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SYNTHESIS OF GUITAR TONES FROM FUNDAMENTAL PARAMETERS RELATING TO CONSTRUCTION

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1. INTRODUCTION

The tone quality of a stringed musical instrument is governed by the vibrational characteristics of its body, but understanding the relationships between these vibrations and the instrument's construction is a long-standing problem in musical acoustics. Although it has received considerable attention from experimental scientists and makers, their results are inconclusive because of the difficulty of adjusting only one parameter at a time. One major obstacle is that no two pieces of wood have identical material properties, even when they are cut from the same tree. Thus it is impossible to make precise copies of instruments let alone introduce deliberate and controlled differences. A more quantitative approach, as proposed here, is to develop a mathematical model of the instrument which predicts the sound radiated to an arbitrary point in space from information relating to the instrument's dimensions, wood properties and string excitation. The musical significance of systematic changes to the structure can then be assessed by ear.

Early work of this research group concentrated on experimental observations of the modes of vibration and sound radiation fields of guitars in both their finished state and also at various stages of construction. Work has also been carried out on the physical interaction between the player and the instrument and on the coupling of strings to the body. The experimental work identified acoustically important components of these systems and allowed us to develop a rudimentary model of the guitar[1]. The purpose of this paper is to introduce the theoretical background of a more advanced model and briefly report on our findings. We use the finite element method to calculate the normal modes of plates and air cavities. Subsequent calculations allow us to determine the input admittance of the body, the coupling between the body and the strings, and also, with some simplifications, to compute the far-field acoustic radiation from the instrument. In effect, we are using information about the construction of the instrument and its material properties to compute the transfer function between the plucking point on the string and an arbitrary point in the acoustic field. This transfer function can be used to generate sounds giving a unique opportunity to directly assess the relationships between guitar tone quality and construction.

2. THEORY

Following the treatment of Gough[2], we consider the transverse vibrations of a damped string of mass m, length L and tension T coupled via the bridge to a single structural resonance of the body. If a sinusoidal driving force $F\exp(j\omega t)$ is applied to a position x_0 along the string, the amplitude of the nth string harmonic is given by

$$a_n = \frac{2}{m} \frac{F\sin(k_n x_0) + Tk_n z}{S(\omega)} \quad \ldots (1)$$

where $S(\omega) = \omega_n^2 - \omega^2 + j\omega^2/Q_s$, $k_n = n\pi/L$, $\omega_n^2 = Tk_n^2/m$ and Q_s is the Q-value of the string resonance.

In comparison with a string mounted rigidly, the yielding support introduces an additional term $\frac{2}{m} Tk_n z/S(\omega)$, which arises from considerations of energy flow into or out of the bridge. The structural resonance itself is treated as a simple harmonic oscillator of resonant frequency ω_b and Q-value Q_b with an effective mass M_b at the bridge. The bridge thus moves with a displacement $z\exp(j\omega t)$ under the action of the string force $Tk_n a_n \exp(j\omega t)$, where

$$z = \frac{Tk_n a_n}{M_b B(\omega)} \quad \ldots (2)$$

and $B(\omega) = (\omega_b^2 - \omega^2 + j\omega^2/Q_b)$.

Eliminating a_n from equations (1) and (2) gives the transfer admittance χ_n between the excitation point and the bridge. Hence

$$\chi_n = \dot{z}/F = j \frac{2\omega \sin(k_n x_0)\, Tk_n}{M_b m\, B(\omega)\, S(\omega) - 2T^2 k_n^2} .$$

χ_n is then summed over all string modes (n) and all body modes (b) to find $\chi(\omega)$.

Now that the velocity at the bridge is known explicitly, the velocity v_e of all other plate elements can be determined from the mode eigenfunctions. The total sound pressure radiated to an arbitrary point r is then calculated by treating each element as a simple source a distance R_e from r and summing over all elements:

$$p(r,\omega,t) = \sum_e j \frac{\omega}{4\pi R_e} \rho_0 A_e v_e \exp j(\omega t - kR_e) ,$$

where $A_e v_e$ is the volume velocity of each element vibrating in a particular mode, ρ_0 is the density of air, $k = \omega/c$ and c is the speed of sound in air

The finite element calculations do not include the interactions between the vibrating plate and the air modes of the cavity. The most important of these is the well-known coupling exhibited between the fundamental mode of the plate (Figure 1a) and the Helmholtz mode of the cavity. This splits the fundamental plate resonance and also introduces significant radiation from the sound-hole.

Incorporating the Newtonian model of Christensen and Vistisen[3] allows us to account for coupling between the fundamental plate mode and the air cavity resonance, of resonant frequency ω_h and Q-value Q_h. Introducing the coupling modifies equation (2), which becomes

$$z = \frac{Tk_n a_n H(\omega)}{M_b [B'(\omega) H(\omega) - \beta^2]}, \qquad \ldots (3)$$

where $H(\omega) = (\omega_h^2 - \omega^2 + j\omega^2/Q_h)$.

The coupling parameter β^2, which is described fully by Christensen and Vistisen, can be determined from quantities obtained explicitly from finite element data and from details of the cavity. We note that ω_b must also be modified to include the added "stiffness" induced by the backing air cavity, giving $B'(\omega)$.

The transfer admittance for the nth string mode coupled to the plate-air system then becomes

$$\chi_n = j \frac{2\omega \sin(k_n x_0) Tk_n H(\omega)}{M_b m [B'(\omega) H(\omega) - \beta^2] S(\omega) - 2T^2 k_n^2 H(\omega)}$$

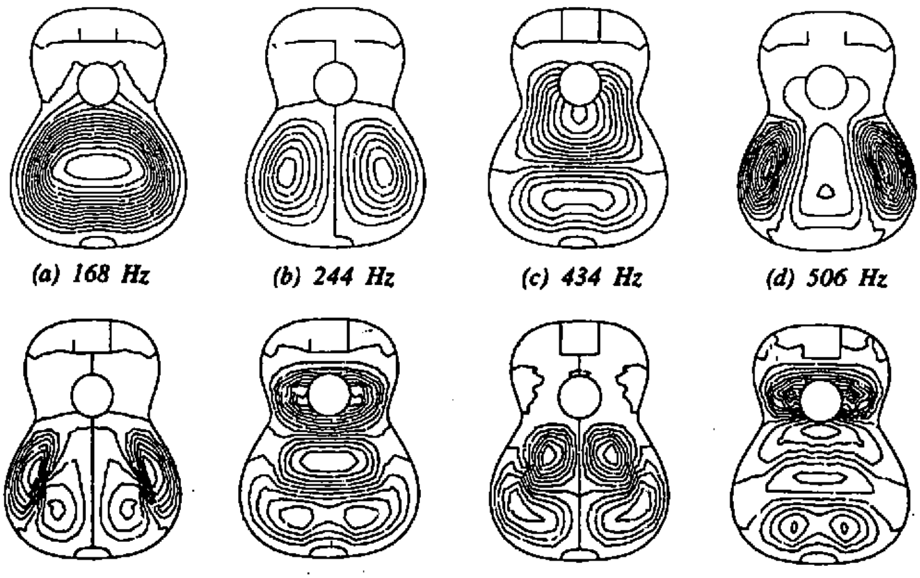
Finally, the sound field is summed as before, but we must now include an additional contribution from the sound-hole. The volume velocity of the sound-hole source is given by

$$A_h v_h = -Av \frac{\omega_h^2 H(\omega)}{(\omega_h^2 - \omega^2) + \omega^4/Q_h^2},$$

where Av is the total volume velocity of the whole plate vibrating in its fundamental mode. Experimental observations show that, to a good approximation, it is valid to retain the same eigenfunction for the fundamental mode of the plate after it has coupled to the air cavity resonance.

3. RESULTS AND DISCUSSION

Finite element calculations were performed using *LUSAS* on a large mainframe computer at the South West Universities Regional Computing Centre. As in our earlier work[1], the model consists of a top plate, complete with struts and bridge, fixed at its edges. Typical output is shown in Figure 1. The eigenfunction data was then reduced for local processing. Numerical solutions were derived for $\chi(\omega)$ and $p(r,\omega)$ on an IBM PC equipped with a maths co-processor and digital-to-analogue interface. Typical results are shown in Figure 2. Note the presence of resonance peaks due to the air cavity and body modes as well as string resonances. The linearly decreasing phase in the pressure response, which dominates the fine structure, merely represents the propagation delay of the signal to the observation point. The complex sound pressure data was then Fourier transformed to obtain impulse responses of the system. Since plucking the string is virtually identical to impulsive excitation, these time-domain signals were used to construct note sequences which could be used for comparative listening tests.



(a) 168 Hz (b) 244 Hz (c) 434 Hz (d) 506 Hz

(e) 612 Hz (f) 672 Hz (g) 738 Hz (h) 864 Hz

Figure 1. Contour plots of top-plate eigenfunctions calculated by the finite element method. The plate is fully strutted and includes the bridge. The plate thickness is 2·6 mm.

SYNTHESIS OF GUITAR TONES

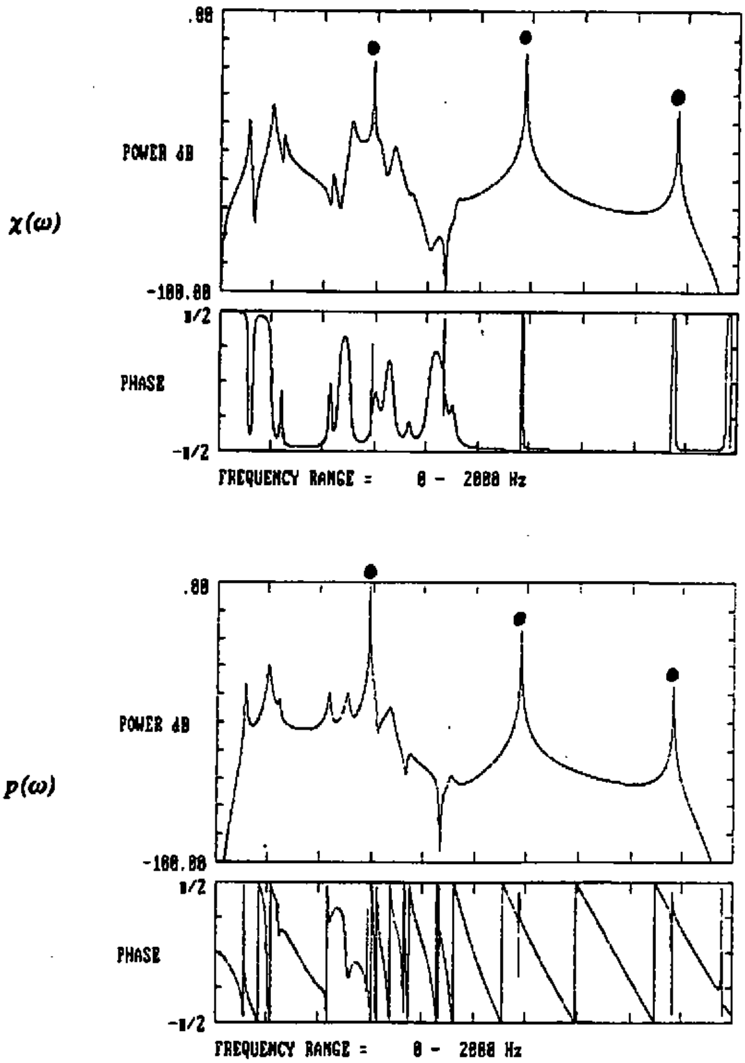
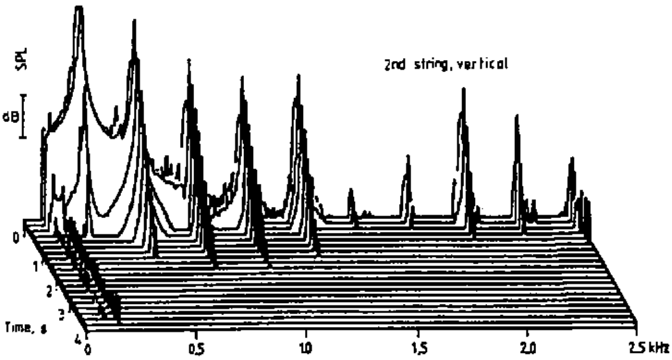
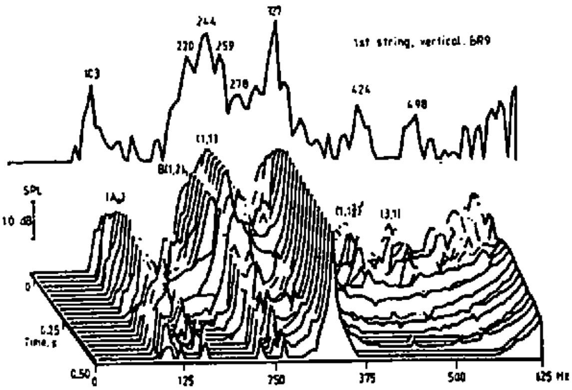


Figure 2. Calculations of transfer admittance $\chi(\omega)$ and sound pressure $p(\omega)$. String partials are marked by dots. A high note (588 Hz) was used in this example so as to clearly differentiate between string resonances and body resonances. $p(\omega)$ is derived 1 m in front of the instrument. Reference levels for the dB scales are arbitrary.

SYNTHESIS OF GUITAR TONES



(a)



(b)

Figure 3. FFT analyses of guitar sounds. (a) Shows the variability of decay rates to be seen amongst the partials. The fundamental decays extremely rapidly because it couples strongly to a powerful body mode. (b) Shows detail of the "body noise" which occurs in the transient. The component at 327 Hz is due to the string; all other components are related directly to body modes.

We have not yet undertaken systematic studies to investigate the relationships between synthesised guitar sounds and parameters of construction, but early experiments involving the thinning of plates from 3·0 mm to 2·2 mm clearly indicate that audible changes occur. The signals are severely bandwidth-limited, mainly because of the lack of reliable mode data above 1 kHz. Nevertheless, they demonstrate features in common with real tones, such as frequency-dependent damping rates of string harmonics, string anharmonicity (due to strong coupling between the string and body) and the presence of "body noise" (Figure 3).

One interesting observation from this work, or from experimental work for that matter, is the apparent similarity between modes of different instruments. What is important, however, are the subtle changes which occur in the vicinity of the bridge. Tiny changes in the positions of the nodal lines can have a profound effect on the transfer admittance and thus substantially modify the sound of the instrument. These sort of changes are likely to be more important than shifts in mode frequencies, for example.

One other aspect we have investigated with the model is the possibility of coupling between other plate modes and the Helmholtz cavity mode. Any plate mode which induces volume changes in the cavity has the ability to couple. We have performed calculations and shown that small frequency perturbations of modes occur due to coupling, and that there are a small additional contributions to the sound field from the sound-hole. For the particular parameters involved in our modelled instruments, these effects would be considered unimportant, but we suggest that this might not be typical. Many classical guitars employ asymmetric cross bracing. The second mode of the top plate (Figure 1b) then becomes asymmetric and induces cyclic volume changes of the cavity. We suggest that in instruments with thin top plates, the resonant frequency of this mode might become low enough to couple significantly to the cavity and enhance the response of the instrument.

4. FUTURE DEVELOPMENTS OF THE MODEL

There are clearly severe limitations with a model of this kind. One of the most apparent is the limited bandwidth (less than about 1.6 kHz). Although there is a good understanding of the function of instruments in this low-frequency range, the upper frequencies have attracted much less attention, though they are very important perceptually. One argument for concentrating on the lower frequencies is that makers are likely to be able to control only the lowest few resonances of the instrument and that rest "has to look after itself". One wonders, however, if the higher partials of guitar sounds are related to more global properties of the instrument, such as material properties or damping rather than mode shapes and frequencies. We are performing statistical studies of these higher partials in an attempt to cast light on the problem.

Our current model takes no account of damping in the material or of radiation damping, both of which are known to have an important influence on the tone and playing qualities of instruments. Q-values used in the calculations have to be derived from typical experimental data (values range from about 70 to 20, the latter being associated with strongly radiating modes). Neither form of damping can be incorporated into our existing finite element model, which is based on a commercial package. Material damping can be included, but not for orthotropic materials, as we have here, which have different damping properties along the material's principal axes. Techniques such as boundary elements can be incorporated into finite element programs to simulate radiation reaction as well as providing a significant improvement in the calculation of radiation fields, but these again are not available in commercial packages in any form suitable for our class of problem. The solution is to write our own code, specific to our problem, to calculate fluid-loaded, damped modes of vibration of musical instrument structures. We have a current project on an *NCUBE* 128-node parallel processor located in Cardiff to tackle this problem.

5. REFERENCES

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[3] O CHRISTENSEN & B B VISTISEN, 'Simple model for low-frequency guitar function', *J. Acoust. Soc. Am., 68(3)*, 758-766 (1980)