Proceedings of the Institute of Acoustics

STRUCTURAL ACOUSTIC POWER TRANSMISSION BY STRIP EXCITATION OF SLENDER BEAMS.

B. A. T. Petersson

Laboratory of Industrial Acoustics, Technical University of Denmark, Building 425, DK-2800 Denmark. (Presently at Institute of Applied Physics, Postbus 155, 2600 AD Delft, The Netherlands)

1. INTRODUCTION

An extension of the mobility concept to cases where the excitation area cannot be considered point-like i.e., small compared with the wavelength, was suggested in [1]. Further investigation of strip excitation in the case of firm contact between the source system and a receiving plate [2] revealed a limited variation in the strip mobility - a mobility defined on power basis - with changes in the force distribution. In order to examine the corresponding variation in strip mobility for beams the variational technique developed in [2] has been adopted to determine the force distribution realising the minimum active power input to the beam. The beam is presumed infinite so that in an overall sense, arbitrary boundary conditions are encompassed.

In addition to the case in which there is no restriction with respect to sign shifts of the force distribution, also a co-phase version is developed and investigated. Such a distribution is practically simple to establish and therefore may be of interest for the design of strip interfaces between source and receiver structures.

In the analysis of strip coupling of elementary structures, two aspects are of major interest; first, the installation of structural acoustic sources on strip-like contact areas on a receiving system and second, the interaction of two passive structural subsystems joined along a strip interface of finite length. The latter aspect is of importance e.g. in conjunction with statistical energy analysis.

2. THEORY

Consider an infinite beam subject to a strip excitation over a finite length, l. The excitation, centered at x=0, is defined by its force distribution $\sigma(x)$, with the net force $F=\int\sigma(x)dx$ and the lateral coupling $\tau(x)=0$. For slender beams i.e., where Euler-Bernoulli beam theory applies the basic condition for minimising the active power input becomes

$$\mathrm{Re}\left[\int_{-l/2}^{l/2} p(x_0)\, K(x|x_0)\, dx_0\right] = E(kl) \tag{1}$$

if the constraints imposed on the force distribution are $p(x)=p(-x)$ and $\mathrm{Im}[p(x)]=0$ i.e., the distribution is taken to be symmetric and real. Hereby $K(x|x_0)$ is the kernel for one-dimensional flexural wave propagation and E is some arbitrary function of Helmholtz number only. This means that the force distribution sought must lead to a uniform real part of the velocity along the excitation region. An admissible choice for the minimising force distribution is

$$p(x) = (F/l)\ (1-\beta x^2)/(1-\beta l^2/12) \tag{2a}$$

where the coefficient β, as determined from the minimisation condition, can be found to be

$$\beta = k^2/[kl\ \cot(kl/2) + (kl^2/2)^2 - 2] \tag{2b}$$

With the minimising force distribution specified the strip mobility is found to be

$$Y_Q = \frac{j\omega}{Bk^3 k\ell[k\ell\ \cot k\ell/2 + \frac{2}{3}(k\ell/2)^2 - 2]}\{(k\ell/2)^2[\frac{2}{3}(k\ell\ \cot k\ell/2 +$$
$$+ (k\ell/2)^2 - 2) - \frac{1}{5}(k\ell/2)^2] - (k\ell\ \cot k\ell/2 + (k\ell/2)^2 - 2)^2$$
$$+ (\frac{1-e^{-k\ell}}{k\ell})(\frac{k\ell}{2}\cot k\ell/2 - \frac{k\ell}{2} - 2)[k\ell(\cot k\ell/2 + \coth k\ell/2) - 4]\} \tag{3}$$

from which is seen that only an imaginary part exists.

Expanding both the cotangent and the hyperbolic cotangent to third order for small arguments, it is found that the imaginary part tends to infinity, the reason being the extreme gradients of the force distribution. The high frequency (or long strip) asymptote is a slightly larger mobility than the ordinary mass mobility of the directly excited part of the beam.

The force distribution in (2) will alter in sign along the excited strip for certain Helmholtz numbers. This means that the stress at positions along the strip will be either in or out of phase relative to the stress at the centre.

In practice, it is simpler to establish a force distribution which is co-phase. Given the minimising force distribution $p(x)$, a co-phase distribution can be obtained for each Helmholtz number, by adding a constant chosen such that it equals the minimum value of $p(x)$ on the interval $[-l/2, l/2]$ if this minimum is negative. Upon introducing the latter condition a co-phase distribution is obtained as

$$g(x) = \frac{F}{l}\,\frac{1}{1+K}\left(\frac{1-\beta x^2}{1-\beta l^2/12} + K\right) \quad (4a)$$

$$K = \begin{cases} 0; & \mathrm{Min}\ (p(x)) \geq 0 \\ \left|\mathrm{Min}\left(\dfrac{1-\beta l^2/4}{1-\beta l^2/12}, \dfrac{1}{1-\beta l^2/12}\right)\right|; & \mathrm{Min}\ (p(x)) < 0 \end{cases} \quad (4b)$$

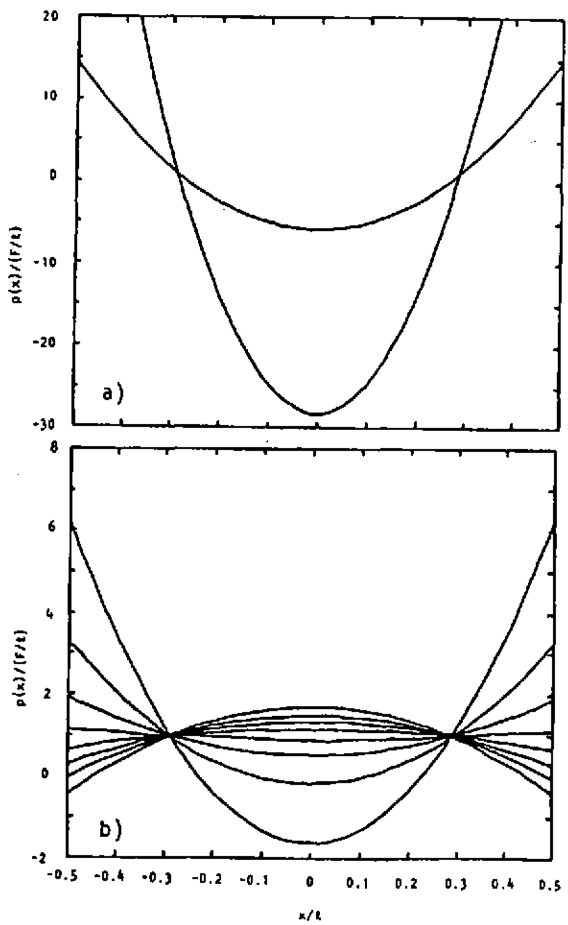

Figure 1. Spatial variation of the force distribution minimising the power input.
a) $kl=1,2$;
b) $kl=3,4,5,6,7,8,9,10$; (from bottom till top curve at x=0).

With the now co-phase stress, g(x), the strip mobility can be found to be

$$\frac{\mathrm{Re}[Y_Q]}{\mathrm{Re}[Y_0]} = \frac{1/(1+K)^2}{[P_\beta - \frac{1}{3}(k\ell/2)^2]^2}\{(\alpha-1)P_\beta[(\frac{\sin k\ell 2}{k\ell/2})^2(\alpha P_\beta-(k\ell/2)^2+2)$$

$$- 2\,\frac{\sin k\ell}{k\ell}]\} \qquad (5a)$$

$$\frac{\mathrm{Im}[Y_Q]}{\mathrm{Im}[Y_0]} = -\frac{(1/1+K)^2}{k\ell/2[P_\beta - \frac{1}{3}(k\ell/2)^2]^2}\{\frac{(\sin k\ell/2)^2}{k\ell/2}[k\ell +$$

$$\cot k\ell/2(\alpha P_\beta - (k\ell/2)^2 + 2)]\,P_\beta(\alpha-1)$$

$$+ \frac{1-e^{-k\ell}}{k\ell}[\alpha P_\beta-(k\ell/2)^2 - k\ell - 2][\alpha P_\beta-(k\ell/2)^2 - 2$$

$$+ k\ell\cot k\ell/2] + 2\alpha P_\beta[(2/3)(k\ell/2)^2-\alpha P_\beta] - \frac{2}{5}(k\ell/2)^4\} \qquad (5b)$$

where both parts are normalised with respect to the corresponding parts of the ordinary point mobility of an infinite beam and $P_\beta = kl\cdot\cot(kl/2) + (kl/2)^2 - 2$ and $\alpha = 1+K(1+\beta l^2/12)$.

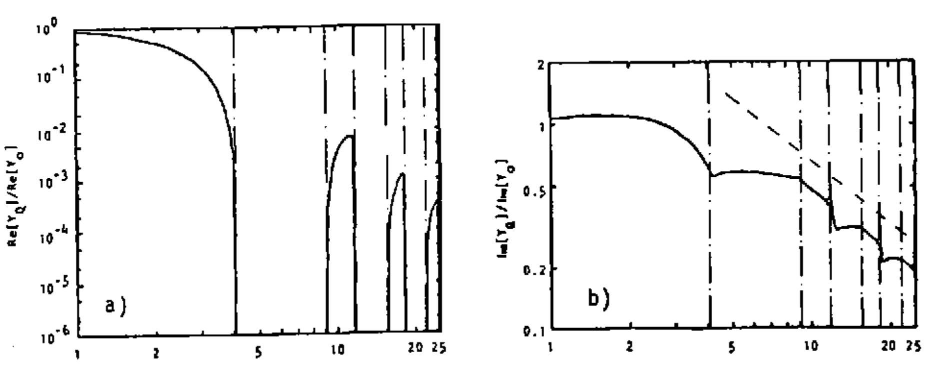

Figure 2. Normalised real (a) and imaginary (b) parts of the strip mobility in the case of the co-phase force distribution.

Again, the two asymptotes for extreme values of Helmholtz number are of interest. For small Helmholtz numbers it can be shown, applying the procedure described in the previous section that the strip mobility tends to the ordinary point mobility for an infinite beam. The asymptote for large values of Helmholtz numbers is that of the non-cophase force distribution. This means that the examined, co-phase force distribution asymptotically is the one giving minimum active input power. The real part of the strip mobility is not identically zero but the results exhibit ranges in which the real part vanishes. The ranges are the same for which the force distributions are identical to those in the case of the truly minimising distribution.

3. EXPERIMENTS

In order to gain additional experience from strip excitation of beams an experimental series was carried out where both a finite and an "infinite" version of a perspex beam were used as test objects. Introductory it was attempted to directly excite the beam via a triangular disc and thereby obtain a true strip excitation, see [3]. Such a technique however, proved inappropriate for the subsequent analysis and interpretation, due to difficulties in both calibration and numerically correcting the results with respect to the mass loading of the test object . Instead a number of transfer accelerances were measured and stored on disc.

For the finite beam a two-accelerometer far-field intensity technique was used (cf. [4]) whereas the simple proportionality between the power input and the magnitude of the transfer mobility squared can be invoked in the case of an infinite beam.

In Figure 3, the measurement results are shown for the finite perspex beam. In this experiment, a uniform force distribution was applied. As expected, there is a distinct decrease in the real part towards high frequencies and a first local minimum at approximately 2.2 kHz. This is in agreement with calculations, predicting a trough at 2215 Hz.

In Figure 4, the normalised real part of the strip mobility of the "infinite" perspex beam is shown in the case of a uniform force distribution for two different lengths of the strip. As for the finite beam, there is a distinct decrease in the real part towards high frequencies and a first local minimum at approximately 3.4 and 6.5 kHz respectively. The frequency of these troughs both correspond to a Helmholtz number of $kl=6.3$ taking into account the effects of shear and rotational inertia in the determination of the wave speed. This is in good agreement with the theoretical results. For the long strip excitation also one may perceive the second trough outside the measurement range which, according to calculations, should occur at 13 kHz.

The co-phase version of the force distribution minimising the input power was applied to the infinite beam and the associated real part of the strip mobility is shown in Figure 5. In this case, the force distribution was chosen such that the first minimum should occur at 5 kHz, corresponding to a Helmholtz number of 5. As demonstrated by the results a pronounced minimum is obtained close to the desired frequency. The overall decrease however, is markedly less than for the uniform distribution. This shows that the co-phase version of the force distribution minimising the power input is highly selective and less advantageous with respect to a suppression of a broad band excitation.

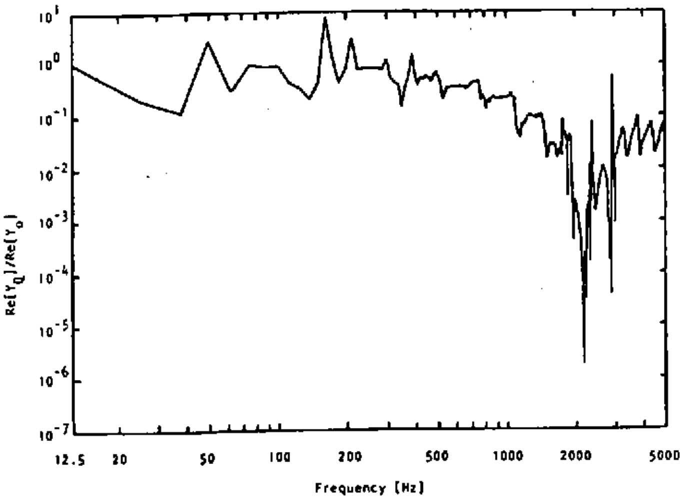

Figure 3. Normalised real part of strip mobility for a perspex beam having free-free end conditions. Length of strip l=0.2 m. Uniform force distribution.

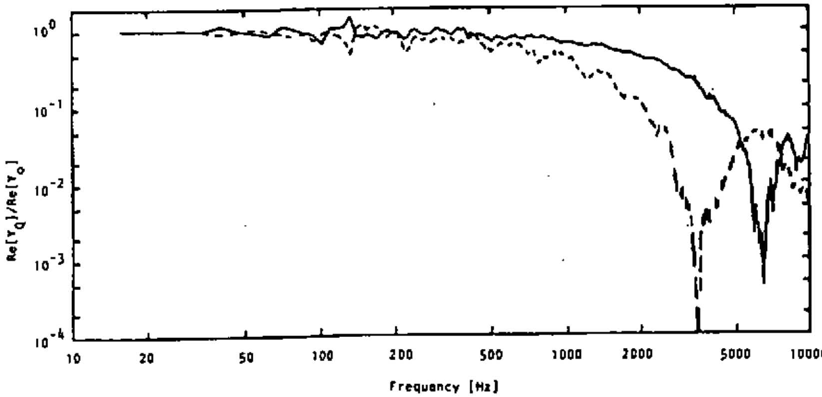

Figure 4. Normalised real part of strip mobility for a perspex beam with the ends embedded in sand. Length of strip l=0.1m (——) and l=0.15m (----). Uniform force distribution.

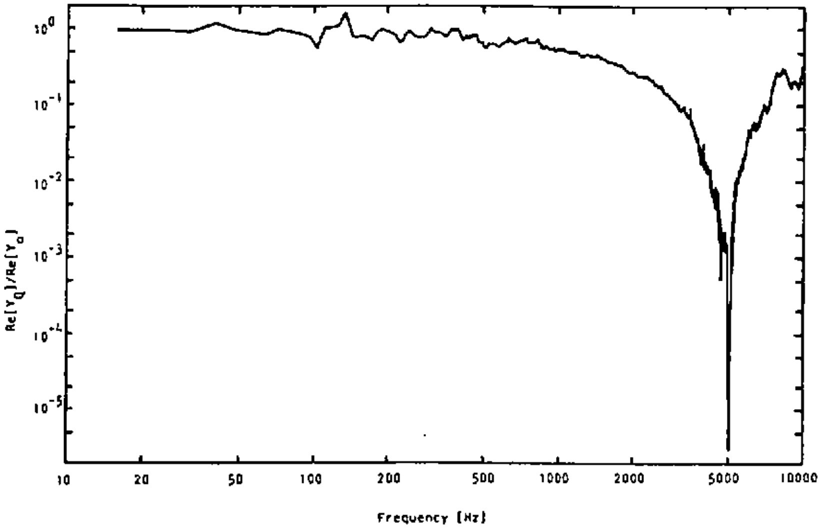

Figure 5. Normalised real part of strip mobility for a perspex beam with the ends embedded in sand. Length of strip l=0.1m. Force distribution minimising the active power input.

Upon comparing the two results for strips of length 0.1m, the co-phase, minimising force distribution can be seen offering a possibility to lower the frequency of the minimum for a given length of the strip. Also, from a comparison of the theoretical and the experimental results for both a finite beam and a simulated infinite one it seems reasonable to state that the practically realisable characteristics by strip excitation of beams are well described by the expressions for the strip mobility derived.

4. CONCLUSIONS

For the case of strip excitation of beams, a force distribution has been derived from a variational approach for which the real part of the strip mobility vanishes. This means that there is no active power transmitted to the beam. The force distribution varies with Helmholtz number and is non-cophase which means that its sign may shift along the strip. A co-phase version of the force distribution minimising the transmission of active power has been developed. Hereby the real part of the strip mobility vanishes within certain ranges which occurs where the adjusted force distribution equals the minimising one. Accordingly, the co-phase version realises bands of no active power input to the beam.

For large Helmholtz numbers both the real and imaginary parts of the strip mobility for the co-phase force distribution asymptotically approaches those of the minimising force distribution. Hence, it can be concluded that the co-phase version, in the limit, constitutes a real valued, co-phase force distribution minimising the power input. The force distribution minimising the active power input as well as the co-phase version, are shown to be selective in frequency with respect to a reduction of the input power. If a broadband suppression is required, the uniform force distribution is preferable.

The experimental results all demonstrate the significance of the length of the strip. The actual technique with which the experiments were undertaken proves that a discretisation of the strip excitation can be applied without loss in performance.

For the case where the phase of the force distribution varies along the strip also, an estimate of the real part of the strip mobility can be found if the phase is assumed to be random. Following [5] and the analysis related to the real part of the effective mobility an analogous statistical reasoning can be invoked. It is thus found that for a phase uniformly distributed on the interval $[-\pi, \pi]$, the real part of the strip mobility is inversely proportional to the length of the strip. Accordingly, it seems reasonable to state that given a net force, a proper design of a strip interface between a source structure and a receiving beam will lead to a substantial reduction in power transmission.

5. REFERENCES

[1] B. Petersson. Strip coupling between source and receiver structures. Proceedings of Inter-Noise, Honolulu, p553, 1984.

[2] P. Hammer & B. Petersson. Strip excitation: Part II: Upper and lower bounds for power transmission. Journal of Sound and Vibration, 129, p133, 1989.

[3] P. Hammer & B. Petersson. Minimised power input by line force excitation. Proceedings of Third International Conference on Recent Advances in Structural Dynamics, p13, Southampton, 1986.

[4] G. Pavic. Measurement of structure-borne wave intensity, part I: Formulation of the methods. Journal of Sound and Vibration, 49, 221-230, 1976.

[5] B. Petersson & J. Plunt. On the effective mobility for the prediction of structure-borne sound transmission between a source structure and a receiving structure; Part I: Theoretical background and basic experimental studies. Journal of Sound and Vibration, 91, p517, 1982.