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INVESTIGATIONS OF MODE COUPLING IN THE GUITAR

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INTRODUCTION

It is of direct interest to the guitar maker to determine quantitatively how the acoustical response of a guitar is related to its physical dimensions and the material properties of the woods used in its construction. Acoustically, the most concise way of describing the response of the instrument is in terms of its normal modes of vibration, but the body of the guitar is a complex mechanical structure, which does not lend itself readily to normal mode analysis. A more satisfactory approach, from the point of view of theoretical modelling and practical analysis, is to consider the instrument to be composed of a number of interconnected parts. The response of the complete instrument can then be described in terms of coupled modes of the individual parts.

The structure is best divided along regions where most modes have nodal lines, and it then breaks into the following parts: the strings, the top plate, the back plate, the sides, the neck and the enclosed air volume of the body. Coupling between these parts occurs when the interconnecting boundaries do not act as idealised fixed boundaries.

To a first approximation, the bridge (an integral part of the top plate) provides a 'fixed' support for the strings. Obviously, if this were so, the body of the guitar would be ineffective in enhancing the radiation of sound from the strings to the surrounding air. Gough [1] has described how a coupling parameter can be derived from measurements made independently on the strings and on the body. This parameter can be used to predict the frequencies and Q-values of the coupled modes. String-to-body coupling in the guitar has been described elsewhere [2,3] and will not be discussed further in this paper.

Because the strings couple directly to the top plate, it is generally accepted that the acoustical response of the guitar is largely governed by the vibrations of this plate. Two examples can be presented to support this statement. Christensen [4] has developed a model which predicts the sound radiation from a guitar in the frequency range up to 600 Hz; the model achieves considerable accuracy even though it assumes that the sound radiation emanates exclusively from the top plate. Our own work at Cardiff (Richardson and Roberts [5]) has included the development of a finite-element model of the guitar. So far, we have modelled only the top plate fixed at its edges, but we nevertheless find that the spatial distributions and frequencies of the modes predicted by theory compare favourably with measurements made on completed instruments.

It would be improper to suggest that the two models described above are accurate representations of the action of the guitar; both models ignore the coupling between the fundamental top-plate mode and the Helmholtz resonance of the air cavity. The importance of this coupling has been outlined by a number of workers [eg 6], and several excellent models exist which are able to predict the resonant response of the guitar in its lowest octave. More sophisticated models explain the formation of a resonance triplet due to coupling between the fundamental top-plate and back-plate modes and the Helmholtz resonance [7].

Rossing et al [8] have investigated modes of a classical guitar and a folk (steel-string) guitar under a variety of boundary conditions. They effectively decoupled certain elements of the body by partially immobilising it in sand. Their results clearly indicate that mode coupling existed between the top plate and other elements of the guitar, but that the degree of coupling was influenced by the construction of each instrument. I have repeated similar measurements on four classical guitars of my own construction, and it is a discussion of these measurements which is presented in this paper. The four instruments were of similar, though not identical, design. I will attempt to relate the differences in coupling in each instrument to differences in their construction. Implications for the accuracy of our finite-element model of the guitar will also be discussed.

EXPERIMENTAL MEASUREMENTS

I used the following methods to measure the resonance frequencies, Q-values and spatial distributions of the modes of each guitar in the frequency range 20 to 1000 Hz. The instruments were excited electromagnetically at a single frequency from an audio oscillator. The frequency of excitation was varied and individual modes were identified in real time by means of speckle interferometry and then recorded holographically. The frequency of excitation was then varied about the resonance frequency of the mode so that the resonance frequency and -3 dB points could be accurately determined from the response of the guitar measured by means of an accelerometer attached to an antinode of vibration. Q-values were subsequently calculated from the -3 dB points and resonance frequencies. In some cases it was necessary to use two drivers so that combination modes could be eliminated [9]. For stability reasons, the guitar was firmly clamped by its neck to a rigid frame attached to the holographic bench. This method of clamping the guitar had very little effect on the modes. The guitar stand allowed the operator to study both plates of the guitar under identical conditions.

In the subsequent discussions, I have categorised modes as (m,n) by counting the number of 'antinodal' or vibrating areas (separated by nodes) across the plate (m) and along the plate (n). Top-plate modes, back-plate modes and air-cavity modes are distinguished as T(m,n), B(m,n) and A(m,n) respectively.

Modes were of three types: motion involved bending of the whole body in either one or two dimensions, or bending of individual plates ('plate' modes). A similar observation was made by Moral and Jansson [10] for the violin. The one- and two-dimensional modes occurred in the frequency range 20 to 50 Hz and were generated as a consequence of the method of clamping the instrument. Vibration involved cantilever and twisting motion of the body. Some less detailed measurements (non optical) made on the four instruments in 'free' states showed that the lowest-frequency mode was always a one-dimensional 'beam' mode with a resonance frequency of about 70 to 75 Hz. In each guitar the mode had nodal lines which ran across the width of the guitar in the region of the bridge and across the second fret on the neck. The mode was a poor radiator of sound, but was detectable in the input admittance at the bridge, and it is conceivable that it could be excited by the strings during the transient part of the note. No other one- or two-dimensional modes were found in this study.

It is the 'plate' modes which are of principal interest in this paper. If coupling between the plates, the sides and the air cavity were not present, modes would exhibit a single resonance. Experimental work on guitars and finite-element predictions of plate modes show that modes of individual plates form a regular hierarchy. Table 1 shows mode frequencies of an isolated guitar top plate, excluding bridge, fixed at its edges. The addition of the bridge greatly perturbs some of the mode frequencies [11]. Similarly, alteration of the plate's design, thickness, strutting or material properties alters the frequencies of the modes, but modes of increasing complexity of bending always occur at increasingly higher frequencies. A similar observation can be made concerning the back-plate modes. In real instruments, where coupling is present, single 'plate' modes are commonly found to exhibit multiple resonances. Although the motion of one plate may appear to be the same or very similar at each of the resonances, the motion of the rest of the instrument is different. Following the methods of Rossing et al [8], I isolated individual parts of the instrument by packing it appropriately with sand bags and made measurements on the plate modes; these results were then compared with measurements made on the instrument in its normal state. When mode frequencies differed or where resonances were split, I observed the motion of the rest of the instrument and attempted to determine the coupling mechanism.

Table 1. Mode frequencies predicted by the finite-element method for a strutted guitar top plate without bridge and fixed at its edges [5].

n	m = 1	2	3	4	5
1	200	260	361	484	643
2	450	612	759	891	
3	589	1100			
4	862				

REGIMES OF COUPLING

(a) Inter-modal coupling

I use the term 'inter-modal coupling' to describe the coupling which can occur between modes of the same plate as a result of a non-zero value of Poisson's ratio. Stretching along one axis of the plate causes a contraction along the other axis and can effectively 'couple' pairs of symmetric or asymmetric modes of similar frequencies [12]. When coupling occurs, there is a considerable disturbance of the frequencies and mode shapes which would have been expected if Poisson's ratio had equalled zero. Unlike with any other type of coupling to be discussed, these modes cannot be decoupled into their 'simple' forms because the modes are, in fact, normal modes of the plate.

Figures 1 and 2 show examples of inter-modal coupling between the T(1,2) and T(3,1) modes. The coupling disturbs the otherwise horizontal and vertical nodal lines to produce a pair of modes, one with a characteristic saddle shape (b), and one with a central raised 'T' section (c). The effects of coupling are more pronounced in the guitar shown in Figure 1.

(a) 360 Hz (b) 420 Hz (c) 450 Hz

Figure 1. Time-averaged interferograms of three modes formed by coupling between the A(1,2) and T(1,2) modes and inter-modal coupling between the T(1,2) and T(3,1) modes. (Guitar BR2.)

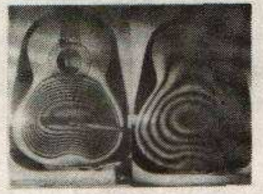
(a) 364 Hz (b) 431 Hz (c) 553 Hz

Figure 2. Time-averaged interferograms of three modes formed by the same coupling described in Figure 1 on another guitar. (Guitar BR11.)

In the frequency range 600 to 800 Hz, inter-modal coupling has been observed between the T(5,1) and T(3,2) modes and a dipole motion of the plate in the upper bout. Some of the higher T(1,n) couple to T(m,1) and T(m,2) modes and can generate a bewildering set of 'odd-shaped' modes which do not fit readily into the assumed hierarchy of mode shapes. The occurence of inter-modal coupling between the modes described has been verified by finite-element predictions. I believe that inter-modal coupling between back-plate modes is also prevalent.

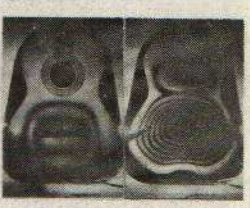
(b) Coupling between the plates and air cavity

Coupling between the T(1,1) mode and A(1,1) mode (the Helmholtz air resonance) has received considerable attention and need not be discussed in detail here. The B(1,1) mode also couples to these modes and forms a resonance triplet in the low frequency range of the guitar. The back-plate mode is generally less strongly coupled to the Helmholtz resonance than the top-plate mode. I have made sets of interferograms of these coupled modes on individual guitars showing the motion of the two plates under identical driving conditions. Figure 3 shows a typical set of results. In Figure 3(a) and 3(b) the driver was attached to the top plate and in Figure 3(c) it was attached to the back plate. The relative amplitudes of the vibrations of the plates at each resonance varies from one guitar to another, and either plate can dominate the motion. Measurements often show a slight splitting of the upper resonance, which is possibly generated as a result of coupling to the B(1,2) mode. The plates vibrate in various phase relationships. At the lowest resonance the motion of the plates is in phase and the whole body swells and contracts. At the middle resonance the plates' motion is out of phase and the whole guitar vibrates like a thick 'sandwich'. The nodal lines are inset from the edges of the plates and the sides participate in the motion. The mass and stiffness of the sides will influence the resonance frequency.



(a) 101 Hz

(b) 193 Hz



(c) 250 Hz

Figure 3. Time-averaged interferograms of the motion of the top and back plates at each of the resonances of the low-frequency resonance triplet formed by coupling between the T(1,1), B(1,1) and A(1,1) modes. (Guitar BR1.)

I have investigated coupling between plate modes and higher-frequency air cavity modes (see Jansson [13]). Strong coupling occurs between the T(1,2) and A(1,2) modes to produce a resonance doublet (Figures 1 and 2). This coupling sometimes raises the position of the central nodal line of the T(1,2) mode and enhances the coupling between it and the strings [5]. Coupling between plate modes and air modes occurs only if the spatial distributions and frequencies of the modes are similar. In the guitars I have investigated, I find no conclusive evidence of coupling between other plate modes and air-cavity modes, although this is not true for all types of guitars [8].

(c) Mechanical coupling

The sides form a direct mechanical link between the top plate and back plate. Figure 4 shows a set of interferograms of the motion of the body of a guitar; all the interferograms were made under identical conditions of excitation (at the top plate). Direct coupling occurs between the two plates by means of a bending mode of the sides. As a result of the coupling (essentially between three modes), the T(5,1) mode exhibits a resonance triplet. In the guitar shown, the first bending mode of the sides occurred at a frequency of about 700 Hz and involved motion on the 'flat' areas on either side of the waist. The next higher mode occurred at approximately double this frequency. Immobilising the sides reduced the coupling between the two plates.

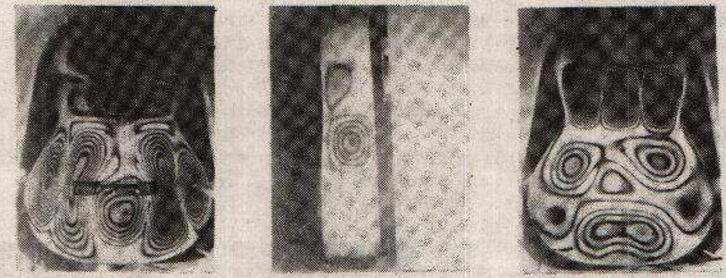


Figure 4. Time-averaged interferograms of the motion of the top plate, back plate and one side of Guitar BR2 when excited at a frequency of 721 Hz.

Below 700 Hz most top-plate modes tend to have zero bending at the edge of the plate and, consequently, couple very weakly to the sides. The same is not true for the back-plate modes, presumably because of the greater rigidity of the plate.

(d) Radiation coupling

Acoustic energy radiated as a result of driving one plate can be partially absorbed by the other plate, which is then set in motion. The primary radiator is a resonant structure, and the spatial distribution and intensity of the sound field falling on the remote plate depends strongly on frequency. I have observed the motion of the back plate of a guitar while driving 'individual' modes of the top plate (and vice versa). The motion of the back plate was dominated by the mode whose resonance frequency was closest to the driving frequency. Often more than one mode was excited, and under these circumstances nodal lines were seen to move when the frequency of excitation was varied and the modes combined in different relative phases [9].

The remotely-driven plate re-radiates sound and creates a back-reaction on the primary plate, which modifies its modes of vibration (principally their resonance frequencies). I observed that immobilising the remote plate induced small shifts (< 1%) in the mode frequencies of the primary plate.

DISCUSSION

It is important to consider the influence of the different types of coupling described above on the response of the guitar. Below about 600 Hz the most important coupling occurred between the plate modes and the air cavity modes. The resonance frequencies of the coupled modes deviated substantially from the frequencies of the decoupled modes. Input admittance curves made at the bridge showed that the response of the instrument at each of the resonances of a resonance doublet or triplet was not the same, but depended on the degree of coupling of the modes. For example, in three of the guitars investigated, the coupling between the T(1,2) mode and the A(1,2) mode was very strong and produced two resonances of the plate of comparable strength. In the fourth guitar (Guitar BR8), coupling between these same modes was very weak, because the resonance frequencies of the modes were far apart. When excited at the top plate, this instrument had a strong response at a frequency of 315 Hz and a second, much weaker, response at 405 Hz. The instrument was built with a Bouchet-style cross strut with relief under the strut on either side of the soundhole. Previous work has shown that a reduction in the stiffness of this strut has a marked effect of the frequency of the T(1,2) mode [5,11].

I found little evidence of coupling between the T(2,1) and A(2,1) modes or B(2,1) and A(2,1) modes. These modes all had very similar spatial distributions and could potentially couple, but their resonance frequencies were well separated and any effects of coupling were too weak to measure. I suspected that coupling occurred between the A(2,1) and T(4,1) modes in three of the guitars under investigation. Interferograms of the T(4,1) modes showed that the outer two vibrating areas of the plate created a larger volume displacement than the inner vibrating areas; the modes, therefore, had dipole components and would be expected to couple to the air cavity mode (of a similar resonance frequency). My measurements were not conclusive because of experimental difficulties. I hope to carry out further investigations of coupling between the plate modes and air cavity modes using a purpose-built test rig with a 'removable' air cavity.

Above 600 Hz the coupling became more complex because of the added contribution of the modes of vibration of the sides. This coupling was very strong at times. The frequency at which this change occurs will depend on the construction of the sides. I found less coupling in guitars which did not have pronounced flat regions on either side of the waist. We can conclude that subtle modifications in the design of the guitar's shape would produce perceptible changes in the instrument's response, even though the modes of the plates themselves would be unaffected. Presumably similar changes could be induced by altering the thickness of the sides or dimensions of the linings. The maker must accept that the response of the instrument is not governed exclusively by the plates themselves, but also by the structure on which they are mounted.

'Radiation coupling' occurred at all frequencies. The absorption and re-radiation of sound by the back plate modifies the sound radiation from the instrument. Likewise, the coupled room modes and the damping of the back plate and sides by the player also affects the response. I have made some measurements of the influence of the player on the modes of guitars. The mode most affected was the middle resonance of the low-frequency resonance triplet (Figure 3b); its resonance frequency can be lowered by as much as 5%.

The guitar maker cannot easily predict the effects of coupling between the various parts of the instrument. Coupling adds yet another complication to his task of trying to produce instruments with a consistent sound quality. One positive improvement would be for the maker to test plates fixed to a rig with an artificial air cavity. The response of the plate would be much more like that found in the completed instrument. Similarly, there are important implications for our finite-element model. The coupling between the plate and the air cavity could be accounted for in a secondary model, but it would be impossible to vary the edge constraint with frequency in any realistic way to model the coupling to the sides. The only alternative would be to model the complete instrument, a prohibitively difficult task. Our work has shown that a a fixed edge produces more accurate results than a hinged edge [5], but this may not be true when modelling the higher-frequency ranges of guitars or for guitars of substantially different construction.

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