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DESIGN OF LOW NOISE MICRO GEOMETRIES FOR HELICAL GEARS ON THE BASIS OF TRANSMISSION ERROR UNDER LOAD

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Gearbox design must satisfy multiple demands that usually involve high power density, low energy consumption and favourable noise emission. Where the layout of loaded gears is concerned, the objectives comprise a high load carrying capacity, low gear losses and low noise mesh design. This has become more important of late since for example engine-generated noise levels that used to mask gear noise are reduced by increasingly electrified drive trains. In order to transmit high loads within the smallest possible space the tooth micro geometry must be modified from pure involute form using so-called standard modifications such as crownings or tip relieves to improve the pressure distribution on the flanks. However, these modifications do not generally grant favourable noise behaviour – it is more often found that these flank forms act to the detriment of the mesh excitation and lead to a conflict of objectives which makes compromises necessary. In this paper a new approach beyond the use of standard modifications is presented that relies on a detailed calculation of the transmission error under load (TE), which can be considered as a measurement of the parametric excitation source. The derived flank micro geometries compensate directly for the TE thus making it possible to ensure the load carrying capacity in a first step and then design a theoretically "optimal" low noise modification in a second subsequent step. Test results with ground specimens on a dynamics test rig show that both TE and acceleration levels are drastically reduced and lead to a superior noise and operating behaviour compared to the also investigated standard modifications. A few challenges associated with this kind of modification are mentioned such as complex manufacturability or compensation for higher mesh orders.

Keywords: Flank Modifications, Mesh Excitation, Transmission Error, Tooth Contact Analysis, Gear Noise Measurements

1. Conflict of Objectives in Designing Gear Flank Modifications

Gearboxes in modern drive engineering are not only reaching their optimisation limits in terms of the best utilisation of the available space, but also regarding efficiency and – of growing importance – noise emission. This also holds for the components. Recently the noise excitation caused by loaded gears is increasingly the focus of research and development because the engine generated noise levels e.g. in automotive applications are decreasing due to the electrification of the drive train, advanced structural damping, etc. Gears with hardened flanks provide in general superb load transmission with high level of efficiencies greater then $99.x\,\%$ of the input power. However, they always will induce vibrations due to elastic deformations of the teeth under load including premature and prolonged meshing, geometric deviations of the ideal flank form, pitch errors, alignment errors and to a lesser extent due to surface roughness and reversal of the friction force at the working pitch diameter.

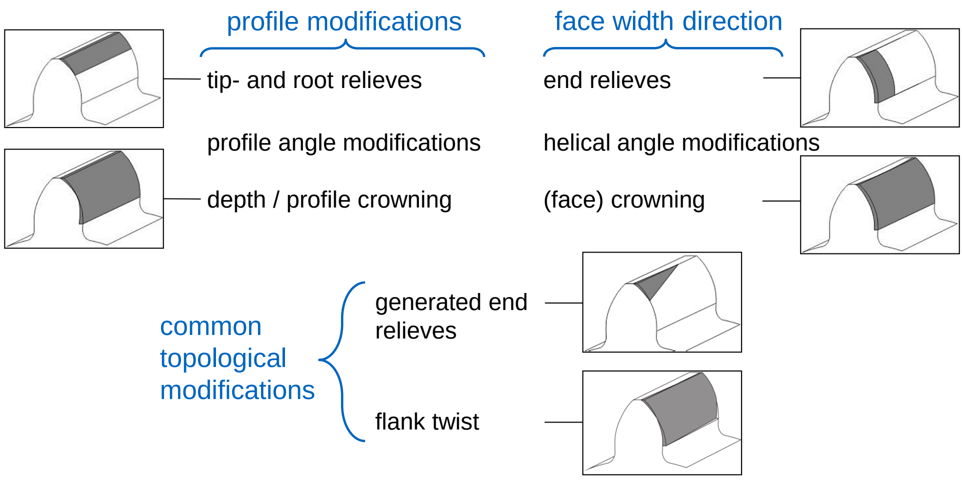


Figure 1: Classification of modifications

This parametric excitation is normally optimised by applying flank modifications. Since the involute only provides ideal transmission when the mentioned deviations are absent, it is common sense to design flank micro geometries that differ slightly from the perfect involute. This is where the conflict of objectives starts. One flank micro geometry must simultaneously satisfy the objectives of high load carrying capacity and low noise meshing while it must be also designed with respect to manufacturability. Flank forms that are focused on optimisation of one these objectives normally result in a specific shape, e.g. a shape that leads to a smooth pressure distribution in the meshing plane and thus grants a high load carrying capacity, whereas such modifications are usually not optimal in respect of low noise excitation over a desired load range. This is a consequence of the optimisation towards different physical values such as

- the local Hertzian contact pressure p_{H} and
- the transmission error under load (TE) x_{e}

with one measure i.e. by using a distinct combination of standard modifications. When standard modifications such as tip relieves or crownings as classified in fig. 1 are used to optimise these objectives simultaneously, compromises in respect of noise excitation or the constructed size of the gearbox relative to the load have to be made. This is where topological modifications offer new design possibilities by separating the optimisation steps into the layout of modifications for a high load carrying capacity at nominal load and into pitch periodic modifications for optimal excitation behaviour at a desired load as it will be shown in this paper.

2. Design of Micro Geometries for Low Noise Excitation

The general steps to derive sound flank micro geometries for loaded gears forms part of a larger design process, including positioning and dimensioning of shafts, bearings, design and elasticity analysis of the housing, etc. (cf. [1, 2]). Considering the design process for the flank micro geometry a tooth contact analysis (TCA) is performed. This is a shortened form of a detailed contact simulation, which includes for the case of the program used here, DZP [3]

- mesh stiffness calculation based on continuum mechanical theory acc. to Weber/Banaschek [4] and Schmidt [5] in respect of the actual contact pattern, resulting in a mesh stiffness c_{zi} for each discretised point,
- deflections of the shaft-bearing-housing system in every spatial direction since they influence the alignment of the flanks,
- highly discrete coverage and calculation of flank form modifications and deviations $x_{\mathrm{f}i}$

Table 1: Main geometry of gear pair for the dynamics test rig at FZG

Parameter		Unit	Pinion	Wheel
Pressure angle	α_n	$\circ$	20.00	
Helical angle	β	$\circ$	21.00	-21.00
No. teeth	z_1/z_2	–	43	45
Centre distance	a	mm	140.00	
Normal module	m_n	mm	3.00	
Face width	b_1/b_2	mm	39.50	39.50
Profile contact ratio	ε_α	–	1.50	
Axial contact ratio	ε_β	–	1.50	
Reference diameter	d	mm	138.18	144.61
Base diameter	d_b	mm	128.74	134.73
Tip diameter	d_a	mm	142.30	148.50

just to name the main components. The transmission error under load (TE) is calculated as

$$x_\mathrm{e}(\varphi) = \frac{F - \sum\limits_i \left[c_{\mathrm{z}i}(\varphi) \cdot x_{\mathrm{f}i}(\varphi)\right]}{\sum\limits_i c_{\mathrm{z}i}(\varphi)} \tag{1}$$

and represents the deviation of the constant transmission of motion evaluated along the line of action. Thus it couples the rotatory direction via

$$x(\varphi) = \varphi \cdot r_\mathrm{b} \tag{2}$$

with the corresponding base radius. It is a quasi-static parameter because there are no inertial forces included in equation (1) and hence the TE results from static formulations of the equilibrium with the load F, derived from the nominal torque. This system of equations is set up for a defined number of mesh positions, that allow the evaluation of the relevant harmonics of the TE, and is then solved. A transformation into the frequency domain

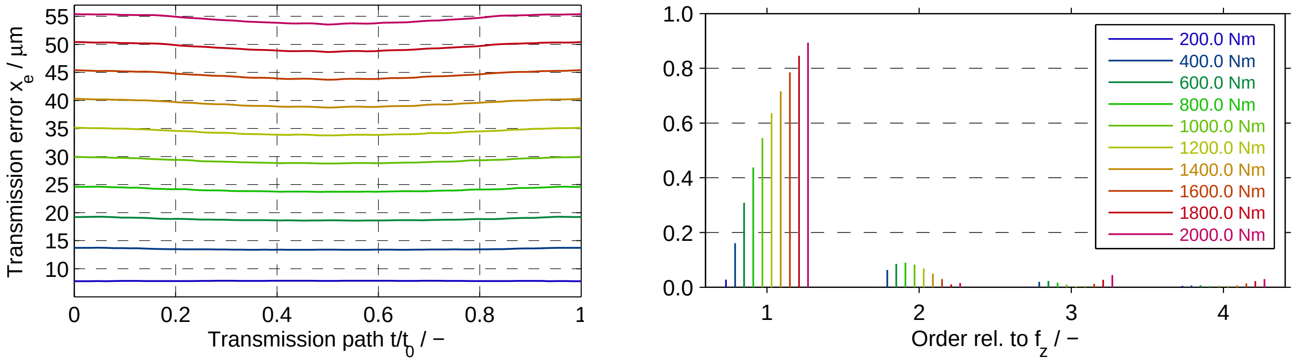
$$x_\mathrm{e}(\varphi(t)) \circ\!\!- X(\omega) = \underbrace{X_0}_{\substack{\text{constant mesh deflection}\\ \text{irrelevant to noise excitation}}} + \underbrace{\sum_{k=1}^{N_{FGL}} X_k \cdot \sin\left(\omega_k\, t + \phi_k\right)}_{\substack{\text{harmonic dissection of}\\ \text{time-variant deflection}}} \tag{3}$$

with the i-th mesh frequency at a shaft (revolutions per minute n) and wheel (number of teeth z) with index j of

$$\omega_{\mathrm{z},i} = 2\,\pi\, i\, \frac{n_j}{60\,\mathrm{s/min}}\, z_j \tag{4}$$

provides the spectral TE which is well suited to determining the excitation-wise critical inconstant portions of the absolute TE with correlation to the mesh-specific orders. After solving equation (1) both the local mesh stiffnesses and the resulting contact deformation are known. This provides the calculation of the load distribution and therefore of the Hertzian contact pressure distribution – which is a key value for balancing the flank load carrying capacity.

In the following sections, flank modifications and results of experimental investigations are shown for test gears that are adapted to meet the specifications of the dynamics test rig at the Gear Research Centre (FZG). The most important gear data are listed in table 1. For this mesh a model of the dynamics test rig (see section 3) is set up and the load distribution is calculated, analysed and then tweaked with the design of a face crowning of 8 microns, which compensates for the slight misalignment in



(a) One mesh period, time domain

(b) Frequency domain

Figure 2: TE for mesh acc. to table 1 with face crowning, no optimisation of mesh excitation

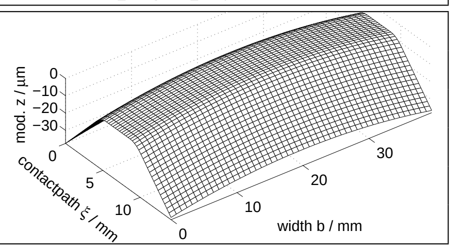
the mesh due to elastic deformations. Using the same model and crowning, the TE is calculated for a variety of loads from 200 Nm to 2000 Nm at the pinion. This range covers partial loading as well as the nominal load of the mesh. From the graphs in fig. 2 it can be seen that the TE is dominated by the first mesh order for loads bigger than 200 Nm.

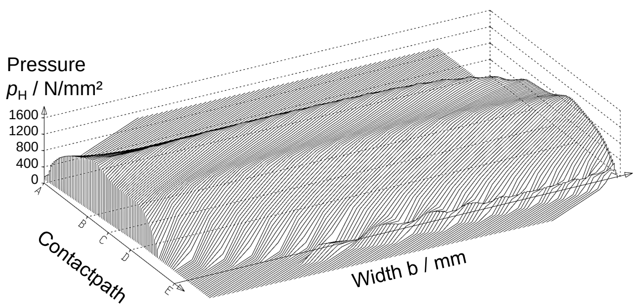
In order to optimise the mesh excitation a target load range must be specified. Looking for example at automotive applications, a partial load is normally chosen for low noise mesh optimisations. Hence a load of 1200 Nm is an appropriate choice for an "excitation load" T_{E} of the test gears (see table 1) and will be focused on in this paper.

2.1 Conventional Approach with Standard Modifications

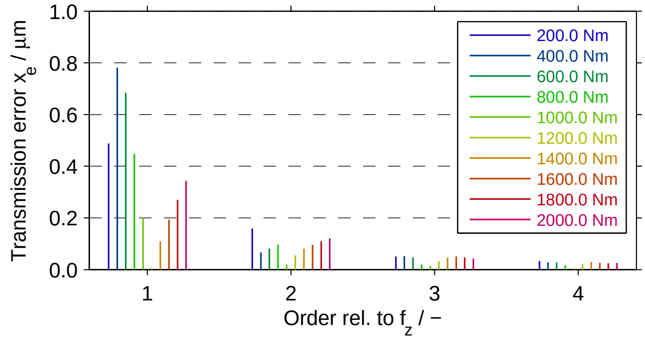
When standard modifications are used to optimise the mesh excitation some rough estimates of the parameters may be applied to obtain a sound starting point for further calculations. Normally mass calculations with a variety of different tip relieves, crownings, etc. are performed and the combination of modifications with the best specific value, e.g. TE of first mesh order, peak-to-peak TE or a similar derived value, is picked. For the case of the examined mesh different standard modifications were compared, dimensioned and four designs actually ground. One of these designs will be presented in this paper. It consists of long tip and root relieves that were designed to reduce the TE of the first mesh order. The parameters are detailed in table 2. Since the tip and root relieves modify the involute flank by almost 30 microns, the load distribution is considerably changed, as fig. 3a shows, and the effective contact ratios under load differ from the nominal values in table 1. This illustrates the major pitfall of standard modifications: Combined objectives of improved load distribution and optimised excitation behaviour cannot be reached independently and compromises must be made. However, for the examined mesh the long tip and root relieves are suited to reducing the TE under load effectively at the chosen target load $\mathrm{T_E}$ of 1200 Nm as fig. 3b shows. Higher mesh orders cannot be optimised separately. Also the amount of modification in order to improve the noise excitation depends heavily

Table 2: Parameters of modification with long tip and root relieves

	Modification	Parameter		Value	Mesh topography
Wheel	Long linear tip relief	Amount	C_{A}	29.5 µm	
		Length	L_{CA}	$33.6\,\%\,g_\alpha$	
	Long linear root relief	Amount	C_{F}	28.2 µm	
		Length	L_{CF}	$33.7\,\%\,g_\alpha$	
Pinion	Crowning	Amount	C_{B}	8 µm	
		Mid	$b/2$	19.75 mm	



(a) Pressure distribution



(b) Spectral TE

Figure 3: Results of TCA for mesh (table 1) with long tip and root relieves, crowning (table 2)

on the main geometry of the mesh [2, 6, 7, 8], first and foremost as the effective length of contact changes.

2.2 New Approach with Pitch Periodic Modifications

A different approach making use of target-specific modification measures is direct compensation for the TE. The resulting flank forms for helical gears are called waveform-modifications [7]. The basic idea is to

1. design modifications for a favourable load distribution
2. perform a tooth contact analysis and derive the spectral TE
3. adapt waveform-modification to the TE of a specific load, e.g. a sine-wave defined as

$$z(\xi, b) = \sum_{i=1}^{N_{\mathrm{wav}}} C_{\mathrm{wav},i} \cdot \sin\left(\frac{2\,\pi}{\lambda_{\mathrm{wav},i}} \cdot \xi + \phi\left(\phi_{\mathrm{wav},i}, \beta_{\mathrm{b}}, b\right)\right) \tag{5}$$

with

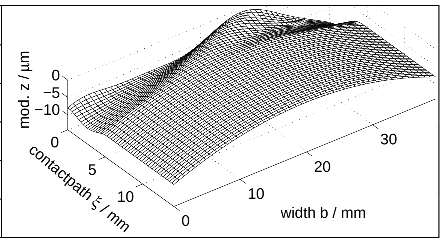
ξ . . . coordinate along path of contact	C_{wav} . . . modification amplitude
b . . . coordinate along face width direction	λ_{wav} . . . modification wavelength
i . . . specific mesh order	ϕ_{wav} . . . modification phase offset
β_{b} . . . base helix angle	

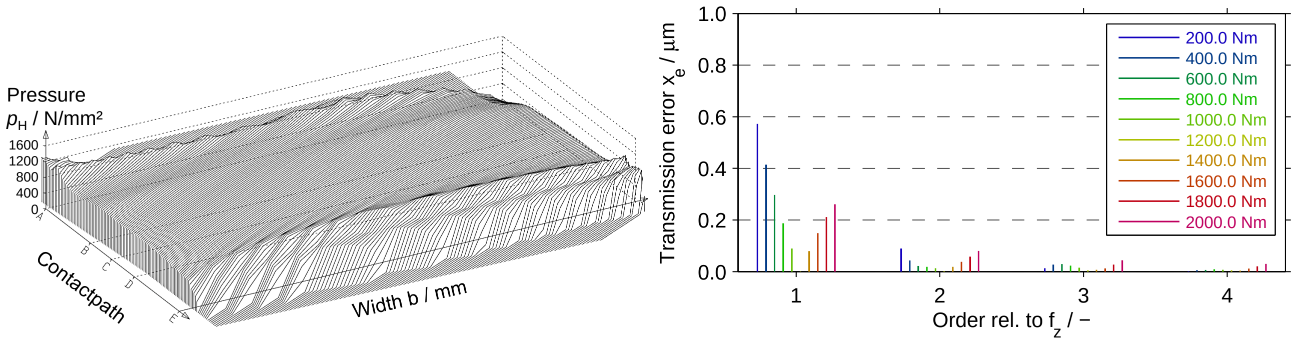
4. recalculate spectral TE and iterate using the adapted modification amplitudes $C'_{\mathrm{wav},i} = C_{\mathrm{wav},i} + \Delta X(\omega_{\mathrm{z},i})$ until the TE of the desired order is below a critical value $X(\omega_{\mathrm{z},i})_{\mathrm{lim}}$
5. check that the load distribution is still favourable which is only necessary if no pure waveform-modifications are applied

This approach is straight forward in respect of the specific optimisation objectives. The waveform modification compensates directly for the TE at the excitation-optimised load T_{E} without altering the load distribution for helical gears as long as pure waveform modifications (cf. [7]) and fully loaded flanks are considered. Thus, the disadvantageous combined optimisation towards load carrying capacity and low noise mesh excitation is avoided. The only drawback is the manufacturability of pure waveform-modifications because the necessary amplitudes to compensate for the TE are normally – e.g. referring again to automotive applications – are very small. As fig. 2b shows, the amplitude to compensate for the TE of first mesh order is $0.65\,\mu\mathrm{m}$ for the investigated mesh. The total height of the waveform modification that must be ground is twice the amplitude (=$1.3\,\mu\mathrm{m}$) which still raises the question of reasonable manufacturability.

In recent research projects funded by FVA/AiF [9] serious effort was put into the development of waveform-like modifications that grant bigger modification amplitudes that can be technically realised in a grinding process while the theoretical advantages of separate design steps and optimal mesh excitation at desired design loads are retained. One possible flank form that is utilised in order to increase the local amount of modification and which still yields a favourable excitation behaviour consists of a partial waveform combined with a (very small) tip relief, called *periodic tip relief*. Table 3 illustrates the resulting parameters and mesh topography in conjunction with a crowned pinion. Compared to the pure waveform-modification the periodic tip relief requires amplitudes at the used tip diameter d_{Na} that are increased by a factor of 5 for the compensation of the first mesh order. This translates to modifications of nearly 7 microns, which lies within a realistic scope for a grinding process.

Table 3: Parameters of periodic tip relief for crowned mating gear

	Modification	Parameter		Value	Mesh topography
Wheel	Linear tip relief	Amount	C_{A}	$3.8\,\mu\mathrm{m}$	
		Length	L_{CA}	$33.5\,\%\,g_{\alpha}$	
	Periodic tip relief	Amplitude 1st order	$C_{\mathrm{wav},1}$	$3.3\,\mu\mathrm{m}$	
		Amplitude 2nd order	$C_{\mathrm{wav},2}$	$0.7\,\mu\mathrm{m}$	
Pinion	Crowning	Amount	C_{B}	$8\,\mu\mathrm{m}$	
		Mid	$b/2$	$19.75\,\mathrm{mm}$	



(a) Load distribution

(b) Spectral TE

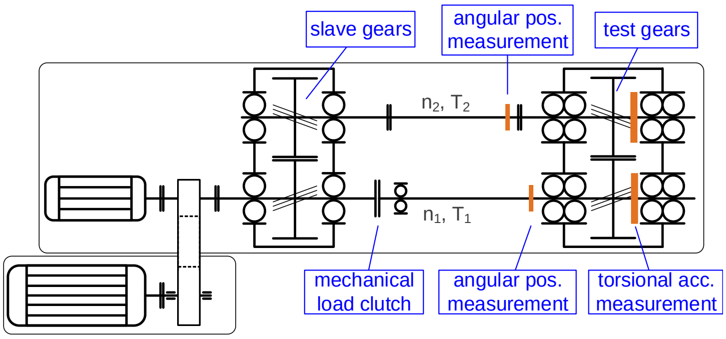
Figure 4: Results of TCA for mesh (table 1) with periodic tip relief and crowning (table 3)

3. Experimental Investigations

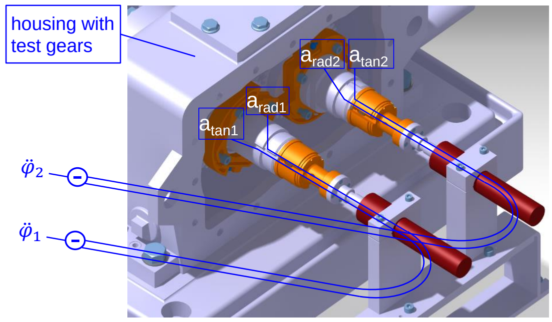
Different standard and pitch periodic modifications were described in detail and manufactured in a continuous generating grinding process by Reishauer AG (Wallisellen, CH). Amongst these specimens were also the modifications presented in this paper. After a tactile topographical measurement the wheels were mounted on the dynamics test rig at FZG as shown in fig. 5 and examined in respect of

- TE at different loads in low speed conditions (n_1 = 60 rpm),
- structure-borne sound at the bearing covers and housing in run-up measurements (n_1 from 500 rpm to 5000 rpm)
- torsional acceleration at both rotating test wheels in run-up measurements (n_1 from 500 rpm to 5000 rpm)

The investigations were each performed at 7 discrete load stages, namely 400 Nm, 700 Nm, 1000 Nm, 1200 Nm, 1400 Nm, 1600 Nm and 2000 Nm at the pinion. Since the test rig comprises a back-to-back layout, the engine only has to feed in the power losses while the mesh can be investigated under its nominal load. The evaluation of the recorded acceleration data is performed with software developed at FZG in recent research projects [10]. On top of the calculated Campbell-diagrams there is an implementation of the applied force level



(a) Schematic



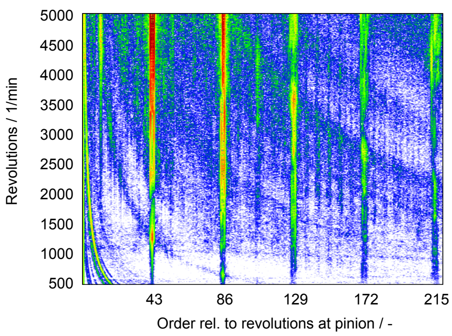
(b) Measurement of torsional acceleration

Figure 5: Dynamics test rig at FZG

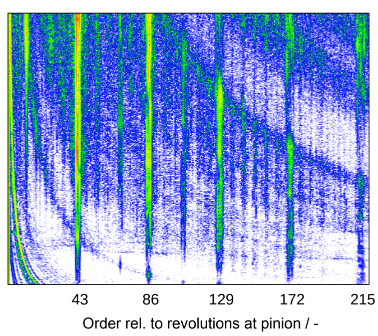
$$L_{\mathrm{A,F}} = 10 \cdot \log\left(\frac{1}{F_{\mathrm{ref}}^2}\sum_{ord_1}^{ord_\mathrm{u}} \overline{F}_{ord}^2\right) \quad , \overline{F}_{ord} = \frac{\int_{n_{\mathrm{l}(ord)}}^{n_{\mathrm{u}(ord)}} F_{ord}\,(n)\;\mathrm{d}n}{n_\mathrm{u} - n_\mathrm{l}} \tag{6}$$

within adjustable lower and upper boundaries in respect of the revolutions per minute and the specified (mesh) orders.

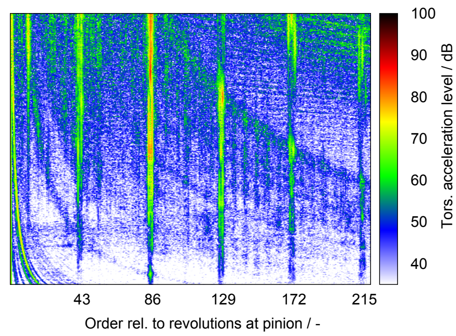
As fig. 6 shows, the torsional acceleration level of the mesh with long tip and root relieves (cf. fig. 6e) is reduced by 8 dB for the first mesh order and 10 dB for the second mesh order in comparison with the mesh that is not optimised in terms of noise excitation (fig. 6d). The mesh with periodic tip relief shows considerable reductions in the torsional acceleration levels for the first mesh order (-18dB) and some improvement also for the second mesh order (-4 dB). In general these results correlate well with the TE measurements [9].



(a) Crowning only – Campbell diagram



(b) Long tip/root relieves + crowning – Campbell diagram



(c) Periodic tip relief + crowning – Campbell diagram

(d) Crowning only – averaged Campbell diagram

(e) Long tip/root relieves + crowning – averaged Campbell diagram

(f) Periodic tip relief + crowning – averaged Campbell diagram

Figure 6: Typical results of torsional acceleration measurements at pinion; specimens were manufactured in a continuous generating grinding process by Reishauer AG

4. Conclusion and Outlook

Optimisation of the micro geometry for loaded gears must balance the conflicting requirements in respect of load carrying capacity, noise excitation and manufacturing. It was shown that the "conventional" approach, where standard modifications are used, may lead to compromises because both the pressure distribution and the transmission error under load (TE) depend on the micro geometry, i.e. on the chosen combination of modifications. The derivation of pitch-periodic waveform modifications offers new design possibilities for helical gears because it separates the design steps. A favourable and distinct optimisation of the mesh excitation is performed after the load capacity objectives are met – without changing the pressure distribution. The downside of this kind of modification, namely the poor manufacturability due to very small amplitudes, can be avoided by new specific flank forms such as periodic tip relief, which was selected for this paper and explained in detail. This flank form theoretically allows full compensation for the TE of specific mesh orders at a desired noise-optimal load that can differ from the load focused on when the load carrying capacity is considered. The amplitudes of the waveform function are increased by a factor of 5 for the first mesh order compared to a pure waveform modification for the illustrated mesh. Experimental investigations have been carried out on the dynamics test rig at FZG with specimens manufactured by Reishauer AG in a continuous generating grinding process. A broad load range could be tested that covers partial loads as well as the nominal load of the shown mesh. Typical results of torsional acceleration levels of a mesh without excitation optimisations, a mesh with standard modifications and a mesh with periodic tip relief adapted to a crowned mating gear are illustrated in this paper. The latter drastically reduced (-18 dB) the acceleration levels of the first mesh order and showed improvements for the second mesh order as well. In comparison to the mesh with equally precisely manufactured standard modifications that is also shown, the periodic tip relief led to improvements of -10 dB in respect of torsional acceleration levels of first mesh order. Further efforts with these kinds of modification could be made in respect of the optimisation of higher order mesh excitations as well as the investigation of the transferability to meshes with different macro geometries.

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