POSITION FIXING OF TRANSPONDERS FROM SURFACE MEASUREMENTS

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1. INTRODUCTION

There are many underwater engineering applications in which an array of acoustic transponders is needed on the sea bed [1-5]. If the applications are for accurate position fixing it is necessary to know their exact coordinates. In practice, the transponders may be dropped onto the sea bed from a ship and therefore these coordinates may not be known accurately. With *intelligent* devices it is possible to determine their separations by programming them to interrogate each other [4]. The problem posed here is how to determine the geometry and orientation of an array from surface measurements only, a practical requirement suggested by an offshore company.

The problem is approached by considering an array of three transponders A, B and C, as shown in Fig.1. The requirement is to determine the distances d_1, d_2 and d_3 between each pair of transponders and the angle θ between the line AB and the direction AN of true or magnetic north.

Attention is limited to the case where the sea bed is a plane, which can be horizontal or slanting, and the sea water is a stratified medium with a known *sound velocity profile* (SVP). Four situations are considered: (i) flat sea bed, constant SVP, i.e. $c(z) = c$; (ii) flat sea bed, any SVP; (iii) slanting sea bed, constant SVP; (iv) slanting sea bed, any SVP.

The practical approach is to make measurements at two points on the surface. The measurements needed in all four situations are illustrated in Fig.2. At a surface observation point O_1, the depth h_1 is measured with a depth sounder and the acoustic propagation times t_{A1}, t_{B1}, t_{C1} along rays O_1A, O_1B, O_1C are measured by interrogating each transponder in turn. At a second surface observation point O_2, the depth h_2 and times t_{A2}, t_{B2}, t_{C2} are measured. The length O_1O_2 and the angle ϕ between this line and true or magnetic north N are also measured. For situations (3) and (4), additional measurements are required to give the directions of each of the transponders as seen from O_1 and O_2, that is γ_{A1}, γ_{B1}, γ_{C1}, γ_{A2}, γ_{B2}, γ_{C2}.

The key feature of the approach is to project all the construction lines O_1A,......, O_2C, O_1O_2, and the angle ϕ onto the plane of the sea bed. The problem is thus reduced to two-dimensional geometry [2], as shown in Fig.3(a). As long as all the projected lines P_1A, P_1B, P_1C, P_2A, P_2B, P_2C, P_1P_2, and the angle ψ are determined, calculation of d_1, d_2, d_3 and θ is straightforward. To illustrate the procedure, d_1 and θ are calculated. The relevant geometry is shown in Fig.3(b). Applying the cosine theorem to triangles AP_1P_2 and BP_1P_2,

$$\cos\angle AP_1P_2 = [P_1A^2 + P_1P_2^2 - P_2A^2] / 2\, P_1A.P_1P_2 \tag{1}$$

$$\sin\angle AP_1P_2 = \sqrt{[1 - \cos^2 \angle AP_1P_2]} \tag{2}$$

$$\cos\angle BP_1P_2 = [P_1B^2 + P_1P_2^2 - P_2B^2] / 2\, P_1B.P_1P_2 \tag{3}$$

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$\sin\angle BP_1P_2 = \sqrt{[1 - \cos^2 \angle BP_1P_2]}$ (4)

Since $\angle AP_1B = \angle AP_1P_2 - \angle BP_1P_2$, then

$\cos\angle AP_1B = \cos\angle AP_1P_2 . \cos\angle BP_1P_2 + \sin\angle AP_1P_2 . \sin\angle BP_1P_2$ (5)

Applying the cosine theorem again to $\triangle AP_1B$ yields

$d_1 = \sqrt{[P_1A^2 + P_1B^2 - 2\, P_1A.P_1B.\cos\angle AP_1B]}$ (6)

Similarly,

$d_2 = \sqrt{[P_1B^2 + P_1C^2 - 2\, P_1B.P_1C.\cos\angle BP_1C]}$ (7)

$d_3 = \sqrt{[P_1C^2 + P_1A^2 - 2\, P_1C.P_1A.\cos\angle CP_1A]}$ (8)

where $\cos\angle BP_1C$ and $\cos\angle CP_1A$ can be calculated by variable replacement ($A \rightarrow B \rightarrow C \rightarrow A$) in equations (1) to (5).

To calculate θ, consider $\triangle AP_1M$ in Fig.3(b), where M is the cross-over point of P_1P_2 with the line drawn from A in the direction of true or magnetic north N. Simple geometric relations give

$\theta = \angle P_1AB - (180^\circ - \angle AP_1P_2 - \psi)$ (9)

where

$\angle AP_1P_2 = \cos^{-1}[\cos\angle AP_1P_2]$ (10)

$\angle P_1AB = \cos^{-1}\{[P_1A^2 + AB^2 - P_1B^2] / 2\, P_1A.AB\}$ (11)

Thus, to locate the transponder positions it is necessary to calculate all the projection lines and angles. In the following sections these are calculated for the situations listed earlier.

2. FLAT SEA BED, CONSTANT SOUND VELOCITY PROFILE

If the sound velocity does not vary with depth, i.e. $c(z)$ = constant, each acoustic ray is a straight line with a length equal to the propagation time multiplied by the sound velocity c. For example, $O_1A = c.t_{A1}$. For a flat bottom, $h_1 = h_2 = h$ and all the projected lines P_1A, P_1B and P_1C are perpendicular to O_1P_1 and O_2P_2, as shown in Fig.2, i.e.

$P_1A^2 = (c.t_{A1})^2 - h^2$ (12)

$P_1B^2 = (c.t_{B1})^2 - h^2$ (13)

$P_1C^2 = (c.t_{C1})^2 - h^2$ (14)

$P_2A^2 = (c.t_{A2})^2 - h^2$ (15)

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$$P_2B^2 = (c.t_{B2})^2 - h^2 \tag{16}$$

$$P_1C^2 = (c.t_{C2})^2 - h^2 \tag{17}$$

It follows that $P_1P_2 = O_1O_2$ and $\psi = \phi$.

3. FLAT SEA BED, ANY SOUND VELOCITY PROFILE

In this situation the acoustic rays are curved lines in general and the first consideration is how to calculate P_1A from the geometry shown in Fig.4. According to two widely known formulae in the theory of sound transmission [5], P_1A may be given in terms of a ray parameter 'a', i.e.

$$a = \sin \eta / c(z) \tag{18}$$

$$t_{A1} = \int_o dz \Big/ c(z) \sqrt{[1 - a_{A1}^2 c^2(z)]} \tag{19}$$

$$P_1A = \int_o a_{A1} c(z)\, dz \Big/ \sqrt{[1 - a_{A1}^2 c^2(z)]} \tag{20}$$

In (19) and (20), a_{A1} is the ray parameter for a ray O_1A having an initial angle η_0. Equation (19) has a general form given by $f_1(a_{A1}) = 0$. This is a known non-linear function that can be calculated by numerical methods. Applying a conventional iterative method, e.g. the Newton-Raphson method, a solution can be obtained for a_{A1}. The approach is to try many values of η_0, then choose the one that leads to a vertical depth h_1 during a specified time t_{A1}. After a_{A1} has been solved, P_1A can be calculated numerically with (20). This method leads to a general procedure for calculating all the projected lines of P_1A,.......,P_2C ($P_1P_2 = O_1O_2$ and $\psi = \phi$ as before):

- Solve a non-linear equation

  $$f_{Ti}(a_{Ti}) \equiv t_{T1} - \int_o dz \Big/ c(z) \sqrt{[1 - a_{Ti}^2 c^2(z)]} = 0 \tag{21}$$

  for a_{Ti}, where T = A, B, C; i = 1, 2.

- Calculate P_iT

  $$P_iT = \int_o a_{Ti} c(z)\, dz \Big/ \sqrt{[1 - a_{Ti}^2 c^2(z)]} \tag{22}$$

4. SLANTING SEA BED, CONSTANT SOUND VELOCITY PROFILE

The geometry related to the calculation of P_1A is shown in Fig. 5, which has the following features:

- The ray O_1A is a straight line whose length is $O_1A = c\ t_{A1}$;
- Four lines on a horizontal plane are drawn from P_1: the true or magnetic north direction P_1N, the direction of A as seen from O_1, the steepest direction (at angle β) and the zero gradient direction of the sea bed;
- The direction of transponder A and the steepest direction have angles γ_{A1} and α respectively with respect to P_1N;
- A_h and A_v are the projection of A onto the horizontal plane and the vertical line O_1P_1.

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The following theorem is now used: if a slanting plane has an oblique angle β along its steepest direction, then the oblique angle δ in a direction at an angle χ to the steepest direction will satisfy

$$\tan\delta = \tan\beta.\cos\chi \tag{23}$$

Applying (23) to the direction of A from P_1 gives

$$\tan(\angle AP_1A_h) = \tan\beta.\cos(\gamma_{A1} - \alpha) \tag{24}$$

$$z_{A1} = h_1 - AA_h = h_1 - x_{A1}\tan\beta.\cos(\gamma_{A1} - \alpha) \tag{25}$$

From $\triangle O_1AA_v$,

$$x_{A1}^2 + z_{A1}^2 = O_1A^2 = c^2 t_{A1}^2 \tag{26}$$

Combining (25) and (26) gives the solution of x_{A1} and z_{A1}:

$$[1 + \tan^2\beta.\cos^2(\gamma_{A1} - \alpha)]\, x_{A1}^2 - [2h_1 \tan\beta.\cos(\gamma_{A1} - \alpha)]\, x_{A1} + [h_1^2 - c^2 t_{A1}^2] = 0 \tag{27}$$

This equation leads to a solution for x_{A1}. In $\triangle P_1AA_h$,

$$P_1A = x_{A1}\sec(\angle AP_1A_h) = x_{A1}\sqrt{[1 + \tan^2\beta.\cos^2(\gamma_{A1} - \alpha)]} \tag{28}$$

After algebraic manipulation, this becomes

$$P_1A = \frac{h_1\tan\beta.\cos(\gamma_{A1} - \alpha) + \sqrt{\{c^2 t_{A1}^2 [1 + \tan^2\beta.\cos(\gamma_{A1} - \alpha)] - h_1^2\}}}{\sqrt{[1 + \tan^2\beta.\cos^2(\gamma_{A1} - \alpha)]}} \tag{29}$$

P_1B, P_1C, P_2A, P_2B, P_2C can each be calculated with a formula similar to (29) by replacing t_{A1}, γ_{A1} by t_{B1}, γ_{B1}......, and h_1 by h_2 where necessary. It is then possible to write a general formula for calculating all six projected lines as

$$P_iT = \{h_i Q_{Ti} + \sqrt{[c^2 t_{Ti}^2 (1 + Q_{Ti}^2) - h_i^2]}\} / \sqrt{(1 + Q_{Ti}^2)} \tag{30}$$

$$Q_{Ti} = \tan\beta.\cos(\gamma_{Ti} - \alpha) \tag{31}$$

where T = A, B, C and i = 1, 2.

To obtain P_1P_2 and ψ, Fig. 6 is used, where O_1 and O_2 are the two surface observation points; $O_1O_3 = O_1O_2$; P_1, P_2, P_3 are the projected points of O_1, O_2, O_3 respectively onto the sea bed; and P_{2h} and P_{3h} are the projected points of O_2 and O_3 onto the horizontal plane which passes through P_1. It is evident that for the trapezium $O_1P_1P_2O_2$ and $\triangle P_2P_1P_{2h}$ the following two equations hold:

$$P_1P_2^2 = O_1O_2^2 + (h_1 - h_2)^2 \tag{32}$$

$$\tan(\angle P_2P_1P_{2h}) = (h_1 - h_2) \big/ O_1O_2 \tag{33}$$

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and when applying (23),

$$\tan(\angle P_2P_1P_{2h}) = \tan\beta.\cos(\phi - \alpha) \quad (34)$$

Combining (33) with (34) gives α as

$$\alpha = \phi - \cos^{-1}\{[h_1 - h_2] / [O_1O_2 \tan\beta]\} \quad (35)$$

This calculation shows that it is not necessary to know α beforehand. From $\Delta P_3P_1P_{3h}$ and $\Delta P_2P_1P_{2h}$

$$P_3P_{3h} = P_1P_{3h}\tan(\angle P_3P_1P_{3h}) = O_1O_2 \tan\beta.\cos\alpha \quad (36)$$

$$P_1P_3 = P_1P_{3h}\sec(\angle P_3P_1P_{3h}) = O_1O_2\sqrt{[1 + \tan^2\beta.\cos^2\alpha]} \quad (37)$$

$$P_2P_{2h} = P_1P_{2h}\tan(\angle P_2P_1P_{2h}) = O_1O_2\tan\beta.\cos(\phi - \alpha) = h_1 - h_2 \quad (38)$$

$$P_1P_2 = P_1P_{2h}\sec(\angle P_2P_1P_{2h}) = O_1O_2\sqrt{[1 + \tan^2\beta.\cos^2(\phi - \alpha)]}$$

$$= O_1O_2^2 + [h_1 - h_2]^2 \quad (39)$$

From trapezium $P_3P_{3h}P_{2h}P_2$

$$P_2P_3^2 = P_{2h}P_{3h}^2 + (P_2P_{2h} - P_3P_{3h})^2 \quad (40)$$

where $P_{2h}P_{3h}$ can be calculated using cosine theorem in $\Delta P_1P_{2h}P_{3h}$ as

$$P_{2h}P_{3h}^2 = P_1P_{2h}^2 + P_1P_{3h}^2 - 2\,P_1P_{2h}.P_1P_{3h}\cos\phi$$

$$= 2\,O_1O_2^2\,(1 - \cos\phi) \quad (41)$$

All three sides of $\Delta P_1P_2P_3$ are now known. The cosine theorem again results in

$$\cos\psi = [\,P_1P_2^2 + P_1P_3^2 - P_2P_3^2\,] / 2\,P_1P_2.P_1P_3 \quad (42)$$

After manipulation, the result is

$$\psi = \cos^{-1}\frac{\cos\phi + (h_1 - h_2)\tan\beta.\cos\alpha / O_1O_2}{\sqrt{[1 + \tan^2\beta.\cos^2\alpha]}\sqrt{[1 + (h_1 - h_2)^2 / O_1O_2]}} \quad (43)$$

It should be noted that when $\beta = 0$, $h_1 = h_2$, the formulae above reduce to (12) through (19), the situation for a flat sea bed.

5. SLANTING SEA BED, ANY SOUND VELOCITY PROFILE

Fig. 5 is also applicable to this situation. The only difference is that the ray O_1A is now a curve; so (25) is still correct but (26) is not. Instead of (25) two acoustic transmission formulae similar

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to (19) and (20) must be used. Therefore the three equations for solving a_{A1}, x_{A1} and z_{A1} are:

$$t_{A1} = \int_0 dz / c(z) \sqrt{1 - a_{A1}^2 c^2(z)} \quad (44)$$

$$x_{A1} = \int_0 a_{A1} c(z) dz / \sqrt{[1 - a_{A1}^2 c^2(z)]} \quad (45)$$

$$z_{A1} = h_1 - x_{A1} \tan \beta . \cos(\gamma_{A1} - \alpha) \quad (46)$$

Inserting (44) into (45) yields

$$h_1 - z_{A1} = \tan \beta . \cos(\gamma_{A1} - \alpha) \int_0 a_{A1} c(z) dz / \sqrt{[1 - a_{A1}^2 c^2(z)]} \quad (47)$$

Equations (44) and (47) have the general form

$$f_1(z_{A1}, a_{A1}) = 0 \quad (48)$$

$$g_1(z_{A1}, a_{A1}) = 0 \quad (49)$$

where functions $f_1(.)$ and $g_1(.)$ are non-linear (but known) and can be calculated numerically. Equations (48) and (49) can be solved by conventional iteration. As long as z_{A1} and a_{A1} are available, x_{A1} can be easily calculated by using (46). The desired projected line P_1A is therefore obtained as

$$P_1A^2 = x_{A1}^2 + (h_1 - z_{A1})^2 \quad (50)$$

The whole procedure for calculating $P_1A, \ldots\ldots, P_2C$ in a general form can be expressed as follows:

- Solve a set of non-linear equations as

$$f_{Ti}(z_{Ti}, a_{Ti}) \equiv t_{Ti} - \int_0 dz / c(z) \sqrt{[1 - a_{Ti}^2 c^2(z)]} = 0 \quad (51)$$

$$g_{Ti}(z_{Ti}, a_{Ti}) \equiv h_i - z_{Ti} - \tan \beta . \cos(\gamma_{Ti} - \alpha) \times \int_0 a_{A1} c(z) dz \Big/ \sqrt{[1 - a_{Ti}^2 c^2(z)]} = 0 \quad (52)$$

for z_{Ti} and a_{Ti}, where T = A, B, C; i = 1, 2.

- Calculate x_{Ti}

$$x_{Ti} = [h_i - z_{Ti}] / \tan \beta . \cos(\gamma_{Ti} - \alpha) \quad (53)$$

- Calculate P_iT

$$P_iT = x_{Ti}^2 + (h_i - z_{Ti})^2 \quad (54)$$

P_1P_2, α and ψ are still given by (32), (35) and (43).

6. SUMMARY

A projection approach has been presented for calculating the geometry of an array of underwater acoustic transponders on the sea bed from two surface measurements only. The algorithms can give the array's configuration, size and orientation with respect to true or magnetic north. Four situations have been considered, those of a flat or slanting sea bed each for either a constant or variable sound velocity profile. The procedure can be divided into three steps:

Measurements:

- the acoustic propagation times from two observation points O_1 and O_2 to each transponder A, B and C: t_{A1}, t_{B1}, t_{C1}, t_{A2}, t_{B2}, t_{C2} (Fig. 2)
- the sea depth at O_1 and O_2: h_1, h_2
- the length and direction of the line connecting O_1 and O_2: O_1O_2, ϕ
- (slanting sea bed only) the directions of each transponder seen from O_1 and O_2: γ_{A1}, γ_{B1}, γ_{C1}, γ_{A2}, γ_{B2}, γ_{C2} (Fig. 2); the oblique angle β along the steepest direction must be known, but the steepest direction σ itself does not need to be known (Fig. 5).

Projection:

The aim is to calculate the projected lines of all the acoustic rays O_1A, O_1B, O_1C, O_2A, O_2B, O_2C, as well as O_1O_2, onto the sea bed (Fig. 2). Also needed is the projected angle of ϕ. The equations for all four situations are summarised as follows:

Flat sea bed, constant SVP	12, 13, 14, 15, 16, 17
Flat sea bed, any SVP	21, 22
Slanting sea bed, constant SVP	30, 31, 32, 35, 43
Slanting sea bed, any SVP	51, 52, 53, 54, 32, 35, 43

Location:

When all the projected lines and angles have been calculated, the whole problem reduces to a plane geometrical one and the algorithms are given by equations (1) through (11), which are common to all four situations. In this paper the calculations have been done for a three-transponder array for the sake of illustration. Because an array of any number of transponders can be considered to be a combination of triangles, the method can easily be extended.

7. REFERENCES

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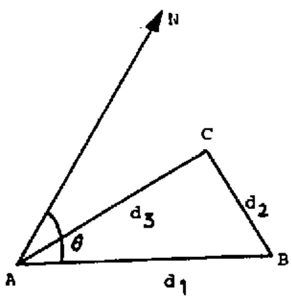

Fig. 1

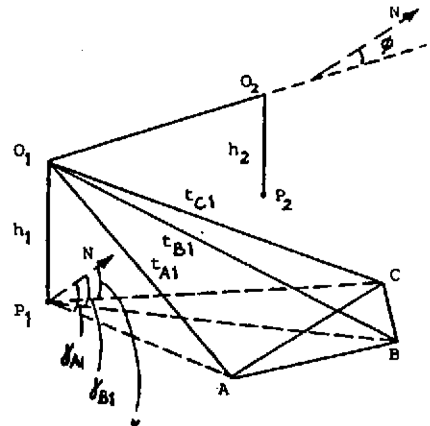

Fig. 2

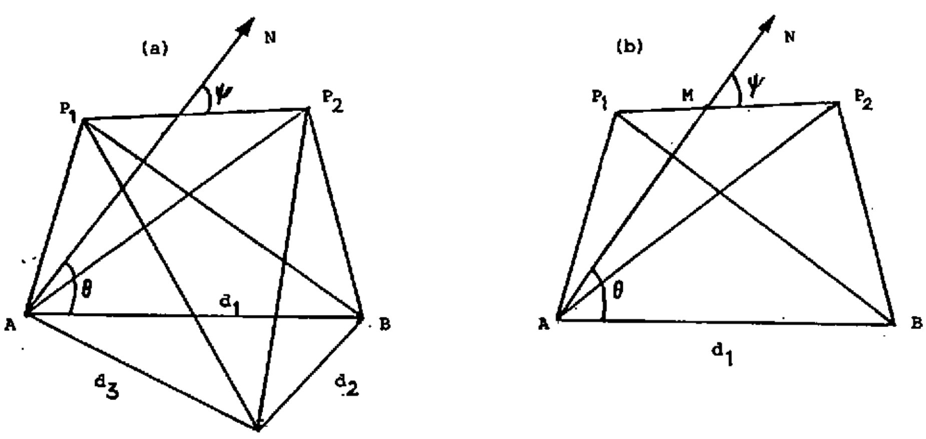

Fig. 3

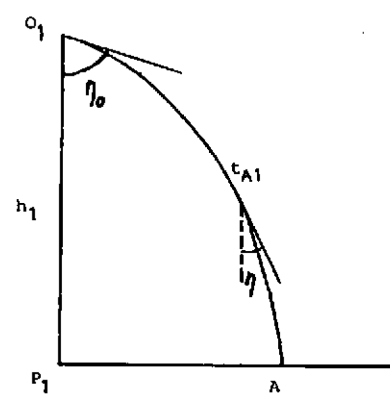

Fig. 4

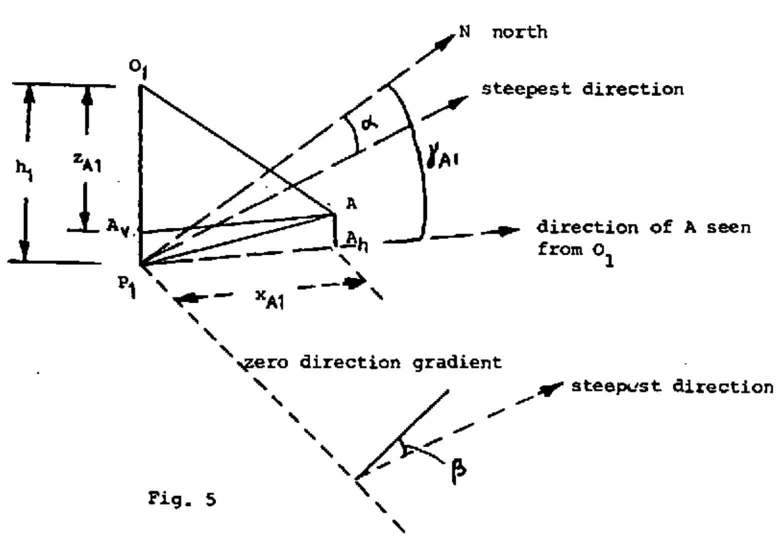

Fig. 5

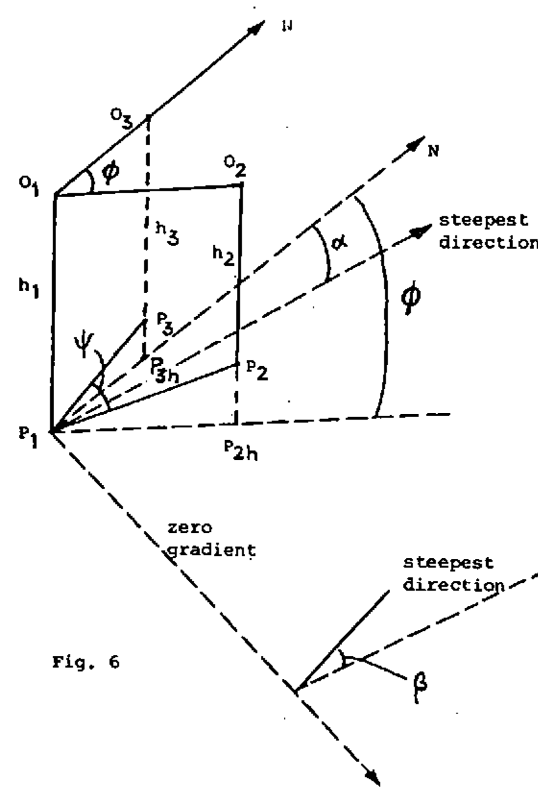

Fig. 6