# RESPONSE OF A SINGLY CURVED, ORTHOGONALLY STIFFENED PANEL TO A RANDOM PRESSURE FIELD

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## 1. Introduction

The prediction of the response of aircraft fuselage structures to acoustic excitation is of interest because of the resulting sound radiation and fatigue problems. A fuselage structure essentially consists of a cylindrical shell stiffened by circumferential frames and longitudinal stringers. This paper is concerned with determining the response of a portion of such a structure to a random pressure field. The model is a rectangular, singly-curved, orthogonally stiffened panel. It has six curved, channel section frames and sixteen straight, Z-section stringers. The response of this model to discrete frequency excitation has previously been measured [1]. Detailed measurements were made of part of the structure between two adjacent frames. A theoretical analysis of this curved, skin-stringer array using the finite strip method is presented in reference [2]. This paper presents a finite element analysis of the complete model. In order to minimize the computing, the model is treated as a two-dimensional periodic structure [3]. The computed response is compared with some recent measurements.

## 2. Response to random pressure field

The excitation used in the experimental investigation can be approximated by a 'frozen' convected random pressure field. In this case it can be shown that the mean square response of the structure $\overline{w^2}$, at the point $\underline{r}$ is given by:

$$\overline{w}^2 = \int_{-\infty}^{+\infty} S_w(\underline{r},\omega)\, d\omega \qquad (1)$$

where $S_w(\underline{r}, \omega)$ is the power spectral density of the response at $\underline{r}$ which in turn is given by:

$$S_w(\underline{r},\omega) = |\alpha(\underline{r},\omega)|^2 S_p(\omega) \qquad (2)$$

where $S_p(\omega)$ is the power spectral density of the pressure field at any point. The function $\alpha(\underline{r},\omega)$ represents the response of the structure at $\underline{r}$ due to an harmonic pressure wave of unit amplitude, frequency $\omega$ and wave-number $\omega/V$ where V is the convection velocity, which is constant for all spectral components. This function is known as the wave receptance function.

## 3. Wave receptance function

The model described in section 1 is a two-dimensional, periodic structure. That is, it consists of a number of identical sub-structures which are coupled together in identical ways in a two-dimensional array to form the complete structure. The sub-structure consists of a singly curved, rectangular panel with a frame segment along one curved edge and a stringer segment along one straight edge.

The response of a finite periodic structure is given by the forced response of an infinite periodic structure plus the free response of the finite structure. This latter component accounts for the reflections from the boundaries. Now the model consists of a large number of identical sub-structures and so the effect of the boundaries will not be significant in the central region. This means that a good approximation to the response away from the boundaries can be obtained by considering the forced response of an infinite, periodic structure only.

Since all the sub-structures are identical, it is only necessary to derive a mathematical model for one sub-structure. In the present investigation the finite element displacement method was used. The singly curved,rectangular panel was represented by a 4 x 4 mesh of singly curved, rectangular elements. These elements have four nodes with nine degrees of freedom at each node, namely

$$u,\ u_y,\ v,\ v_x,\ v_y,\ w,\ w_x,\ w_y,\ w_{xy}\ [4].$$

u, v, w are the components of displacement in the x, y, z directions, which are the straight, curved and normal directions respectively. The frame and stringer segments were each represented by 4 thin-walled, open section beam elements. Each element has two nodes with the same nine degrees of freedom as the shell element. However, since these elements are attached to the shell along discrete lines, the straight beam does not contribute to the degrees of freedom $u_y$, $v_y$, and the curved beam does not contribute to the degree of freedom $v_x$.

The equations of motion of a single substructure take the form

$$\left[(1 + i\eta)\ [K] - \omega^2\ [M]\ \right]\ \{q\} = \{F\} \tag{3}$$

when harmonic time dependence, $e^{i\omega t}$, is assumed. [K] and [M] are the stiffness and inertia matrices, $\eta$ the structural loss factor and {q} a column matrix of nodal degrees of freedom. {F} is a column matrix of equivalent nodal forces which arise from the eight neighbouring substructures and the pressure distribution.

$$p(\underline{r},t) = e^{i(\omega t - \underline{k}.\underline{r})} = e^{i(\omega t - k_1 r_1 - k_2 r_2)} \tag{4}$$

where $k_1$, $k_2$ and $r_1$, $r_2$ are the components of the wave-number $\underline{k}$ and the position vector $\underline{r}$ in the straight and curved directions respectively.

In the following analysis the degrees of freedom, {q}, are partitioned into sets corresponding to interior, left, right, bottom, top and the four corners, that is

$$\{q\} = \lfloor\ q_I\ q_L\ q_R\ q_B\ q_T\ q_{LB}\ q_{RB}\ q_{LT}\ q_{RT} \rfloor^T \tag{5}$$

The equivalent nodal forces are also partitioned in a similar manner.

RESPONSE OF A SINGLY CURVED, ORTHOGONALLY STIFFENED PANEL TO A RANDOM PRESSURE FIELD

As the pressure wave (4) propagates it will induce a wave motion in the structure which propagates in the same direction and with the same wave form as the pressure wave. Thus, both the forces due to the pressure field and the structural displacements will satisfy the following relationships

$$\{\phi_R\} = e^{i\varepsilon_1}\{\phi_L\} \quad , \quad \{\phi_T\} = e^{i\varepsilon_2}\{\phi_B\} \tag{6}$$

where $\phi$ represents either $f^e$ or $q$, $\varepsilon_1 = -k_1\ell_1$ and $\varepsilon_2 = -k_2\ell_2$ where $\ell_1$ and $\ell_2$ are the lengths of the substructure in the straight and curved directions. Considering equilibrium between adjacent substructures and using the relations (6), the equation of motion (3) reduces to:

$$\left[ (1 + i\eta) \left[\overline{K}(\varepsilon_1,\varepsilon_2)\right] - \omega^2 \left[\overline{M}(\varepsilon_1,\varepsilon_2)\right] \right] \{q\} = \{\overline{F}\}^e \tag{7}$$

where

$$\left[ \overline{K}(\varepsilon_1,\varepsilon_2) \right] = \left[W^1\right] \left[K\right] \left[W\right] \tag{8}$$

and

$$\left[ \overline{M}(\varepsilon_1,\varepsilon_2) \right] = \left[W^1\right] \left[M\right] \left[W\right]$$

Both $[W^1]$ and $[W]$ are functions of $\varepsilon_1$ and $\varepsilon_2$. The displacement and force vectors are defined by

$$\{\overline{q}\} = \lfloor q_I \quad q_L \quad q_B \quad q_{LB} \rfloor^T \tag{9}$$

and

$$\{\overline{F}\}^e = \lfloor F_I \;\; 2F_L \;\; 2F_B \;\; 4F_{LB} \rfloor^T \tag{10}$$

These forces are due to the external pressure field only, since the forces due to adjacent substructures have been eliminated. Finally, the response of the structure to the harmonic pressure field is given by the solution of equation (7). This gives the wave receptance function for the node points referred to in (9).

## 4. Free wave propagation

The physical interpretation of the response of a periodic structure to a random pressure field is aided by considering the propagation of free waves through the structure.

It can be shown [3] that the equations of motion for free wave propagation in an undamped structure is

$$\left[ \left[ \overline{K}(\mu_x,\mu_y) \right] - \omega^2 \left[ \overline{M}(\mu_x,\mu_y) \right] \right] \{q\} = 0 \tag{11}$$

where $[\overline{K}(\mu_x,\mu_y)]$ and $[\overline{M}(\mu_x,\mu_y)]$ are defined by (8) with $\mu_x,\mu_y$ replacing $\varepsilon_1,\varepsilon_2$. $\mu_x,\mu_y$ are propagation constants in the x, y directions. These constants are in general, complex quantities which are functions of the frequency $\omega$.

RESPONSE OF A SINGLY CURVED, ORTHOGONALLY STIFFENED PANEL TO A RANDOM PRESSURE FIELD

Whenever $\mu_x / \mu_y$ is a rational number (that is, $\mu_x = n_1\mu$, $\mu_y = n_2\mu$ where $n_1$ and $n_2$ are integers), equation (11) can be reformulated as an eigenvalue problem for $\mu$ corresponding to a given value of $\omega$. The solutions to this eigenvalue problem show that free waves can only propagate without attenuation in certain frequency bands. Outside these bands waves cannot propagate and the motion is attenuated. Knowing this fact, the variation of $\omega$ with $\mu_x$, $\mu_y$ within the propagation bands can be obtained by solving equation (11) directly.

## 5. Experimental investigation

The model was rubber mounted at its four corners onto a vertical, rigid frame which was placed in front of a bank of four 100 watt loudspeakers. The testing was carried out at near grazing incidence. Baffles were used to reduce the intensity of sound reaching the rear side of the model. The exciting pressures were measured by a microphone which was positioned 5.08 cm away from the centre of the panel. The response of the model was measured by means of strain gauges situated on a cluster of three panels at the centre.

## 6. Results

The measured power spectral density of the pressure field indicates significant excitation in the range 100-600 Hz. Both measurement and calculation show that the major response at the centre of a panel occurs in the range 300-400 Hz.

## 7. References


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4. S.P.LIM 1981 (In preparation) Finite element study of acousto-structural interaction.