THREE DIMENSIONAL NUMERICAL MODELLING OF SOUND PROPAGATION OVER NOISE BARRIERS OF ARBITRARY UNIFORM CROSS-SECTION

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1.INTRODUCTION

The boundary integral equation method (BIEM) is well-known in the context of time-harmonic acoustic scattering [1-8] in two and three dimensions. The method has been applied to the propagation, in two dimensions, of acoustic waves over a plane of homogeneous impedance [3], a plane perturbed by a discontinuity of inhomogeneous impedance [4-5], or over a cutting [6] or roadside noise barrier [7-8]. In these two-dimensional approximations cartesian axes 0xyz are adopted, with the x- and y-axes horizontal; the source is assumed to be a coherent isotropic line source, parallel to the y-axis, and there is no variation in any of the significant physical variables in the y-direction. This two-dimensional situation with a coherent line source does not of course directly model any actually occurring outdoor sound propagation problem; a traffic noise stream, for example, is correctly modelled by an incoherent line source or by a line of incoherent point sources. Nevertheless the two-dimensional predictions have been related to experimental measurements with a point source with some considerable success [4,8]. In this paper with a view to obtaining a more exact model of the performance of outdoor noise barriers, though with more computational cost, the full three-dimensional problem of propagation from a point source over an infinitely long noise barrier is addressed.

Below, this problem is formulated mathematically, first as an impedance boundary value problem for the Helmholtz equation, then reformulated as a boundary integral equation for the unknown pressure on the boundary. The numerical solution, using a simple boundary element method (BEM), of this integral equation is discussed. Results are presented comparing predictions of barrier insertion loss with this model with those from a two-dimensional prediction using s coherent line-source.

2. FORMULATION OF EQUATIONS

THE BOUNDARY VALUE PROBLEM

Attention is restricted throughout to time harmonic propagation ($e^{-i\omega t}$ time dependence), in a homogeneous medium, which is at rest in the absence of sound wave, and is characterised by the wavenumber $k = 2\pi/\lambda > 0$. Cartesian axes 0xyz are adopted, with the x- and y-axes horizontal. The source is assumed to be a monopole point source and there is no variation in the barriers cross-section along its length in the y-direction. Figure 1 illustrates a typical situation. D is the region of propagation and ∂D, the boundary, consisting of ground and barrier surfaces, is assumed locally reacting. The surface of the barrier is denoted by Γ. The acoustic pressure $p(\mathbf{r})$ at receiver point $\mathbf{r} = (x, y, z)$ due to the source at $\mathbf{r_0} = (x_0, 0, z_0)$ satisfies the following BVP:

$$(\nabla^2 + k^2)p(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r_0}), \quad \mathbf{r} \in \mathrm{D}$$

the impedance boundary condition

$$\frac{\partial p(\mathbf{r})}{\partial \mathbf{n}} = ik\beta(\mathbf{r})\mathrm{p}(\mathbf{r}), \quad \mathbf{r} \in \partial\mathrm{D}$$

and Sommerfeld radiation conditions. The normal is assumed to point out of the region of propagation, that is, into the barrier. We assume that β, the normalised surface admittance satisfies $\beta(\mathbf{r}) = 0$ or $Re\beta(\mathbf{r}) > 0$ at each point $\mathbf{r} \in \partial\mathrm{D}$. We assume that the admittance is constant, $\beta = \beta_c$ everywhere except on the barrier Γ.

THE BOUNDARY INTEGRAL EQUATION

Let $G_{\beta_c}(\mathbf{r}, \mathbf{r_0})$ be the solution to the same problem but in the absence of the noise barrier, i.e. $G_{\beta_c}(\mathbf{r}, \mathbf{r_0})$ is the pressure at $\mathbf{r}$ when the source is at $\mathbf{r_0}$ and propagation is over a homogeneous plane with constant admittance, β_c.The Thomasson approximation for $G_{\beta_c}(\mathbf{r}, \mathbf{r_0})$ [10] is evaluated accurately as a series expansion with a suitable number of terms taken according to the distance $|\mathbf{r} - \mathbf{r_0}|$. By applying Greens second theorem [9] to the two functions $p(\mathbf{r})$ and $G_{\beta_c}(\mathbf{r}, \mathbf{r_0})$ an integral equation is obtained over the whole domain $\overline{D}$.

$$\epsilon(\mathbf{r})\mathrm{p}(\mathbf{r}) = \mathrm{G}_{\beta_c}(\mathbf{r}, \mathbf{r_0}) + \int_\Gamma \frac{\partial \mathrm{G}_{\beta_c}(\mathbf{r}, \mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})}\mathrm{p}(\mathbf{r_s}) - \mathrm{ik}\beta(\mathbf{r_s})\mathrm{G}_{\beta_c}(\mathbf{r}, \mathbf{r_s}))\mathrm{ds}, \qquad \mathbf{r} \in \overline{\mathrm{D}}$$

where $\epsilon(\mathbf{r}) = 1$ for $\mathbf{r} \in \mathbf{S}$ and $\frac{1}{2}$ for $\mathbf{r} \in \Gamma$.

A difficulty in the numerical solution of this integral equation is that the kernel function, $\frac{\partial G_{\beta_c}(\mathbf{r}, \mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})}$, tends to infinity as $\mathbf{r}$ tends to $\mathbf{r_s}$. This difficulty is resolved by applying the modification of Burton [12]. The modified integral equation on the barrier is given below where the barrier surface is denoted by Γ.

$$\begin{aligned}\frac{1}{2}p(\mathbf{r}) = \mathrm{G}_{\beta_c}(\mathbf{r}, \mathbf{r_0}) \quad &+ \quad \int_\Gamma \frac{\partial G_{\beta_c}(\mathbf{r}, \mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})}p(\mathbf{r_s}) - \frac{\partial \mathrm{G}^*(\mathbf{r}, \mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})}\mathrm{p}(\mathbf{r})\mathrm{ds} \\ &- \quad ik\int_\Gamma \beta(\mathbf{r_s})\mathrm{G}_{\beta_c}(\mathbf{r}, \mathbf{r_s})\mathrm{p}(\mathbf{r_s})\mathrm{ds} + \mathrm{p}(\mathbf{r})\int_\Gamma \frac{\partial \mathrm{G}^*(\mathbf{r}, \mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})}\mathrm{ds}. \quad (1)\end{aligned}$$

The function $G^*(\mathbf{r},\mathbf{r_s}) = -\frac{1}{4\pi}\left(|\mathbf{r}-\mathbf{r_s}|^{-1} + |\mathbf{r}-\mathbf{r}'_s|^{-1}\right)$ is the principal singularity of $G_{\beta_c}(\mathbf{r},\mathbf{r_s})$. From Gauss' theorem, the last integral can be integrated exactly, giving

$$\int_\Gamma \frac{\partial G^*(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} ds = -\frac{1}{2}$$

for $\mathbf{r} \in \Gamma$. Thus

$$\begin{aligned} p(\mathbf{r}) = \mathbf{G}_{\beta_c}(\mathbf{r},\mathbf{r_0}) \;\; &+ \;\; \int_\Gamma \frac{\partial G_{\beta_c}(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} p(\mathbf{r_s}) - \frac{\partial \mathbf{G}^*(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} \mathbf{p}(\mathbf{r}) \mathbf{ds} \\ &- \;\; ik \int_\Gamma \beta(\mathbf{r_s}) \mathbf{G}_{\beta_c}(\mathbf{r},\mathbf{r_s}) \mathbf{p}(\mathbf{r_s}) \mathbf{ds}. \end{aligned}$$

In order that the integral equation above can be solved numerically the region over which we integrate must be truncated for some finite value of y. Thus we replace the infinite barrier surface Γ in equation (1) by a truncated surface of length $2A$, $\Gamma_A = \{\mathbf{r} = (\mathbf{x},\mathbf{y},\mathbf{z}) \in \Gamma : |\mathbf{y}| \le \mathbf{A}\}$. We denote the solution of equation (1) with Γ replaced by Γ_A as p_A. Thus p_A satisfies the equation:

$$\begin{aligned} p_A(\mathbf{r}) = \mathbf{G}_{\beta_c}(\mathbf{r},\mathbf{r_0}) \;\; &+ \;\; \int_{\Gamma_A} \frac{\partial G_{\beta_c}(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} p_A(\mathbf{r_s}) - \frac{\partial \mathbf{G}^*(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} \mathbf{p_A}(\mathbf{r}) \mathbf{ds} \\ &- \;\; ik \int_{\Gamma_A} \beta(\mathbf{r_s}) \mathbf{G}_{\beta_c}(\mathbf{r},\mathbf{r_s}) \mathbf{p_A}(\mathbf{r_s}) \mathbf{ds}. \end{aligned} \qquad (2)$$

We now describe the numerical method (boundary element method) for solving the truncated integral equation over the barrier. Once this BIE has been solved, by a numerical method, to give the pressure distribution on the boundary, the pressure at the point $\mathbf{r}$ in D can be found by numerical integration of (2).

3. NUMERICAL METHOD FOR SOLVING BIE'S

Let Γ_A be divided into $M * N$ elements, $\Gamma_A^j, j = 1, ..., MN$, by putting a rectilinear mesh mesh generated with M sub-divisions in the y-direction and N sub-divisions in the x-z plane. Let $\mathbf{r_j}$ denote the mid-point and A_j the area of an element j. The elements are numbered in figure 2. To obtain accurate results the sides of each element must be ≤ 0.2 wavelengths

The boundary integral equation (2) is solved by a product mid-point rule approximation [13], it follows that :

$$p_A(\mathbf{r_j}) = \mathbf{G}_{\beta_c}(\mathbf{r_j},\mathbf{r_0}) + \sum_{l=1}^{MN} \mathbf{I_{jl}} \qquad (3)$$

for $j = 1(1)MN$, where $\mathbf{r_j}$ is the midpoint of element Γ_A^j and:

$$I_{jl} = \int_{\Gamma_A} \frac{\partial G_{\beta_c}(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} p_A(\mathbf{r_s}) - \frac{\partial \mathbf{G}^*(\mathbf{r},\mathbf{r_s})}{\partial \mathbf{n}(\mathbf{r_s})} \mathbf{p_A}(\mathbf{r})d\mathbf{s}$$
$$-ik \int_{\Gamma_A} \beta(\mathbf{r_s}) \mathbf{G}_{\beta_c}(\mathbf{r},\mathbf{r_s}) \mathbf{p_A}(\mathbf{r_s}) d\mathbf{s}. \quad (4)$$

For $j \neq l$, the approximation can be made that

$$I_{jl} \approx \left(\frac{\partial G_{\beta_c}(\mathbf{r_l},\mathbf{r_j})}{\partial \mathbf{n}(\mathbf{r_j})} p_A(\mathbf{r_l}) - \frac{\partial \mathbf{G}^*(\mathbf{r_l},\mathbf{r_j})}{\partial \mathbf{n}(\mathbf{r_j})} \mathbf{p_A}(\mathbf{r_j}) \right) r \int_{\Gamma_A^j} \frac{1}{r} ds$$
$$-ik\beta(\mathbf{r_l}) \mathbf{G}_{\beta_c}(\mathbf{r_l},\mathbf{r_j}) \mathbf{p_A}(\mathbf{r_l}) \mathbf{r} \int_{\Gamma_A^j} \frac{1}{\mathbf{r}} d\mathbf{s} \quad (5)$$

where $r = |\mathbf{r_l} - \mathbf{r_j}|$, and for $j = l$

$$I_{jl} \approx -ik\beta(\mathbf{r_l}) \mathbf{r} \mathbf{G}_{\beta_c}(\mathbf{r_l},\mathbf{r_j}) \mathbf{p_A}(\mathbf{r_l}) \int_{\Gamma_A^j} \frac{1}{\mathbf{r}} d\mathbf{s}. \quad (6)$$

These approximately satisfied set of N linear equations for the values of p at the midpoints of the boundary elements can be written in standard form for $j = 1(1)MN$,

$$\sum_{l=1}^{MN} a_{jl} p(\mathbf{r_l}) = \mathbf{G}_{\beta_c}(\mathbf{r_0},\mathbf{r_j}) \quad (7)$$

where the elements of the matrix $[a_{jl}]$ are given by equations (5) and (6). The integrals in equations (5) and (6) can be evaluated exactly [12].

The matrix $[a_{jl}]$ is block Toeplitz of order $M * N$, with block-entries of order N. The computational cost of using Gaussian-Elimination for this method is $O(M^2N^2)$ to set up the governing equations and $1/3M^3N^3$ manipulations to solve the set of linear equations. With algorithms that use the block Toeplitz structure [14,15], equations (7) can be solved using $O(4M^2N)$ storage with $O(6M^3N^2)$ multiplications.

RESULTS

Results calculated using the above methods are shown in figure 3 in which insertion loss defined is by $IL = -20 \log \frac{p(r)}{G_{\beta_c}(r,r_0)}$. The source is located $5.6m$ from the centre of the barrier. The barrier is rigid and has square cross-section (in the x-z plane), 1m high and 1m thick. The two-dimensional model discretises this cross-section, whereas the three-dimensional model must take the length of the barrier into account. The insertion loss calculated by the two-dimensional model at a receiver position 50m from the centre of the barrier, at 100 Hz, is $3.5dB$. For the three-dimensional model the number of subdivisions in the x-z plane were the same as the two-dimensional model. The subdivisions in the y-direction were constant $0.1m$ lengths. The insertion loss was underestimated for a small number of sub-divisions, due to sound waves diffracting around the sides of the barrier. Thus comparable results are only produced when the barrier is of considerable length relative to the height and width of the barrier. This results in very large systems of equations to be solved directly.

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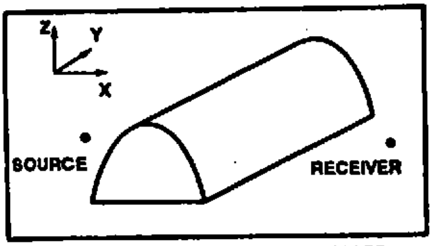

FIGURE 1 TYPICAL SITUATION MODELLED.

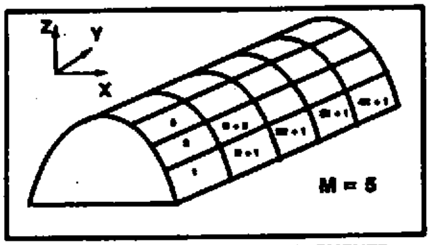

FIGURE 2 NUMBERING OF THE ELEMENTS

FIGURE 3 Comparison of three-dimensional model with two-dimensional model at 100 Hz