ADDITION versus MULTIPLICATION in SONAR ARRAY SIGNAL PROCESSING

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1. INTRODUCTION

When attempting the difficult task of comparing sonar array signal processors account must be taken of the application for which the sonar system is intended. A particular processor may be optimum for a certain task but could prove quite unsuitable under a different set of operating conditions. Against this background a contrasting theoretical and practical study has been made of the ways in which various combinations of target type and environmental factors can influence the performance of two types of processor. The examples selected are incorporated in the Birmingham University within pulse sector scanning sonar[1] where the choice provided is between conventional square-law detection and a multiplicative (or correlator) arrangement where the sum of one half of the array elements is multiplied by that of the other half prior to low pass filtering.

2. THEORETICAL CONSIDERATIONS

Important practical considerations in a comparison of directional systems are their ability to detect targets against an interfering background, the accuracy with which single target bearing can be estimated, the resolution achieved for closely spaced target groups and mapping the energy distribution across broad or diffuse targets. Commonly used figures of merit for array processors are output signal to noise ratio and array beamwidth for a single point source, plane wave input. Although these criteria are often sufficient guide to the optimization of a given configuration[2] they are unreliable indicators for two different configurations.

Both Welsby and Tucker[3] and Arndt[4] compared the detection performance of additive and multiplicative arrays in terms of output signal to noise ratios. Such an analysis is invalid, however, since a rigorous

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statistical treatment must take account of the full probability density functions for combinations of signal and noise at the receiver output. Using the well known Neyman-Pearson detection model where probability of detection is plotted against input signal to noise ratio for a given false alarm rate, Shaw[5] illustrates that the additive square-law processor offers marginally better performance. This same conclusion has been published by Miller[6] but his results are presented in terms of the much more informative "receiver operating characteristics" or ROC curves. Here probability of detection is plotted with false alarm rate and input signal to noise ratio as variables. The detectors are then contrasted by calculating minimum detectable signals (MDS) in the respective cases where these are the input signal to noise ratios required to ensure a given probability of detection along with a specified maximum probability of false alarm. A typical result shows the MDS for a 10 element broadside additive array to be reduced by no more than 1 dB below the value for the corresponding multiplicative array.

Miller also points out that at low input signal to noise ratios single target directional responses can become misleading if employed to compare the bearing estimation potential of additive and multiplicative arrays. The reason is that account must be taken of the efficiency with which each detector smooths the interfering noise signals. In reference 6 results gained from direct statistical analysis are given to demonstrate that even though the multiplicative array has a beamwidth about 70% that of an additive array, under certain conditions the additive array is the better bearing estimator. In this context bearing estimation is understood in terms of obtaining a minimum bearing variance from signals embedded in noise.

At high input signal to noise ratios the narrower beamwidth of the multiplicative processor does offer superior single target delineation. Moreover the first sidelobes of the multiplicative directional pattern are of negative polarity and can be removed by rectification. In this way target points are highlighted on an intensity modulated display. Unfortunately these resolution advantages do not translate directly to a situation where more than one coherent target is present in the sonar search area. Multiple target directional patterns cannot simply be made up from a superposition of the individual single target directional responses because both processors are non-linear devices. Thus unwanted cross-product terms occur in the outputs together with the wanted single target responses.

Freedman[7] shows how the magnitude of the cross-product terms in an additive processor are critical in controlling resolution of closely spaced targets. In turn it can be shown that the magnitude of the cross-products are dependent on the vector relationship between the target echoes[8]. This applies to additive and multiplicative arrays since multiplicative resolution is also dictated by signal cross-modulations. So if target amplitudes remain fixed, multiple target resolution can be decided by the value of the phase angle between the incoming acoustic signals. By then assuming that all values of phase angle are equally likely a probability of target resolution can be computed in terms of a range of phase angles which allow a given resolution criterion to be achieved. In Fig. 1 a joint probability of resolution and detection is plotted against target separation in units of $p = \pi d.\text{Sin}\theta/\lambda$ where d is the array element spacing, λ is the acoustic wavelength and θ is the angle of arrival of an incident wavefront relative to the normal to the array face. Two fixed, equal strength targets have been taken and the resolution criterion is a requirement for a 3 dB dip in the directional pattern mid-way between the two target peaks. An amplitude or detection threshold of unity corresponds to each array's mainlobe output for a single target input.

It is apparent from the results of Fig. 1 that at very low threshold settings the resolution performance of the multiplicative array is superior to that of the additive array except at small target separations where the multiplicative pattern tends to become negative at the true target positions. As the threshold increases the superiority of the correlator array decreases until at high thresholds no value of target separation yields performance superior to the additive array. Further investigation into two target resolution in reference 8 reveals that when unequal target ratios are introduced the multiplier's performance again suffers due to amplitude suppression of the smaller target peak. On the other hand the straightforward resolution advantage of the correlation processor (when amplitude is ignored) increases as more stringent resolution criteria are imposed in terms of greater target peak to null ratios.

Thus a multiplicative array can offer superior resolution to an additive array on a statistical basis when amplitudes are ignored. If relative response amplitudes are taken into account any pure resolution advantage of the multiplier is soon nullified; the target pair is often not detected on the sonar display due to the amplitude suppression characteristic of the receiver. Fig. 2 demonstrates that these findings also apply to more complex targets. The diagram shows additive and multiplicative array directional responses for two extended targets made up of ten discrete target points each separated by $\pi/32$ on the p scale. Target amplitudes were derived by independent sampling from a normal Rayliegh probability density function whereas phases were obtained from a rectangular probability distribution between limits 0 and 2π. The negative excursions of the multiplicative patterns are not shown. At the high threshold level indicated there is little difference in the data displayed by either processor. As the threshold is lowered the superior resolution of the multiplier can be seen in examples of narrower target peaks and greater separation between target peaks. Accompanying this extra fineness of detail there is also an increasing loss of displayed target data due to amplitude suppression of some targets.

3. RESULTS

It would appear from the preceding theoretical discussion that multiplicative processing offers little advantage over normal square-law additive processing. The additive array has superior target detection capability and at low signal to noise ratios it can be the better estimator of target bearing. At high signal to noise ratios the narrower beamwidth of the multiplicative array does represent greater resolving power but when used with closely spaced, coherent target groups the associated amplitude suppression characteristic is a serious drawback. However a correlator processor has been in use on a Birmingham University analogue scanning sonar since 1959 and has always proved very successful in practice. Indeed it is often preferred to the additive alternative by operators in the field. What then are the reasons for this success? The answer lies in the particular sonar design, in aspects of its normal operational environments and in the echoing properties of the targets it deals with.

The Birmingham sonar is a high resolution equipment employing a 1° scanned beam over a 30° sector and a 10° beam in the non-scanned plane; the scanning frequency is 10kHz which allows a range resolution of about 15cm. An acoustic frequency of 500kHz is used which limits the working range for most targets to between 100 to 150m. Therefore present day usage of the sonar centres around short range, high definition studies such as fish behaviour and stock assessment in shallow and confined waterways. A practical investigation into this type of usage revealed that the B-scan oscilloscope display commonly showed wanted targets against a background of heavy multipath together with direct bottom and/or surface reverberation. A further marked feature of the displays was appreciable scintillation of both wanted and unwanted targets alike.

It is the very high acoustic frequency of 500kHz that accounts for the level of signal intensity fluctuations encountered by the equipment. At this frequency formation of most target echoes can be described using the multiple scattering hypothesis where reflected signals are composed of many individual contributions each with a random amplitude and phase. If only slight changes take place in the transmitter/target/receiver geometry then appreciable alteration of this vector addition can be brought about and a noise-like signal results. These physical pertubations can occur due to movement of the transmitter/receiver, changes in target aspect or wave motion in the water surface. The following examples, involving target aspect variations only, demonstrate the echo formation process.

Single scan video output waveforms from the additive processor are given in Fig. 3. The target was a pair of small (approximately 6 in. x 4 in. x 2 in.), air-filled, rectangular metal boxes separated by an angular distance of $\pi/32$ on the p scale. For the four groups of results shown the time interval between individual frames was 5s while the period separating individual frames was 0.5s. Even though the targets were observed by divers to be moving only very slowly, the short timescale of the fluctuations emphasizes the significance of relatively small physical changes in target aspect. Using the central, vertical line as a reference, the obvious assymmetry in the variation in the directional patterns reveals that both amplitude and phase fluctuations were occurring. In fact the returns from each target appear truly randomised. This was substantiated by measuring the probability density function of the amplitude fluctuations from a single target and fitting a standard Rayleigh distribution to the results.

A presentation of results similar to Fig. 3 is used in Fig. 4. Here the target was an extended sheet of metalwork with a rough surface texture. The rectangular structure covered about 5 bearing resolution cells of the scanning sonar at a range of 20m. Weights and bouys were used to keep the target suspended in mid-water where it too underwent only very slight changes in attitude due to its considerable inertia. The video output outlines again highlight the complex nature of the echo formation process and the high sensitivity of this process to small amounts of target motion. On a single scan basis the vector addition of signal contributions across the target leads to the formation of localised areas of high intensity reflected energy. As the target moves, these highlights fluctuate in both time and space producing a signal with Gaussian statistics in each resolution cell.

Similar measured results are contained in reference 8 for fluctuations induced by transmitter/receiver motion and surface waves. What is the relevance of these signal variations to relative array processor performance? In terms of a probability of target resolution and detection, the existence of alterations in signal amplitude as well as phase introduces a new mechanism for resolution whereby targets can be displayed either together or one at a time on successive scans. That this does happen in practice is clearly seen in Fig. 3. Fig. 5 results were therefore computed to show a probability of resolution and detection versus target separation as in Fig. 1 but in this case the two target amplitudes are independently sampled from a Rayleigh probability density function. To achieve detection only one of the target peaks need exceed the given threshold while the resolution criterion is maintained as a 3 dB dip. A threshold setting of unity corresponds to the magnitude of either processor output when the input is a single target with an amplitude at the peak of the Rayleigh distribution. In contrast to the results for fixed amplitude targets, use of the multiplicative processor results in better performance from low threshold settings (0.1) through to high values (1.3) although there is a decrease in the relative gain as the threshold is raised.

Fig. 6 investigates the effect of integrating video patterns such as those in Fig. 4 for an extended target. The target was simulated on a digital computer using 12 target points, each spaced by $\pi/32$ and having an independent Rayleigh amplitude and random phase. When either 5, 10, 20 or 50 scans are integrated only the maximum amplitude arising at each angular sampling position appears in the final response. Once as few as 5 patterns are combined in this way the multiplicative and additive responses begin to appear very similar. The amplitude suppression effect in the multiplier as demonstrated in Fig. 2 is quickly reduced. This tendency increases with the number of samples included until little difference can be detected in the two final results after 50 integrations. When using the short range Birmingham scanning sonar it is unlikely that fluctuating target echoes will be completely uncorrelated from scan to scan. But the practical results indicate that video integration will be effective in producing target imaging results as just discussed within only very short, and thus operationally meaningful, time periods.

Perhaps the main advantage found when using the multiplicative array in the field involves operations such as searching for fish target movements against a high background of stationary bottom reverberation. Firstly the bottom returns must be rendered truly stationary by synchronizing the range and bearing scanning waveforms on the sonar. If this is not done then complex, fixed targets are sampled at different positions on successive scans and at high frequencies appreciable alterations in the target echo intensity can occur[9]. When the range and bearing waveforms are synchronized this effect is eliminated and a low detection threshold is used to maximize the probability of fish detection. The results of Fig. 2 now become very relevant since as the detection threshold is lowered an increasing amount of background clutter is introduced into the display. In such circumstances the multiplicative array has an important role to play as the much more effective suppressor of the unwanted background reverberation.

As a final point it should be noted that in a different context unsynchronized sonar scanning waveforms can be a distinct advantage in the functioning of a multiplicative versus an additive processor. It has been shown that the multiplier benefits in resolution performance and in target imaging ability when fluctuating signal conditions prevail. Consequently, when fixed target geometry situations exist, non-synchronization of the sonar scanning operations can be used to introduce useful amounts of target scintillation to the sonar data.

4. CONCLUSIONS

To compare array signal processors it has been shown that it is necessary to carefully consider the use to which the processor is to be put. Practical and theoretical results have been given to demonstrate that a multiplicative processor can be usefully incorporated in a high resolution within pulse sector scanning sonar. Contrary to the results of a classical theoretical investigation of additive and multiplicative arrays, the multiplicative arrangement can prove the more attractive scheme of demodulation in several commonly experienced field search situations.

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9. Practical results showing the effect of range to bearing scan synchronization can be found in reference 8.

Fig. 1 Two target resolution: Equal strength targets with amplitude threshold.

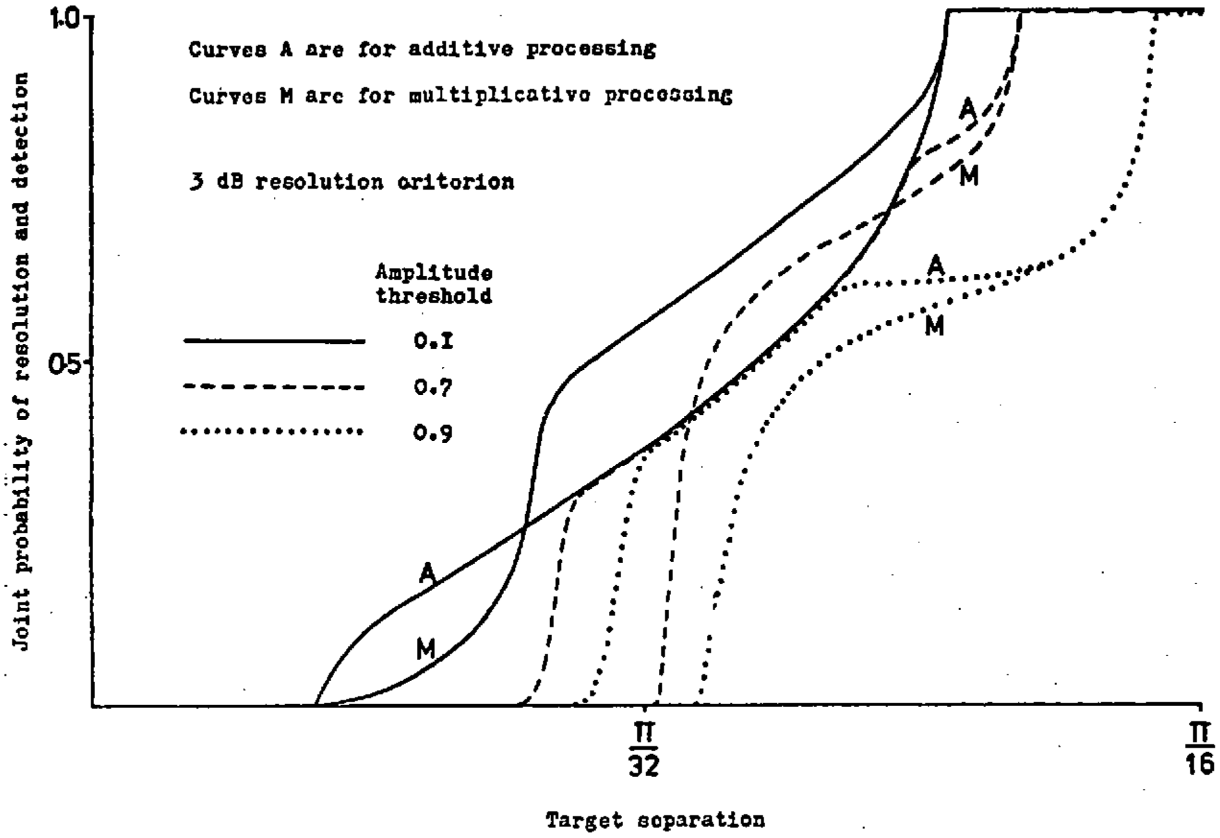

Fig. 2 Multiple target directional responses showing effect of varying amplitude threshold.

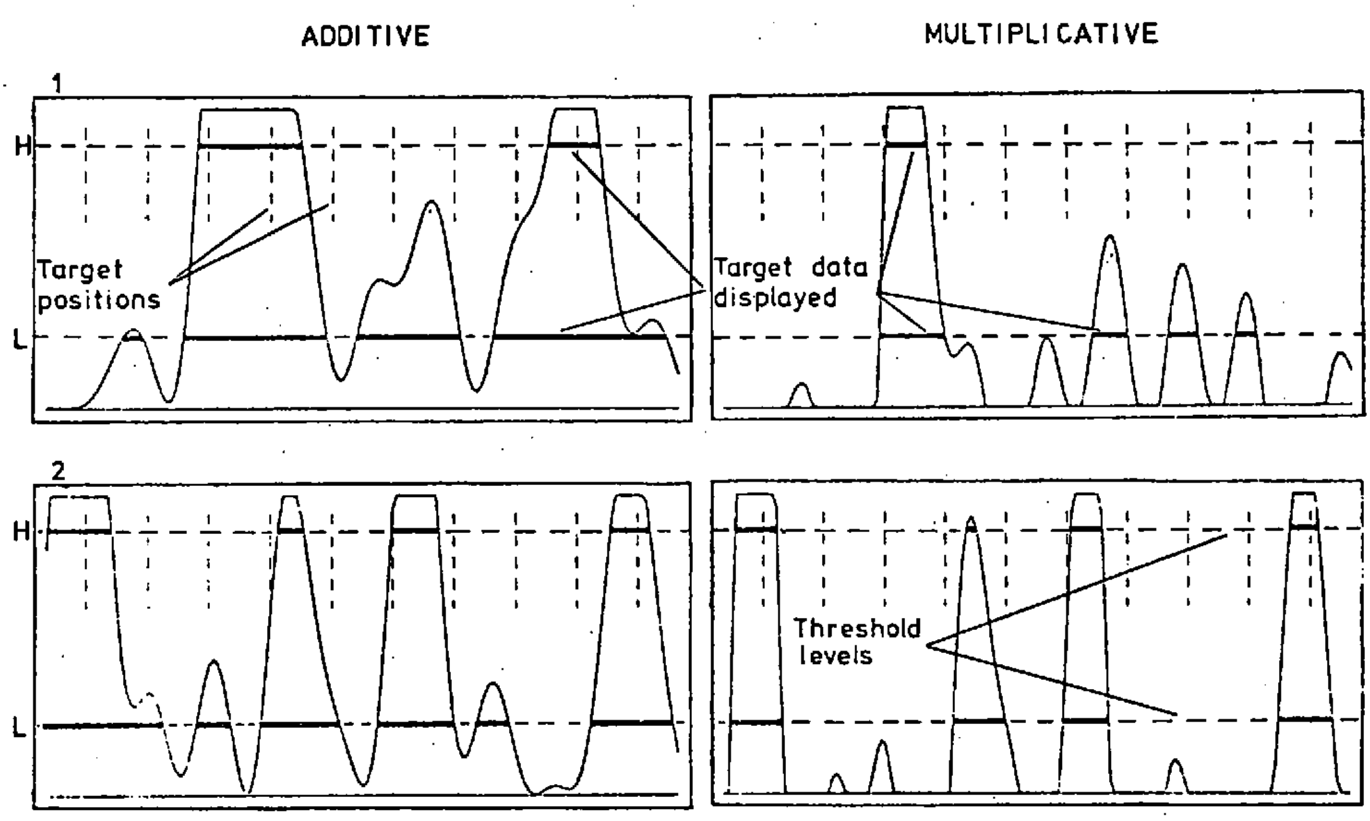

Fig. 3 Sets of single scans of metal box targets.

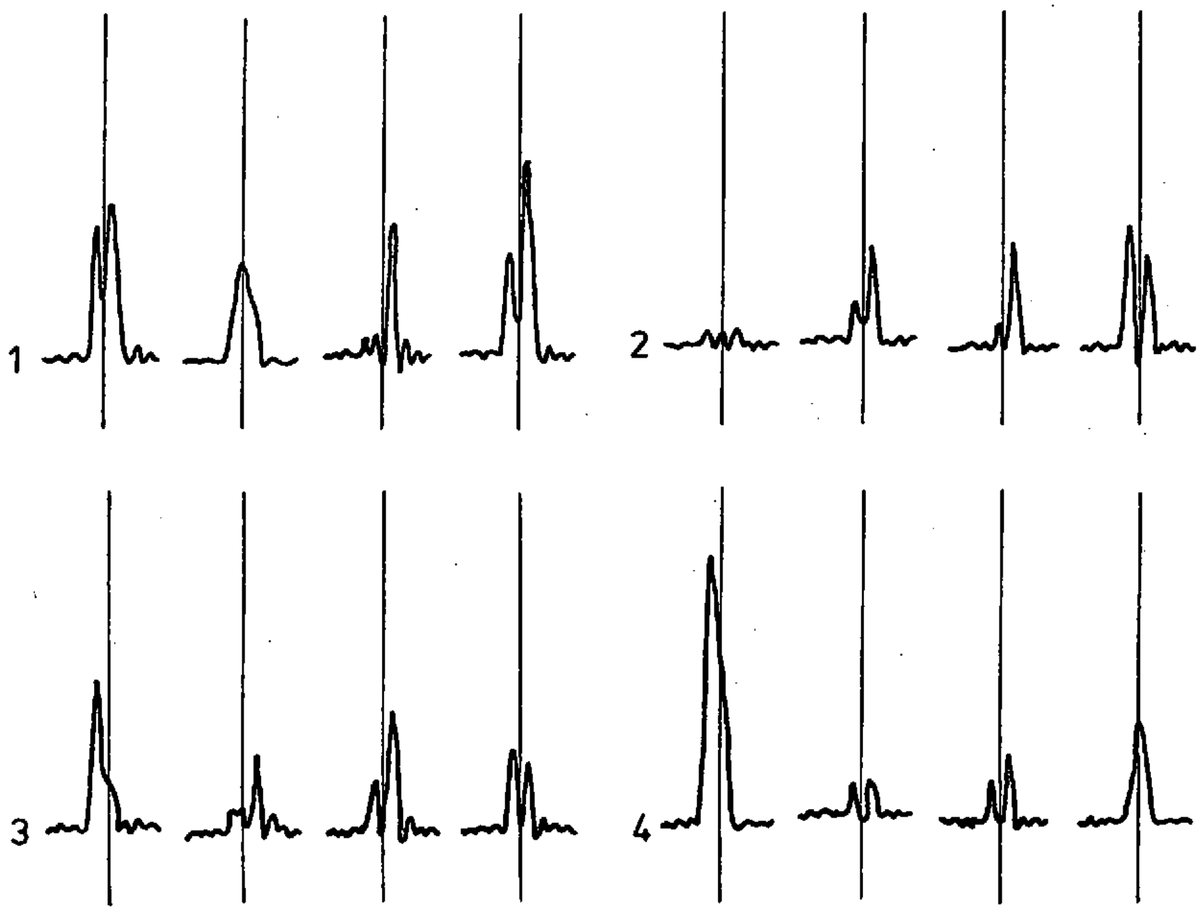

Fig. 4 Sets of single scans of extended target.

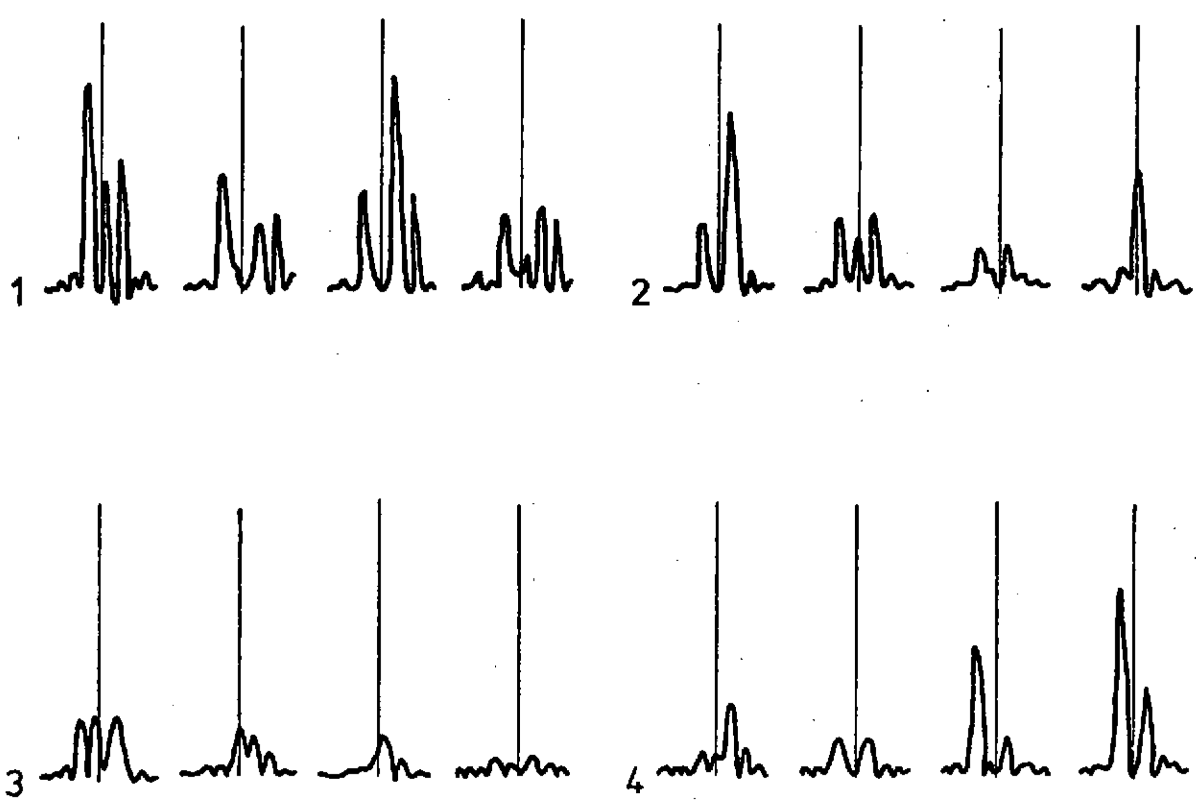

Fig. 5 Two target resolution: Equal power Rayliegh targets, varying amplitude threshold.

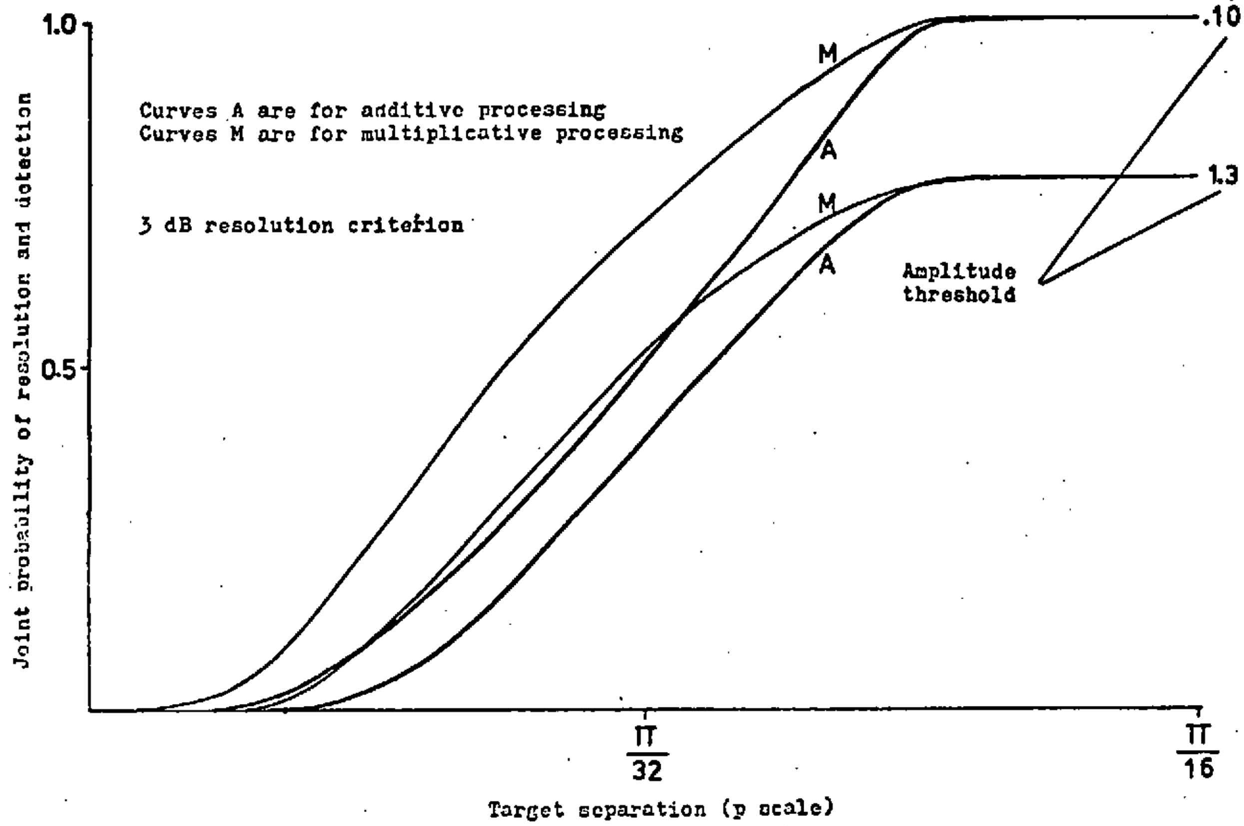

Fig. 6 Comparison of integrated multiple target directional responses.