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LARGE EDDY SIMULATION OF SOUND RADIATION AND SCATTERING BY TURBULENT FLOW IN SLIGHTLY COMPRESSIBLE MEDIUM

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1. INTRODUCTION

Sound scattering and sound radiation by localized vortices are widely investigated in connection with importance of these processes for a number of problems in aeroacoustics (turbulent flow noise abatement [1,2]), in atmospheric and ocean acoustics (as a model of sound interaction with large scale turbulence [3]), nonlinear acoustics (this processes could produce harmonics from synchronism and thus could prevent the generation of shock waves [4]), and even in the theory of superfluidity (scattering of vortices determines the mutual friction force between the superfluid and normal components of the medium [5]), and astrophysics (as a possible mechanism of energy transfer in solar coron [6]). From the formal point of view the radiation and scattering of sound by vortices is a kind of nonlinear process responsible for interaction between the potential (acoustics waves) and rotational (vortices) components of motion of compressible medium [7]. In the present paper we shall review the fundamental laws governing the processes of sound radiation and scattering by vortices in a slightly compressible medium, i.e. in the case when characteristic vortex velocity is considerably less than speed of sound:

$$v \ll c \ . \qquad (1)$$

Flows in the form of localized vortices have been studied in detail [8] in hydrodynamics and are often used to obtain both analytical and numerical results (in particular, a solvable model of turbulent flow). Such model approach to the problems involved allows to obtain analytical description of typical nonlinear process occurring in real turbulent flow (stretching of vortex tubes, collapse and coupling of vortices and so on) and consequently analytical description of "elementary"sound radiation and scattering processes in such flow.

2. SOUND RADIATION BY VORTICES

Here we consider the radiation of sound by vortices in free space. The modern approach to the problem was suggested by Mohring [9] (see also [10]), who, based on Lighthill equation, obtained the farfield sound pressure in terms of vorticity $\Omega(\mathbf{r},t)=\mathrm{curl}\,\mathbf{v}$:

$$p(R,t) = \frac{\rho}{c^2}\,\frac{n_i n_j}{R}\,Q_{ij}(t-R/c)\ , \qquad (2)$$

where $\mathbf{R}$ is the radius vector of the observation point, $R=|\mathbf{R}|$, $n=\mathbf{R}/R$ is the unit vector in the direction of $\mathbf{R}$, and

$$Q_{ij}(t) = \frac{1}{12\pi} \int \xi_i [\xi \times \Omega] \, d\xi \ .$$

The integration is carried out here over the vortex flow domain. Equation (2) together with the familiar equation of dynamics of vortices in incompressible fluid [8]

$$\frac{\partial \Omega}{\partial t} + \text{curl } [v \times \Omega] = \nu \Delta \Omega \qquad (3)$$

allows to investigate theoretically the process of sound radiation by vortices. Substituting different expressions for vorticity distribution $\Omega(r,t)$ (i.e. the solution of equation (3)) in equation (2), corresponding to dynamics of particular vortex system, one can calculate corresponding acoustic emission. For a system of point vortices in inviscid fluid [11]

$$\Omega(r,t) = \sum_{\alpha} p^{(\alpha)} \delta(r - r^{(\alpha)}) + \nabla \Phi \ , \qquad (4)$$

where $p^{(\alpha)}(t)$ - vortices intensity, $r^{(\alpha)}(t)$ - their trajectories, Φ - potential component associated with the need to satisfy the condition $\text{div}\Omega=0$ in three-dimensional case [11], the dynamics of vorticity $\Omega(r,t)$ reduces only to calculation of functions $p^{(\alpha)}(t)$, $r^{(\alpha)}(t)$. In a plane case $p^{(\alpha)}=(0,0,p^{(\alpha)})=\text{const}$, $\Phi=0$ [8] and situation is considerably simplified. Due to multiple analytic solutions describing point vortices dynamics in plane case availability (two-vortex system rotating as a whole, head-on collisions between two identical vortex pairs, one vortex near a corner and so on) [8], there exist a lot of publications where radiation of sound by such system has been considered (see [12,13] and references). The sound radiation by system of point vortices in collapse regime is of particular interest. It was obtained, that sound radiation power in this case obeys equation [13]

$$W(t) = W(0) \left| 1 - t/t_* \right|^{-\alpha} \ , \ \alpha = 3 \qquad (5)$$

where W(0) is the power radiated at t=0, t_* - time of collapse. Thus the power of sound radiation generated in the collapse of vortices is described by an explosive-type of function, i.e. it becomes infinite in a finite time. The regimes of collapse-type motions are discussed intensively nowadays in application to the dynamics of turbulent patches in ocean. The result (5), which was obtained on the basis of an elementary (discrete) model of collapse shows, that the region of a turbulent patch can serve as a strong source of ocean noise in this case. Sound radiation increase predicted before in [13] was confirmed in our numerical experiments [14] for turbulent patch in stratified medium. Results obtained can be generalized on three-dimensional case (three-dimensional point vortices are called vortons [11]). Acoustic intensity explosive-type increase (5) takes place for a system of vortons as well, but with $\alpha=14/3$ [13]. Another characteristic nonlinear effect which together with collapse largely determines the dynamics of vortex perturbations in three-dimensional case, is the stretching of vortex tubes [ ]. Using vorton formalism it could be shown, that the characteristic vorticity produced due to stretching of vortex tubes increases exponentially (intensities of vortons $p^{(\alpha)}\sim\exp[-st]$ where $s>0$ is determined by the initial configuration of vorton system). From equations (2), (4) one can obtain, that the total sound radiation intensity increases exponentially too [13]. Of course, the monotonic increase of the acoustic radiation intensity occurs only in the initial stage of the collapse and stretching vortex tubes possesses, because the basis assumption of the theory (1) is eventually violated. In all other papers the generation of strong sound waves by vortices occurs only when feedback effect of sound on vortices is taken into account [13]. Thus, using of localized eddy simulation in the problem of vortex sound , allows to prove the conjection that due to nonlinear hydrodynamical effects intensive acoustic emission is really produced. The problem of proving conjectures of this kind is known as the dynamo problem in magnetohydrodynamics [14]. Another class of solvable problems of sound generated by localized vortices is connected with approximate solution of equation (3). This solution corresponds to nonlinear large scale vortex perturbations, propagating along shear flow - vortex solitons [13,15] (the solitons may be considered as a simple model of coherent structures in turbulent flows). For the jet-type flow it could be shown, that the dynamic of axial vortex perturbation is governed by well known Korteweg-de Vries equation [16]

$$v_t + U_o v_z + v v_z + \nu v_{zz} + \beta v_{zzz} = 0, \qquad (6)$$

with the functions Q_{ij} from (2) being represented in the form [13,17]

$$Q_{zz}(t) = \int_{-\infty}^{+\infty} z v(z,t)\, dz \quad , \quad Q_{ij} = 0, \text{ if } i \neq z,\ j \neq z\ . \qquad (7)$$

Here v is amplitude of velocity perturbation along jet axis, U_o - velocity on the jet axis, $\beta=U_o d^2 \ln 2/2=\text{const}$, d - jet radius. Equation (7) is directly derived from equation (3) and other equations of motion [13]. The system of equations (2), (6), (7) can be used to investigate sound radiation theoretically for any processes involving nonlinear vortex perturbations (6) [13,17], viz.: various collision phenomena, the decay of an arbitrary perturbation into solitons, dissipation, etc., since the corresponding solution of equation (7) (i.e. the form of the functions v(z,t) entering into (2) and (7) for these processes) are well known [16]. The results concerning the sound generation by other kind of vortices (Kirchhoff vortex, vortex rings, hypocycloidal vortices and so on) may be found in [13,18,19,20]. Some experimental results are described in [20,21].

3. SOUND SCATTERING BY VORTICES

Let a high frequency (ω_o>>v/c, where v is characteristic vortex velocity, a - vortex size) plane sound wave be incident on a system of vortices. The scattered field is then described by the following expression in the general case at large distances from the system (see, e.g., [22-27]):

$$\frac{p_s}{p_i} = f(n,n_o)\ \frac{e^{ik_oR}}{R},$$

where p_i and p_s are the amplitudes of the sound pressure in the incident and scattered waves,

$$f(n,n_o) = \frac{i\omega_o}{2\pi^2}\ \frac{(nn_o)([n_o n]\Omega(q))}{q^2} \qquad (8)$$

is the scattering amplitude, c is the sound velocity, k_o is the incident wave vector, $\omega_o=ck_o$ is the wave frequency, $n_o=k_o/|k_o|$, $R=|R|$ is the radius of the point at which the scattered field is determined (the origin is located inside the system of vortices), $n=R/|R|$; $q=k-k_o$; $k=k_on$; and

$$\Omega(q) = \int \Omega(r)\ e^{iqr}dr$$

is the Fourier component of the vorticity $\Omega(r)$. From expressions (8) one can see, that the back-scattering ($k=-k_o$) and scattering in perpendicular direction ($k\perp k_o$) are always equal to zero. In the case $k=k_o$ (forward scattering) amplitude f is finite, if $\Omega(q=0)=\int\Omega(y,t)dy=0$. This integral corresponds to circulation around vortices, and, generally speaking, is non-zero. That is why scattering amplitude may be irregular and Born approximation is not valid. Such irregularity takes place if Born approximation is applied to the problem of sound scattering by trailing-edge vortex [22,23] and may be eliminated by special "renormalization" procedure [28]. The scattering cross section of vortex can be calculated by means of equations (8):

$$\sigma = \int |f|^2 d\theta,$$

where $d\theta$ is an element of solid angle. By means of Born approximation, sound scattering by different kinds of vortices (Hill vortex, vortex rings, vortons, vortex solitons) was calculated. As a result, the following estimates for total cross sections have been obtained

$$\sigma \cong a^2 M^2 (ka)^{2p} \qquad ka \ll 1$$

$$\sigma \cong a^2 M^2 (ka)^{2p-2} \qquad 1 \ll ka \ll 1/M$$

where p=1,2,3... and depends on vortex structure, M=v/c - Mach number. It may be shown, that the general criterium of Born approximation validity here is $kaM \ll 1$ [ ]. In reference [29] so called eikonal approximation valid for intermediate short-wave case $ka \gg 1$ was additionally developed. Finally, in the limit $ka \to \infty$ ray approach is successful and it would be more accurate to consider the scattering of sound as a refraction on a vortex. The system of equations describing the refraction of sound ray in moving medium is well known [30]

$$\frac{d\mathbf{m}}{dl} = \frac{1}{c}\,[\Omega \mathbf{m}], \qquad (9)$$

where **m** is unit vector along the ray, l - ray length. From (9) one can see, that sound is refracted only by vortex component of velocity. Therefore, if there is a system of localized vortices in moving medium, each of them will behave as a lens with respect to sound rays. From equation (9) it may be easily estimated, that the focus distance L for such "vortex" lens is $L \cong a/M$. Ray pattern corresponding to sound refraction on Hill vortex was calculated in [31]. Vortex displacement changes sound field scattered by vortices. Particularly, Doppler frequency shift $\Delta\omega = k_o U_o$ arises, where U_o is velocity of vortex displacement. If there is intrinsic inner vortex mode of oscillation with characteristic frequency Ω_n, the spectrum of sound field scattered by vortex will be enriched by combination of frequencies $\omega_o \pm \Omega_n m$ (m=0, ±1, ±2) [18]. Finally, let us briefly consider scattering in the case $\omega_o \cong v/a$, i.e. when resonance between incident sound waves and inner hydrodynamic motion in vortices occurs at resonance. Effective energy exchange between incident waves and vortices is possible, so the incident wave intensity is could be even increased (so called superreflection effect). The effects mentioned above may be used for acoustical diagnostics of vortices and control of turbulent flow parameters. Some experimental results concerning sound scattering are presented in [23,24].

4. CONCLUSION

In the present paper the main effects concerning sound radiation and scattering by vortices were considered. The scope of the consideration was restricted to rather simple situations: homogeneous isotropic fluid, low velocities of vortices, purely hydrodynamical approach, absence of boundaries (some generalizations of the approaches developed were considered in [32,33]). Nevertheless, one may hope that the laws discussed above, although established on the basis of such an elementary model, are of sufficiently general nature and could be used in a number of cases to obtain simple estimates of similar phenomena in more complex hydrdynamic systems (the atmosphere, the ocean, superfluids, plasma etc.).

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