COMPARISON OF ADAPTIVE AND DIRECTIONAL OPTIMAL SENSOR WEIGHTS IN ISOTROPIC NOISE FIELDS

Benjamin A. Cray and Ivars P. Kirsteins

Newport Division, Naval Undersea Warfare Center, 1176 Howell Street, Newport, Rhode Island 02841-1708
email: Benjamin.cray@navy.mil

The question addressed here is whether an adaptive beamformer, in an ideal spherically isotropic noise field, generates the same, unique, optimal array shading coefficients as that obtained from a single highly-directional sensor (via direct analytical Taylor series expansion of the pressure field). If so, then the adaptive process naturally forms first, and higher, order gradients between the orthogonal sensor channels. The optimal weight set (J. Acoust. Soc. Am. **113** (3), March 2003) for a first-order directional (vector) sensor, are $\overline{w} = \{w_0, w_x, w_y, w_z\} = \{1, 3a_s, 3b_s, 3c_s\}$, where $a_s = a(\theta_s, \phi_s)$, $b_s = b(\theta_s, \phi_s)$, $c_s = c(\theta_s, \phi_s)$ are the direction cosine angles at the chosen azimuth and elevation steering. These optimal weights provide a single vector sensor with a directivity index of 6 dB. For a second-order directional (Dyadic) sensor, an optimal weight set provides a directivity index of 9.5 dB. Both first and second order directional sensors are examined here and compared to optimal adaptive array processing. For simplification, the directional point sensors are compared to two and three element linear arrays, aligned along the z-axis.

Keywords: Directional sensors, Adaptive Beamforming, Optimal Shading Weights

1. Introduction

Directional acoustic receivers, such as sonobuoys or vector sensors, are devices that measure, at a collocated point, acoustic pressure and the nth-order spatial gradient of pressure. The output of the receiver is the power sum of acoustic pressure and each independently weighted gradient. However, before generating the weighted sum, each gradient must be converted to an equivalent value of acoustic pressure. This is easily accomplished by multiplying by a scale factor inversely proportional to the acoustic wavenumber. Examples of such devices are given in references [1,2].

2. Comparison for the first-order directional sensor

The power sum of the weighted and scaled pressure-gradients, of any order *n*, may be written [2] as

$$B(\phi) = \left| \sum_{n=0}^{N} w_n c^n(\phi) p(z;t) \right|^2, \tag{1}$$

where the gradients are taken only along the z-axis and $c^n(\phi) = \cos^n(\phi)$. For example, for $n = 1$, and ditching the time dependence,

$$B(\phi) = \left| w_0\, p(z) + w_1 \frac{1}{(ik_0)^1} \frac{\partial p(z)}{\partial z} \right|^2 = \left| \sum_{n=0}^{1} w_n c^n(\phi) p(z) \right|^2. \tag{2}$$

The optimal weights (derived in [2]) in spherical isotropic noise, for the first order sensor, at endfire, are $\overline{w} = \{1,3\}$, which provides a directivity index of, $N_{DI} = 6\ dB$.

The question addressed here is whether an adaptive scheme, in the same noise field, generates the same, unique, optimal weights. For a two-element array, with the elements separated by a distance, *d*, the adaptive beam weights, $\overline{w}_{adp} = \{w_{a0}, w_{a1}\}$, would produce an adaptive beam,

$$B_{adp}(\phi) = \left| w_{a0}\, p_1(z) + w_{a1}\, p_2(z) \right|^2 . \tag{3}$$

Therefore, it is necessary to derive the equivalence between the two weight sets analytically, and then compare to the weights generated with adaptive beamforming. Let

$$p(z) = \frac{p_2 + p_1}{2}, \tag{4}$$

and

$$\frac{\partial p(z)}{\partial z} = \frac{p_2 - p_1}{d} . \tag{5}$$

Then, from (2),

$$w_0\left(\frac{p_2 + p_1}{2}\right) + w_1 \frac{1}{(ik_0)^1}\left(\frac{p_2 - p_1}{d}\right) = w_0\left(\frac{p_2 + p_1}{2}\right) - w_1 \frac{i}{2\pi}\left(\frac{\lambda_0}{d}\right)(p_2 - p_1). \tag{6}$$

Rearranging, and defining a ratio, $a = \lambda_0/(2\pi d) = 1/(k_0 d)$, then

$$w_0\left(\frac{p_2 + p_1}{2}\right) + w_1 \frac{1}{(ik_0)^1}\left(\frac{p_2 - p_1}{d}\right) = \left(\frac{w_0}{2} + w_1 ia\right)p_1 + \left(\frac{w_0}{2} - w_1 ia\right)p_2 , \tag{7}$$

Hence

$$w_{a0} = \frac{w_0}{2} + w_1 ia \qquad and \qquad w_{a1} = \frac{w_0}{2} - w_1 ia , \tag{8}$$

and with the substitution of the known optimal weights

$$w_{a0} = \frac{1}{2} + 3ia \qquad and \qquad w_{a1} = \frac{1}{2} - 3ia . \tag{9}$$

The weight set shown in (8) could also be normalized, such that, $w_{a0} = w_0 = 1$, then

$$\widehat{w}_{a0} = 1 \qquad and \qquad \widehat{w}_{a1} = 1 - 4i\frac{w_0 w_1 a}{w_0^2 + 4w_1^2 a^2} . \tag{10}$$

The normalized adaptive weight set for <u>two</u> collinear elements then would be

$$\widehat{w}_{a0} = 1 \qquad and \qquad \widehat{w}_{a1} = 1 - 12i\frac{a}{1 + 36a^2} , \tag{11}$$

and since $1 << 36a^2$,

$$\widehat{w}_{a1} \approx 1 - i\left(\frac{1}{3a}\right) \approx 1 - i\left(\frac{2d}{\lambda_0}\right). \tag{12}$$

3. Summary

Directivity increases proportional with measurement of the n^{th}-order pressure gradient. As the order increases, the set of weights, for each gradient, which optimizes directivity change dramatically. Unit weights are far from optimal, with the consequence of diminishing gains with each additional gradient.

It has been shown here, and in the companion presentation, that optimal directivity gains, using adaptive and directional beamforming of a single directional sensor and two and three element uniform linear arrays, are nearly identical. Optimal gains obtained using higher order directional sensors have been compared as well, though not presented, and again equivalence was attained.

These optimal gains are unique and require equivalent shading weight coefficients – subsequent inspection of the adaptive and directional weight sets revealed that they are indeed identical.

Hence, adaptive beamformers will naturally form spatial pressure gradients, as a directional sensor inherently does, where the gradients are known to be proportional to physical properties, such acoustic particle velocity.

REFERENCES

1 Hines, P. A., Rosenfeld, A. L., Maranda, B. H., and Hutt, D. L., Evaluation of the endfire response of a superdirective line array in simulated ambient noise environments, *Proc. of Oceans 2000*, Providence, RI, pp. 1489-1494.

2 Cray, B. A., Evora, V. M., and Nuttall, A. H., Highly Directional Acoustic Receivers, *J. Acoust. Soc. Am.*, **113** (3), March 2003.