the new boundary element program (CUTIE) with the boundary element program for noise barriers (BARIE). For the results from program CUTIE the cross-section is the polygon FCBAGHIL in Figure 2. For the program BARIE this cross-section is approximated by the polygon EDCBAGHIJK.

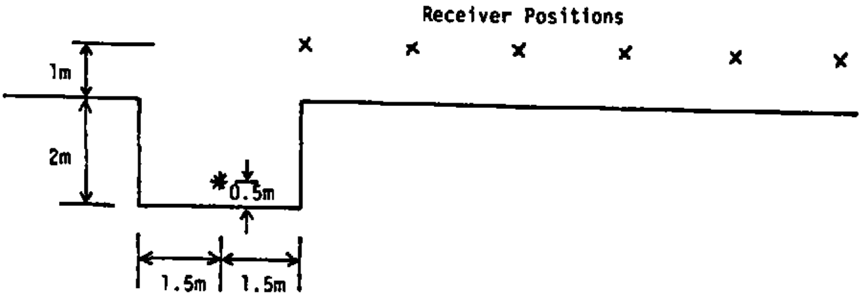

Figure 4. The geometry for the boundary element calculations in Figure 5.

SOUND PROPAGATION FROM A CUTTING

FIGURE 5

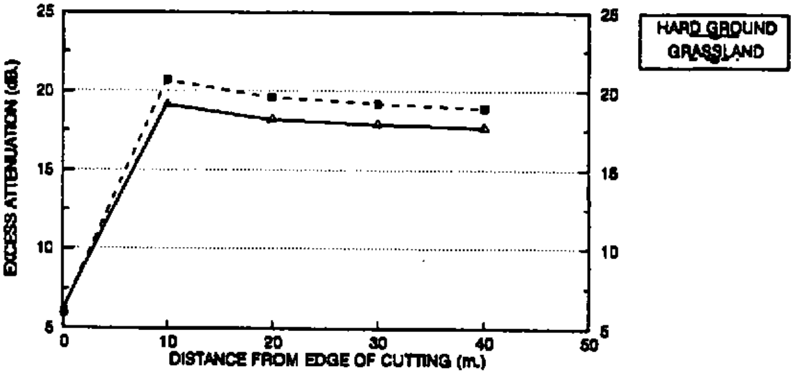

Figure 5. Boundary element prediction of propagation from a cutting across rigid ground or grassland. The geometry is as indicated in Figure 4 and the frequency is 100Hz.