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MODES OF VIBRATION AND RADIATION FIELDS OF GUITARS

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1. INTRODUCTION

In the last few years, musical acoustics research at Cardiff has concentrated on trying to establish relationships between the construction of guitars and their acoustical function. Investigations have involved practical studies of real musical instruments and theoretical modelling of the vibrational modes of guitars and their associated radiation fields. The latter has involved the development of a finite element model to calculate modes of vibration of the wooden structure in terms of the instrument's dimensions and the properties of the materials used in its construction. Radiation fields are then calculated using the boundary element method, which gives an accurate predication of the sound radiated from all vibrating surfaces of the instrument. A string can be coupled to this model, enabling the time-varying response of the instrument to be determined as a result of a plucking force applied at an arbitrary point along the string. The model can, in effect, be played.

This paper will present a review of these investigations, concentrating mainly on aspects of the numerical modelling and its predictions. The model has highlighted the important role of the low-order modes of the guitar. These have been shown to be the primary agents for radiation of both low- and high-frequency sounds. Their contribution of strong anharmonic components in the transient part of the note is also an important aspect of the recognition of individual instruments.

2. MODES OF VIBRATION OF GUITARS

The function of the body of a guitar is to enhance the transfer of energy from the strings to the surrounding air. String vibrations couple to the transverse vibrations of the wooden structure, which in turn cause significant local pressure changes in the air and radiate sound. The most important motions of the instrument are those which induce volume changes of the instrument.

For ease of analysis, it is useful to divide the structure mentally into parts and talk about "top-plate", "back-plate" or "air-cavity" modes[1]. Undoubtedly, the most important vibrating element of the guitar is the top plate. This is a light-weight structure which is directly coupled to the strings via the fixed bridge. Typical mode shapes are shown in Figure 1. The precise shapes, frequencies and Q-factors of the modes depend critically on the design of the instrument and on the material properties of the parts. Somewhat similar vibration patterns

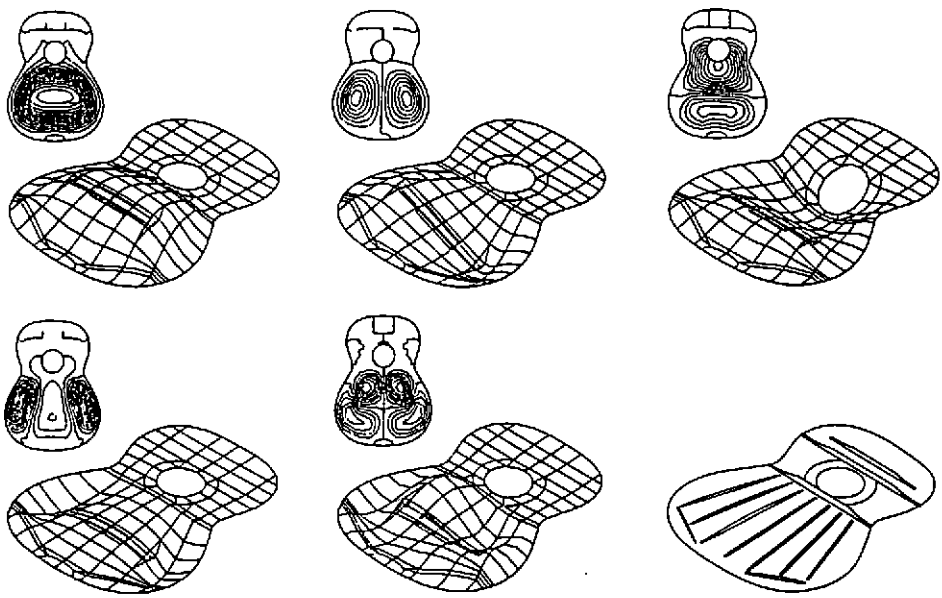

Figure 1. Contour and relief plots of guitar top-plate modes calculated using the finite element method. The plate is strutted as shown and includes a bridge.

may be observed on the back plate. Air cavity modes are also important, particularly for extending the low-frequency range of the instrument. The modal density of guitars is typically one or two modes per 100 Hz with Q-factors ranging from 20 to 60, depending mainly on the radiation efficiency of the mode.

Although studies of the vibrations of the body alone are of interest, they miss the point that the body is really a mediator between the string (and the player) and the outside world. Efficient transfer of energy from the string depends firstly on good coupling to the body and secondly on good radiation efficiency of the body modes. The general forms of the modes vary surprisingly little from instrument to instrument, but it is, of course, the detailed differences which are important for distinguishing one instrument from another. It is this fine detail which is of concern in our studies. The model described in this paper is capable of producing sounds as a function of plucking force applied at the string. The synthesised tones can be evaluated psychoacoustically, allowing relationships to be determined between the sound of the instrument and its construction. The effects of specific modifications to the structure can thus be evaluated.

3. THE NUMERICAL MODEL

The basic elements of the model have been reported previously[2]. It is based on a system comprising a flexible body, including an air cavity, coupled to a lossy string. Finite element analysis (FEA) is used to calculate the normal modes of the body, based solely on information about the dimensions of the instrument and the material properties of the component parts. As well as predicting the surface velocities, FEA can also be used to determine the input admittance at the bridge, information which can be used to couple the body to the transverse modes of the string[3]. For correct mechanical action at low frequencies, it is essential to couple the structural modes to the Helmholtz air cavity resonance[4]. This also provides the velocity of the "plug" of air in the sound-hole, which is required for the subsequent calculations of acoustic radiation from the instrument.

We have used a commercial FEA package for the mode calculations[5]. Limitations in computing resources dictate that the structure of the body has to be simplified. We have found it convenient to work with an accurate model of the top plate attached to rigid sides. Since the back plate is coupled via the Helmholtz cavity mode, it is sufficient to include a back plate which exhibits only its lowest mode. Whilst these simplifications clearly represent a loss of accuracy, comparisons between real and model structures indicate that the most important features are retained. Further descriptions of the methods involved and the influence of systematic variations in constructional parameters are given by Walker[6].

At this stage the model predicts the complex transfer admittance, $\chi(\omega)$, between the excitation point on the string and the bridge. Used in conjunction with the predicted eigenfunctions, $\chi(\omega)$ can be used to determine the velocities of each surface element over the complete instrument, including the sound-hole "plug" of air. Our previous model[2] used this information to calculate sound radiation by treating each element as a simple source of sound. This method gave reasonable approximations for the radiation directly to the front of the instrument, but it was unable to account for radiation from the back and sides and gave poor results for sound radiated towards the player, for example. Significant improvements have been made by incorporating the boundary element method (BEM) for the calculation of sound radiation from the structure[7].

The BEM is based on a numerical implementation of the Helmholtz Integral Equation (HIE):

$$\int_s \left(\mathbf{p}(\mathbf{x}_s)\frac{\partial \mathbf{G}(\mathbf{r})}{\partial n_s} - \mathbf{G}(\mathbf{r})\frac{\partial \mathbf{p}(\mathbf{x}_s)}{\partial n_s} \right) dS = \varepsilon\, \mathbf{p}(\mathbf{x}_f)\ ,$$

where $\mathbf{p}(\mathbf{x}_f)$ is the complex acoustic pressure at the field point f with position vector $\mathbf{x}_f$, $\mathbf{p}(\mathbf{x}_s)$ is the complex acoustic pressure at the source point s with position vector $\mathbf{x}_s$,

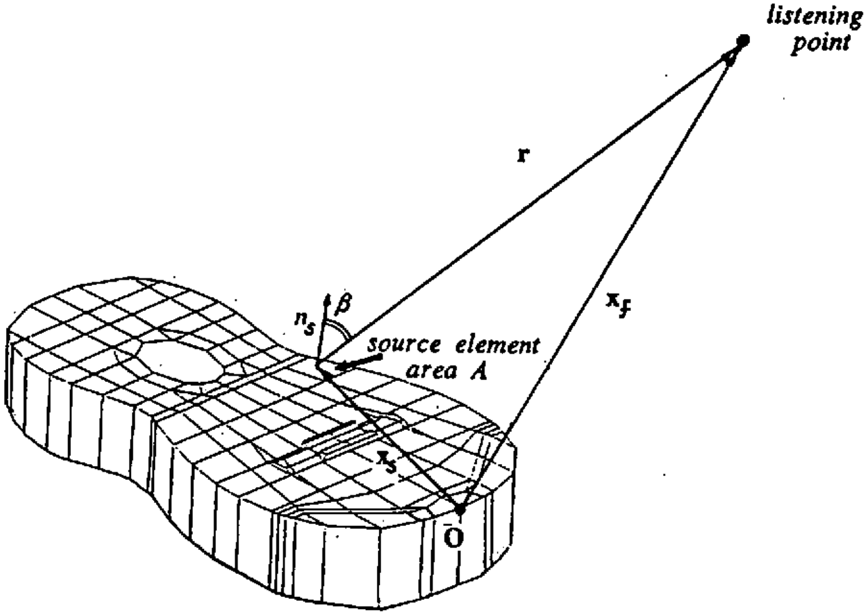

Figure 2. Geometry associated with the boundary element method.

$\mathbf{r} = (\mathbf{x}_f - \mathbf{x}_s)$, and $G(\mathbf{r})$ is the free-space Green's function. The value of ε depends on the position of f, and it is sufficient to note here that $\varepsilon = 1$ for regions exterior to the surface, and that $\varepsilon = \frac{1}{2}$ for points on the vibrating surface. Other terms are defined with reference to Figure 2, namely n_s is the outward unit vector normal to the surface, $\partial G / \partial n_s = G(ik + 1/r)\cos\beta$, and $\partial \mathbf{p} / \partial n_s = \nabla \mathbf{p} \cdot n = -i\omega\rho v_n$ where v_n is the surface normal velocity.

The second term of the HIE basically represents the acoustic pressure due to a point source located at $\mathbf{x}_s$. The first term is more difficult to interpret. It can be thought of as the influence of the rest of the radiating surface on the simple-source terms. The equation is not intrinsically difficult to solve. Information about the surface velocities is obtained from FEA results. The listening point is first placed on the surface at each element in turn, summing the HIE over all elements. This yields values for the surface pressures. Once the values for $\mathbf{p}(\mathbf{x}_s)$ are known, the HIE can then be evaluated at exterior points. There are, of course, numerical problems associated with these calculations. For example, when the listening point coincides with the source point the equation becomes singular. There are also problems with the determination of ε at edges or at surface points of high curvature. Procedures for dealing with these problems, plus an extensive coverage of the method as a whole, are given by Brooke[8].

4. RESULTS AND DISCUSSION

Figure 3 shows $|\chi(\omega)|$ and $|p(\omega)|$ of the system when excited by a harmonic force applied at a typical playing position on the first string (fundamental 330 Hz). Because we are looking into the instrument through the string, we see not only resonances of the body but also of the string itself. The three tall peaks are basically "string resonances"; the remaining, smaller peaks, with much lower Q-factors, are "body resonances". However, these two systems are coupled and exhibit phenomena such as mode splitting when the frequencies of string and body modes coincide. Whilst the two curves are similar in form, note that body modes which are antisymmetric about the centre-line of the instrument do not radiate on the axis of the guitar. If the complex pressure is Fourier transformed, the impulse response of the system can be determined. This time domain signal can be regarded as the equivalent of a note produced by a very short-duration pluck. Although longer pluck interactions can be included, preliminary investigations have shown that the plucking mechanism is modelled reasonably accurately by a narrow spike, rather than a step function as might be expected. These modelled tones display all the characteristics of a real guitar note, including inharmonicity of the string resonances (due to their coupling to the body) and the short-lived anharmonic components which result from impulsive excitation of the body modes. The latter are clearly audible in the early part of the sound, and psychoacoustical experiments by Brooke[8] have shown that they are more important than high frequency information for distinguishing between individual instruments. When one considers the short duration of many notes played in a musical context, one appreciates the importance of these transient phenomena.

Figure 4 shows radiation fields of various modes of the guitar. Normally polar responses like these are calculated at the resonance frequency of each mode. Such plots show that most modes have, broadly speaking, either "monopole" or "dipole" radiation fields. Whilst the radiation of these modes at resonance is of interest, the majority of energy conveyed from the string to the body is usually well above the resonance frequencies of the modes shown. In Figure 4, therefore, we have displayed the radiation from the instrument due to excitation by a string resonance at 990 Hz. Although the resonance frequency of the T(4,2) mode is near to 990 Hz, and therefore might be expected to be the primary radiator of energy at this frequency, closer examination shows that a number of other modes are more efficient radiators. Similar results are obtained at other frequencies.

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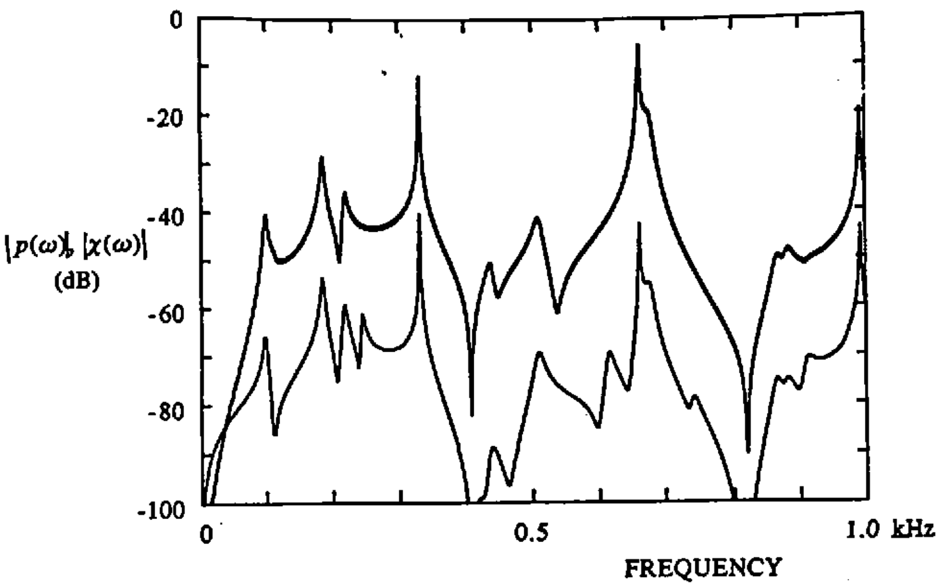

Figure 3. Calculations of the amplitudes of transfer admittance $\chi(\omega)$ (thin line) and radiated sound pressure $p(\omega)$ (thick line). The latter is calculated one metre in front of the centre of the bridge.

One very important conclusion, therefore, is that much of the radiated sound from higher string resonances is radiated through these low-order modes, even though they are excited well above resonance. The reason for this is simple. Modes such as the T(1,1) are readily excited by the strings and, in turn, couple strongly to the surrounding fluid. When excited well above resonance, they are in a mass-controlled region, and, for maximum efficiency, it is important that the maker should ensure that they have a low effective mass. Makers deliberately select materials for soundboards which have high stiffness and low density. By ensuring high stiffness, the plate can be worked as thin as possible to achieve a low effective mass whilst still retaining reasonably high mode frequencies. What is less clear at the moment is how the maker can modify the structure to aid coupling of the strings to modes such as the T(1,2) and T(3,1), which also potentially have strong "monopole" radiation fields. Our FEA work and practical studies have shown that the positions of the nodes and antinodes of these modes are extremely sensitive to construction, particularly the design of the bridge and the dimensions the cross-grain struts. One of the longer term aims of this work is to understand how such modes can be modified in order to maximise the response of the instrument.

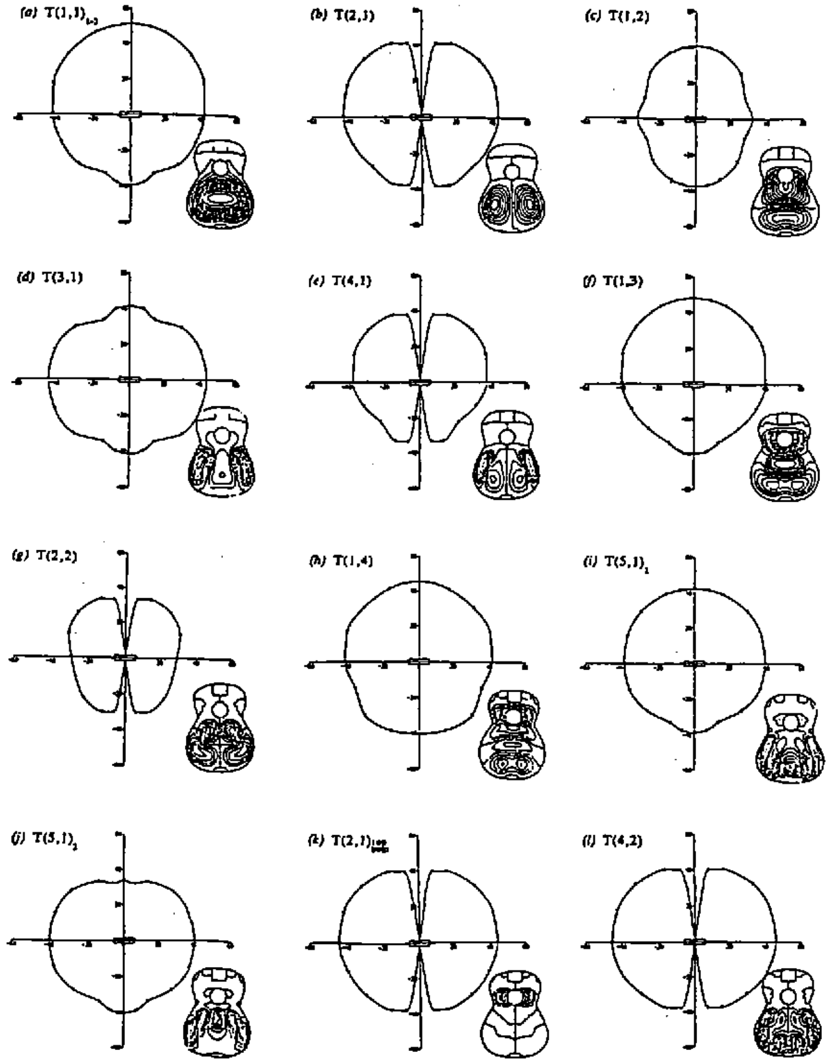

Figure 4. Polar plots of the sound pressure radiated by guitar modes coupled to a 990 Hz string mode. The same excitation force is used in each figure. Resonance frequencies of the modes range from about 100 Hz to 1 kHz.

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