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COMPUTATIONALLY EFFICIENT MODEL FOR CALCULATING THE 2.5D GREEN'S FUNCTIONS OF A TUNNEL BURIED IN A LAYERED HALF-SPACE SYSTEM

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Noise and vibration pollution caused by underground railway traffic in urban areas become a major issue of concern due to their effects on the quality of life and comfort of the inhabitants. An accurate and computationally efficient model is required to predict the ground-borne vibration and to evaluate performance of countermeasures before their implementation. In this paper, a computationally efficient model to calculate vibration levels induced by underground railways buried in a layered half-space is presented. It is assumed that near-field dynamic behavior of the tunnel-soil system is only influenced by the dynamics of the tunnel itself and the layer that contains it. Thus, it is assumed the other layers and the free surface do not affect the dynamic behavior of the tunnel-soil system close to the tunnel. The two-and-a-half-dimensional (2.5D) tunnel-soil interface vibration displacements are calculated by using the Pipe-in-Pipe (PiP) model in which tunnel wall and surrounding soil are modeled using thin shell theory and elastic continuum theory, respectively. Then, the 2.5D Green's functions for a homogeneous full-space are employed to find a set of equivalent forces that can reproduce the tunnel-soil interface displacements. Finally, the far-field vibration displacements of the layered half-space are calculated by employing computationally efficient method which calculates the 2.5D Green's functions for a layered half-space based on the stiffness matrix method in Cartesian coordinates. The results are compared with the ones calculated by using a well-established model proposed by Hussein et al. (JSV, Vol. 333 (25), pp. 6996-7018, 2014) in which the stiffness matrix method in cylindrical coordinates has been used to calculate the 2.5D Green's function for a layered half-space. It is shown that not only the results of two methods are in a good agreement but also the new method is computationally more efficient in comparison with the existing one.

Keywords: 2.5D Green's functions, stiffness matrix method, layered half-space

1. Introduction

Underground railway-induced vibration must play an important role in the construction planning of new railway lines in order to avoid non-compliance of noise and vibration laws or regulations.

Thus, computational models for predicting the levels of vibration induced by new railway infrastructures in the nearby residential buildings and evaluating the efficiency of vibration countermeasures are of significant interest [1]. Numerous models have been proposed for this aim, and they can be categorized as numerical, empirical and analytical models. Regarding to the frequency range of interest, experimental measures showed that the dominant frequencies for perceptible vibration generated by the passage of underground trains is 10-100 Hz [2].

Three-dimensional (3D) numerical models, such as Finite Element-Boundary Element (FE-BE) models, are powerful tools to estimate vibration levels induced by railway infrastructures with high accuracy despite structural complexity. However, these models demand large computational costs. Their computational efficiency can be improved by assuming the geometry of the system to be periodic or invariant in the longitudinal direction of the tunnel. The assumption of a periodical structure is normally addressed by employing the Floquet transform [3, 4] and a 2.5D approach is normally applied to take advantage of the invariancy assumption. [5, 6].

Empirical prediction models present easy, cheap and fast ways to predict underground railways vibration. They are based on a large set of experimental measurements and some simple empirical means to interpolate or extrapolate the vibration levels in any desired receiver [7, 8]. Accordingly, these models can only be used for sites very similar to the existing ones on the experimental measurements database of the empirical model. Hybrid modeling can be used to overcome the limitation of those models, by combining numerical and empirical methodologies. [9].

Semi-analytical approaches require far less computational resources than numerical models and they have more flexibility in comparison with the empirical models. One of the most well-established semi-analytical models is the Pipe-in-Pipe (PiP), which was developed by Forrest and Hunt [10, 11]. It is a 2.5D semi-analytical model in which the tunnel and the surrounding soil are addressed using thin shell theory and elastic continuum theory, respectively. A comparison between the results obtained using the PiP model and 2.5D FE-BE model was presented in the work of Gupta et al. [12]. The results show a good agreement between the responses obtained by both models and they also show a remarkable decrease in computational time with using the PiP model. A methodology that uses the PiP model to compute the ground-borne vibration induced by a railway tunnel buried in a homogeneous half-space was developed by Hussein et al. [13]. Later, this methodology has been extended for a tunnel embedded in a layered half-space [14] and it has been complemented and named as the fictitious force method.

In this paper, the application of a new method for the computation of the 2.5D Green's functions of a layered half-space to the fictitious force method is studied in terms of its accuracy and computational efficiency. The paper is organized as follows. In the next section, the fictitious force method is described briefly. Then, in Section 3, the new and common methods for obtaining the 2.5D Green's functions of a layered half-space are explained, and the computational efficiency of the new method are pointed out. Finally, in Section 4, the 2.5D Green's functions obtained using the new method are applied within the fictitious force method to calculate 2.5D Green's functions of a tunnel buried in a layered half-space.

2. The fictitious force method

In the following, the fictitious force method is explained briefly; A detailed overview of the methodology can be found in [15]. Consider a tunnel embedded in a layered half-space. The near-field dynamic behavior of a tunnel-soil system can be assumed to be the one associated to a tunnel embedded in a full-space with the mechanical parameters of the layer that contains the tunnel in the original model. Besides, the model is defined in the basis of a 2.5D approach, which means that the system is assumed to be invariant along the longitudinal direction, therefore, the 3D problem can be decomposed into a set of two-dimensional (2D) models which depend on the wavenumber along the invariant direction. The 2.5D Green's functions of a tunnel buried in a layered half-space related to a force applied on the tunnel invert can be computed by employing the fictitious force method, which consists of the three following steps:

1. **Calculating tunnel-soil interface displacements**: The tunnel-soil interface displacements due to a point load on the tunnel invert is calculated using the PiP model [10]. It is assumed that the tunnel is embedded in a homogeneous full-space with the mechanical parameters of the layer that contains the tunnel.
2. **Calculating equivalent forces**: A set of equivalent forces which are able to reproduce the tunnel-soil interface displacements in a full-space (without the embedded tunnel) are determined. 2.5D Green's functions for a homogeneous full-space, developed by Tadeu and Kausel [16], are employed.
3. **Calculating responses at receiver-points due to the equivalent forces**: The response at the receiver-points can be computed through multiplying the equivalent forces by the required 2.5D Green's functions of the layered half-space. In the fictitious force method, these 2.5D Green's functions of the half-space are evaluated by means of the stiffness matrix method in cylindrical coordinates [17, 18].

The third step is computationally more expensive than the two first steps. The computational costs of the third step can be decreased with employing an alternative method to calculate 2.5D Green's functions for a layered half-space, which is explained in the next section.

3. 2.5D Green's functions for a layered half-space

In this section, 2.5D Green's functions for a layered half-space evaluated through the stiffness matrix method in cylindrical and Cartesian coordinates are briefly explained and compared.

The 3D Green's functions for a layered half-space calculated by means of the stiffness matrix method in cylindrical coordinates [17] is the most common method used in the literature ([15, 14, 6]) as a basis to obtain the 2.5D Green's functions of the system. In this method, first, the global stiffness matrix of the layered half-space in the frequency-radial wavenumber domain (ω, k_r) is obtained from the assembly of elemental stiffness matrices of layers and homogeneous half-space of the system, in a similar way to the Finite Element (FE) in the finite element method. Afterwards, the Green's functions can be obtained in the (ω, r, θ) domain by applying an inverse Hankel transformation and Fourier series expansion in the radial and circumferential directions, respectively. Taking into account the relation between cylindrical and Cartesian coordinates, the Green's functions can be found in the (ω, x, y) domain. Finally, to reach the domain of the 2.5D Green's functions, which is (k_x, y, ω) if x the direction of invariancy, one can apply a Fourier transform along x.

Recently, explicit expressions of the 3D stiffness matrices for a layered media in Cartesian coordinates have been developed by Arcos et al. [19]. These 3D stiffness matrices have been employed by the authors to develop computationally more efficient method to calculate 2.5D Green's functions for a layered half-space [20]. To obtain the 2.5D Green's functions for a layered half-space with an arbitrary distribution of receiver-points and sources using the new methodology, the next steps must be followed:

1. Adding virtual interfaces for each source or receiver-point not placed on any of the original layer interfaces.
2. Computing the element stiffness matrices of layers and homogeneous half-space in the (ω, k_x, k_y) domain using the recently developed explicit expressions in Cartesian coordinates.
3. Assembling global stiffness matrix.
4. Inverting the global stiffness matrix to compute the Green's functions in (k_x, k_y, ω) domain.
5. Applying an inverse Fourier transform in the y direction to get the 2.5D Green's functions.

As shown, the proposed methodology requires fewer numerical operations to compute the 2.5D Green's functions in (ω, k_x, y) domain in comparison with the commonly used method which needs an inverse Hankel transform and a Fourier transform to do the same task. Therefore, the computational efficiency of the fictitious force method can be improved by employing the present method in its third calculation step.

4. Results and discussion

In this section, an algorithm that computes the 2.5D Green's functions of a layered half-space based on the new presented method is used in the third step of the fictitious force method to calculate far-field displacements of a tunnel buried in a layered half-space. After this, the results are compared with the ones obtained through the methodology that uses the stiffness matrix method in cylindrical coordinates to obtain the required 2.5D Green's functions. Consider Fig. 1, in which a tunnel with outer radius and thickness of 3 m and 0.25 m, respectively, buried in a layered half-space is presented. The mechanical parameters of the tunnel and the soil are given in Table 1. Noteworthy, complex-valued Lamé parameters are used in order to replace the elastic model with viscoelastic one; D_p and D_s are hysteretic damping ratios for P- and S-waves.

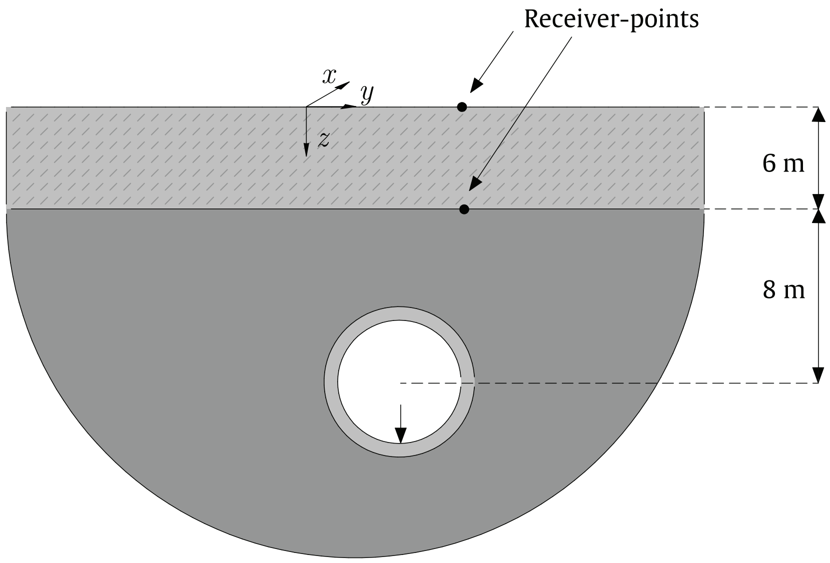

Figure 1: Tunnel embedded in a layered half-space which consists of a layer over a half-space.

Table 1: Parameters used to model the tunnel and soil.

Tunnel parameters	Values
E (GPa)	50
ν (-)	0.3
ρ (kg m^{-3})	2500
D_p (-)	0.03
D_s (-)	0.03
Tunnel depth (m)	14

Soil parameters	1$^{\text{st}}$ layer	2$^{\text{nd}}$ layer
E (MPa)	44.6	28.6
ν (-)	0.49	0.49
ρ (kg m^{-3})	1980	1980
D_p (-)	0.06	0.06
D_s (-)	0.06	0.06
Layer thickness (m)	6	Inf

A numerical example has been carried out in the basis of this system. The PiP model has been employed to calculate the displacement fields at 20 positions at tunnel-soil interface due to a load applied at the tunnel invert. The tunnel is assumed to be embedded in a full-space with mechanical parameters of the 2$^{\text{nd}}$ layer. Then, the virtual forces have been computed at $r = 1$ m at 20 positions by means of 2.5D Green's functions for the full-space [16]. Finally, the responses at the receiver-points due to the virtual forces have been computed using the 2.5D Green's functions for the layered half-space evaluated by means of the stiffness matrix method in Cartesian and cylindrical coordinates. The latter has been computed using the ElastoDynamics Toolbox (EDT) [21]. The amplitudes of the three displacement fields at the receiver-point, placed at $(y_{rp}, z_{rp}) = (10, 6)$ m, are plotted versus k_x in Fig 2-a for a frequency of 60 Hz. $|H_{ij}|$ represents the response in the i direction due to the force acting along the j direction. Their related phase values, φ_{ij}, are plotted in Fig 2-b. As it can be seen, the H_{xz} is antisymmetric but H_{yz} and H_{zz} are symmetric with respect to the k_x. The same comparisons has been provided for the receiver-point placed at $(y_{rp}, z_{rp}) = (10, 0)$ m. The results are shown in Fig 3. Comparisons at other frequencies also give the good levels of agreement.

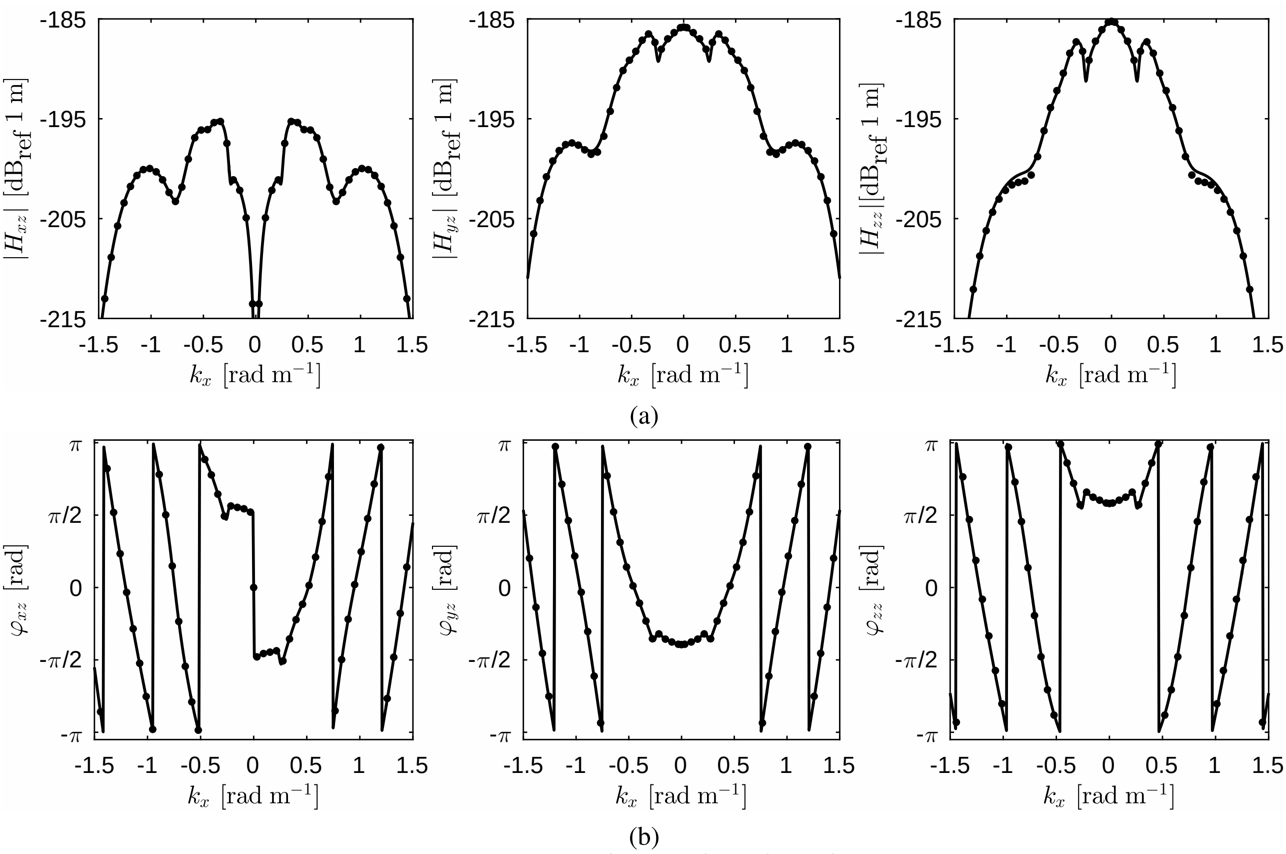

Figure 2: Amplitude (a) and phase values (b) at $(y_{rp}, z_{rp}) = (10, 6)$ at 60 Hz. Employing 2.5D Green's function for a layered half-space in Cartesian (line) and cylindrical (dotted) coordinates.

5. Conclusions

The fictitious force method is an extension of the PiP model to calculate the vibration induced by a tunnel embedded in a layered half-space. It has three main steps: calculating the tunnel-soil interface displacements, obtaining a set of virtual forces that can reproduce these displacements in a full-space environment and finally calculating the response at the desired receiver-points by multiplying the virtual forces by their associated 2.5D Green's functions in the half-space. The Green's functions for a layered half-space are commonly calculated in the (ω, k_r) domain, then numerical operations are applied on them to compute 2.5D Green's functions which are in the (ω, k_x, y) domain. In the present paper, a recently developed method for obtaining the 2.5D Green's functions for a layered half-space has been employed in order to improve the computational efficiency of the fictitious force method. In the proposed method, the stiffness matrix method in Cartesian coordinates has been employed, therefore, the Green's functions for layered half-space are calculated directly in (ω, k_x, k_y) domain. Consequently, only one inverse Fourier transform is needed to calculate the 2.5D Green's function for a layered half-space, which are in the (ω, k_x, y) domain, while the method based on the stiffness

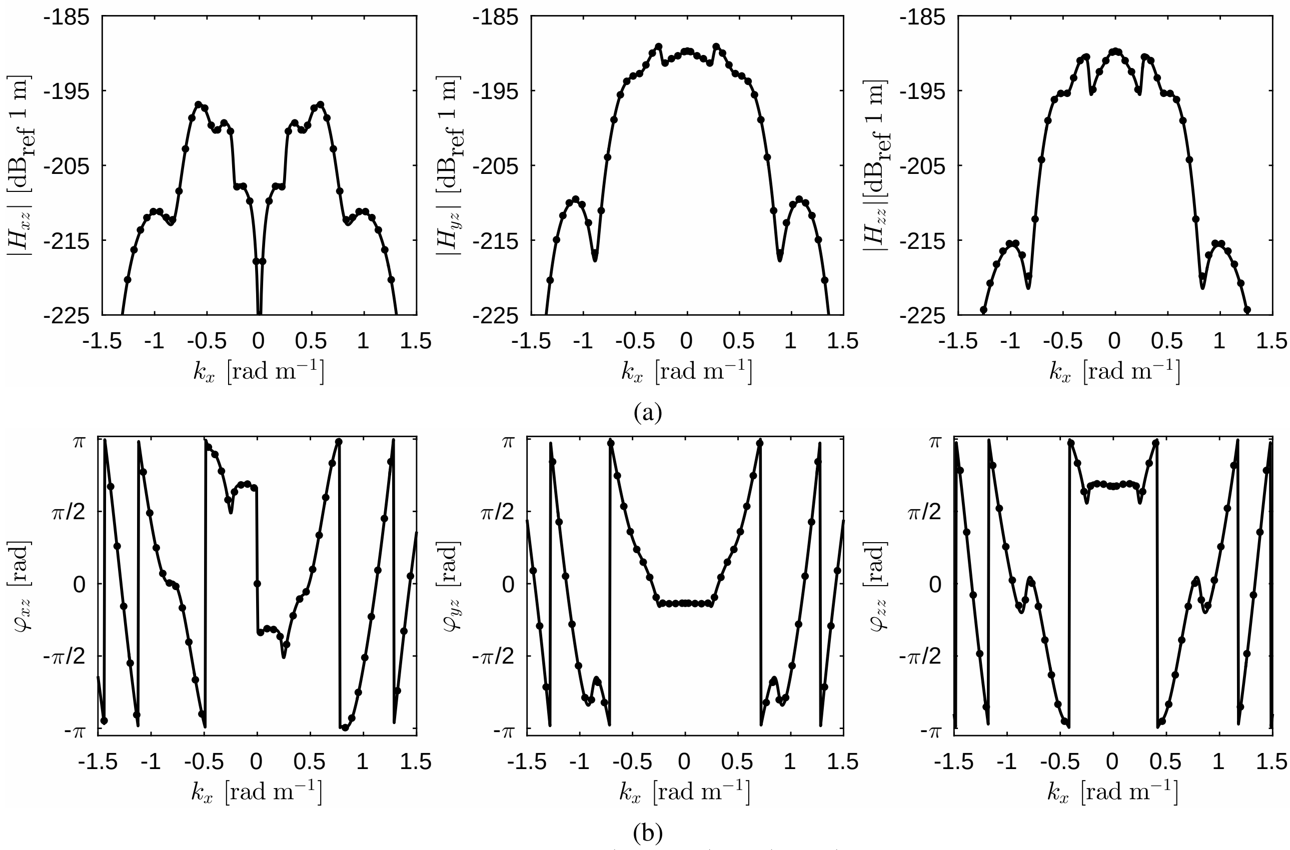

Figure 3: Amplitude (a) and phase values (b) at $(y_{rp}, z_{rp}) = (10, 0)$ at 60 Hz. Employing 2.5D Green's function for a layered half-space in Cartesian (line) and cylindrical (dotted) coordinates.

matrix method in cylindrical coordinates needs two more transformation. Thus, it is clear that in a general point of view the new methodology is more computationally efficient than the previous one because of the fewer numerical transformations. The results of applying the fictitious force method using both methodologies are compared and a very good agreement is obtained.

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