Proceedings of The Institute of Acoustics

A NUMERICAL APPROACH FOR ROUGH SURFACE SCATTERING

B.J. Kachoyan and C. Macaskill

Dept. of Applied Mathematics, University of Sydney, Sydney, N.S.W. 2006, Australia.

INTRODUCTION

Acoustic scattering from the ocean surface is a problem of considerable interest and complexity. The surface wave amplitudes clearly require a statistical description and hence there are close analogies with problems such as optical reflection from random rough surfaces. In addition, however, the sea surface varies in time and it is also necessary to consider a wide range of surface wavelengths, from low-frequency swell to high-frequency capillary waves.

The Kirchhoff approximation has long been used for such problems and has on the one hand, the virtue of relative simplicity but, on the other, the deficiency of an ill-defined range of applicability. Moreover, there are cases where the approximation always breaks down (large surface amplitude/wavelength ratios, incident waves with small angle of incidence) and then one must use more exact theories that can take account of multiple interactions of the reflected acoustic wave with the sea-surface, and attendant large fluctuations in the acoustic pressure field. (It may be noted here that the presence of large pressure fluctuations does not necessarily imply that the Kirchhoff approximation is invalid).

It is a straightforward matter to formulate the reflection problem exactly as an integral equation on the surface, and indeed one can go further and work out, under certain approximations, integral equations for the various moments of the scattered field (e.g. Ito [1]). Given an arbitrary rough surface, however, it is not so straightforward to solve the integral equation for the pressure. For the special case of a regular sinusoidal surface (spatially one-dimensional and time-independent) Holford [2] has derived an eigenfunction expansion procedure and McCammon and McDaniel [3] have used this same technique to obtain reliable solutions to the basic integral equation for a wide range of parameters. This method uses the periodicity of the surface to obtain an integral equation over only one period of the sinusoid, thus allowing convenient numerical evaluation. For more general aperiodic surfaces, this is not possible, but one can exploit the local nature of the fluctuations, particularly close to the surface. Since constructive interference generates intensity peaks, contributions from distant, and hence uncorrelated parts of the surface, tend to be less significant than they are in problems involving regular surfaces. This property makes it possible to solve reflection problems using only a fairly limited portion of the surface, even for plane wave illumination. In this paper we consider the class of time-independent, one-dimensional, randomly modulated surfaces, where the amplitude function obeys Gaussian statistics. For the sea-surface Gaussianity is a reasonable approximation (see, e.g. Phillips [4]). The time-independence is clearly unrealistic, but the present model extends trivially to the time varying case, so long as the time taken for the acoustic wave to propagate is small compared to the time scale of the surface waves. This requirement is generally satisfied in practice and so time appears only parametrically in the wave equation. The current method involves inverting an NxN matrix, where typically N must be between 64 and 256, for adequate resolution in practical situations. Thus extension to the 2-D spatial problem is not trivial, although increased localization effects may help.

The overall approach envisaged for the acoustic wave-field is to solve for a number of realizations of the rough surface and then to average over these realizations. Such Monte Carlo methods have been used successfully for may types of problem, in particular, wave propagation in extended random media (see, e.g. Macaskill and Ewart [5]). In the present preliminary work, we concentrate on the basic problem of evaluating the reflected acoustic field for a single realization of the rough surface.

Direct collocation of the integral equation is not a useful approach, due to the very high resolution required in order to deal with high frequency acoustic waves. This problem is overcome in the present paper by transforming to the spectral domain. The method is essentially a re-working of the Holford [2] approach in terms of discrete Fourier transforms. The method appears quite general, and in fact may have applications for other types of integral equations, although this possibility has not been explored.

The body of the paper describes, in some detail, the numerical technique used, emphasizing the setting-up of the matrix equation that is inverted to find the Fourier transform of the pressure gradient on the rough surface. From these values the pressure at any point in the medium can be found, and a technique is given for doing this. To test the numerical method, the sinusoidal surface is treated first, and satisfactory agreement with previous results of McCammon and McDaniel [3] is obtained. Results for a particular realization of a surface with a Gaussian correlation function are then presented, and compared with Kirchhoff results for a variety of incident angles and surface amplitudes. The effects of multiple scattering at the surface are clearly apparent in some of these results and shadowing is properly treated.

FORMULATION AND NUMERICAL SOLUTION OF THE INTEGRAL EQUATION

We non-dimensionalize the analysis by scaling the horizontal coordinate (range) by L, the vertical coordinate (depth) by kL^2 and the surface by h. L is the correlation length and h^2 the variance of a surface realization (which is taken to have zero mean) and k is the wavenumber of the incident wave.

The pressure field away from a surface $z = \frac{h}{kL^2}\zeta(x)$ can then be written as

$$p(x,z) = p_{inc}(x,z) + p_s(x,z) \ , \tag{1}$$

where the incoming wave p_{inc} is given by

$$p_{inc} = e^{i\hat{k}(\alpha_0 x - \hat{k}\gamma_0 z)} \tag{2}$$

with $\hat{k} = kL$, $\alpha_0 = \cos\theta$, $\gamma_0 = \sin\theta$, θ being the incident angle measured from the horizontal.

The scattered field is (Meecham [5])

$$p_s = \frac{\hat{k}}{4}\int_{-\infty}^{\infty} \Psi(x')H_0^{(1)}\left(\hat{k}|\underset{\sim}{r}-\underset{\sim}{r}'|\right) dx' \ , \tag{3}$$

where $\underset{\sim}{r} = (x,\hat{k}z)$, $\underset{\sim}{r}' = (x',\hat{h}\zeta(x'))$, $\hat{h} = h/L$

and $$\Psi(x) = \frac{i}{\hat{k}}\left(-\frac{1}{\hat{k}}\frac{\partial p}{\partial z} + \hat{h}\zeta'(x)\frac{\partial p}{\partial x}\right)_{z = \frac{h\zeta(x)}{kL^2}} .$$

It can also be shown (Meecham [5], Holford [2]) that $\Psi(x)$ itself satisfies a Fredholm second kind integral equation on the surface, namely

$$\Psi(x) + \hat{k}\hat{h}\int_{-\infty}^{\infty}\Psi(x')K(x',x)dx' = 2\Psi_{inc}(x) \ , \tag{4}$$

where $$\Psi_{inc} = -(\gamma_0 + \alpha_0\hat{h}\zeta'(x))e^{i\hat{k}(x\alpha_0 - \hat{h}\zeta(x)\gamma_0)}$$

and $$K(x',x) = \frac{i}{2}\frac{H_1^{(1)}(\hat{k}\rho)}{\rho}[\zeta(x') - \zeta(x) - (x'-x)\zeta'(x)] \tag{5}$$

with $$\rho^2 = (x'-x)^2 + \hat{h}^2[\zeta(x') - \zeta(x)]^2 .$$

Note that the integral equation is singular due to the limits of integration but the kernel itself is not, having the behaviour

$$K(x',x) \sim \frac{1}{2\pi\hat{k}}\frac{\zeta''(x)}{1+(\hat{h}\zeta'(x))^2} \text{ as } \rho \to 0 .$$

The choice of collocation points in any discretization of the kernel is thus not crucial.

For convenience we introduce a new unknown $\bar{\Psi} = e^{-i\hat{k}\alpha_0 x}\Psi$ where $\bar{\Psi}$ can be interpreted as a slowly varying envelope. The form of (4) is unchanged but the kernel is now $\bar{K} = Ke^{i\hat{k}\alpha_0(x'-x)}$. Similarly (3) becomes

$$p_s(\underset{\sim}{r}) = \frac{\hat{k}}{4}\int_{-\infty}^{\infty} e^{i\hat{k}\alpha_0 x'}\bar{\Psi}(x)H_0^{(1)}\left(\hat{k}|\underset{\sim}{r}-\underset{\sim}{r}'|\right)dx'. \qquad (6)$$

For simplicity we henceforth drop the overbar.

To solve (4) for $\Psi(x)$ we generalize the method of Holford [2] to deal with non-periodic surfaces. More specifically, a method is needed which deals with arbitrary surfaces which in addition may be known only at a discrete set of points. To model a sea surface, for example, where the spectrum is given, FFT techniques are required to generate a realization of the "surface". Accordingly, we discretize and approximate (4) by letting (without loss of generality since we are free to choose the horizontal origin)

$$x = x_j = j\Delta x \qquad j = 0,1,\ldots,N-1$$
$$x' = x'_\ell = \ell\Delta x \qquad \ell = 0,1,\ldots,N-1 .$$

Then

$$\Psi_j + \Delta x\sum_{\ell=0}^{N-1} K_{\ell j}\Psi_\ell = 2\Psi_{inc_j} \qquad (7)$$

where $\Psi_j = \Psi(x_j)$ and $K_{\ell j} = K(x'_\ell, x_j)$

Defining a discrete transform pair in the normal way,

$$\hat{\Psi}_n = \frac{1}{N}\sum_{j=0}^{N-1}\Psi_j e^{-2\pi ijn/N} \qquad (8)$$

$$\Psi_j = \sum_{n=0}^{N-1}\hat{\Psi}_n e^{2\pi inj/N}, \qquad (9)$$

and applying the operator $\frac{1}{N}\sum_{j=0}^{N-1} e^{-2\pi ijn/N}$ to (7) we obtain

$$\hat{\Psi}_n + N\Delta x\sum_{m=0}^{N-1}\hat{\Psi}_m Q_{mn} = 2\hat{\Psi}_{inc_n} \qquad (10)$$

where $$Q_{mn} = \frac{1}{N^2} \sum_{j=0}^{N-1} \sum_{\ell=0}^{N-1} K_{\ell j} e^{-2\pi i j n/N} \, e^{2\pi i \ell m/N} \qquad (11)$$

The matrix equation (10) may be inverted to find the $\hat{\Psi}_p$ with the matrix elements calculated using a 2-D Fast Fourier Transform. As in the sinusoidal case (Holford [2], Uretsky [7]), the pressure field may be calculated directly from the transformed normal derivative $\hat{\Psi}$ to give

$$p_s(x_j,z) = \sum_{r=0}^{N-1} R_r e^{i\hat{k}\alpha_0 x_j} \, e^{i\hat{k}^2\gamma_r z} \, e^{2\pi i r j/N} \qquad (12)$$

where $$\gamma_r^2 = 1 - \left(\frac{2\pi r}{\hat{k}N\Delta x} + \alpha_0\right)^2 \quad \text{for} \quad r = 0,1,\ldots,\frac{N}{2} - 1 ,$$

$$1 - \left(\frac{2\pi(r-N)}{\hat{k}N\Delta x} + \alpha_0\right)^2 \quad \text{for} \quad r = \frac{N}{2},\ldots,N-1 . \qquad (13)$$

The reflection coefficients, R_r, are given by

$$R_r = \frac{1}{N^2} \sum_{m=0}^{N-1} \hat{\Psi}_m \frac{1}{2\gamma_r} C_{r-m}(\hat{k}\hat{h}\gamma_r) \qquad (14)$$

where $$C_r(\tau) = \sum_{n=0}^{N-1} e^{-i\tau\zeta(x_n)} \, e^{-2\pi i r n/N} .$$

A more detailed description of this analysis will be given elsewhere.

Specialization of these results to a truly periodic surface, for example a sinusoid of wavelength Λ, is made by letting $L = \Lambda$, $N\Delta x = \Lambda$ and $x'_\ell = \ell\Delta x + q\Lambda$ where $q = -NQ,\ldots,0,\ldots,NQ$, where the surface is implicitly truncated to (2NQ+1) periods. The matrix equation (7) remains unchanged in form but $K_{\ell j}$ is now defined as $\sum_{q=-NQ}^{NQ} K(x'_\ell,x_j)$. The formulation is then equivalent to those of Holford [2] and McCammon and McDaniel [3].

RESULTS AND DISCUSSION

It is well known that acoustic energy is scattered by a sinusoidal surface in certain discrete non-specular directions given by the Bragg angles. In the present formulation, this is equivalent to exciting only some of the real reflection coefficients R_r in (12), the rest being zero or negligible. For a general surface, however, we do not necessarily expect the scattered energy to be propagated in only a few directions. All modes may be excited in an essentially continuous distribution of energy with angle with perhaps a "smearing" around some preferred directions. The reflection coefficients, however, must satisfy energy conservation (Uretsky [7]), namely that the reflected energy E must satisfy

$$E \equiv \mathrm{Re}\left\{\sum_r |R_r|^2 \frac{\gamma_r}{\gamma_0}\right\} = 1 ,$$

bearing in mind that this should not be the sole criterion for the convergence of any numerical scheme.

Table 1 shows a comparison of some E values of McCammon and McDaniel [3, Table 1] with the present values for a sinusoidal surface with 16 points per wavelength (N = 16). Unfortunately, McCammon and McDaniel gave no indication of how many kernel terms (NQ in the previous section) they used - here NQ = 150 for the figures in Table 1. Moreover, they calculated the reflection coefficients analytically from the values for the transformed normal derivative $\hat{\Psi}_p$ so a direct comparison is of limited usefulness. The curves of reflection coefficient obtained, however, were essentially identical to those of McCammon and McDaniel [3], so that agreement with the sinusoidal case can be claimed with some confidence.

θ (deg)	Results of [3]		Present Results
	FS	KA	FS
15	1.001	2.288	1.016
45	0.999	0.817	1.006
75	1.0001	0.802	1.0003

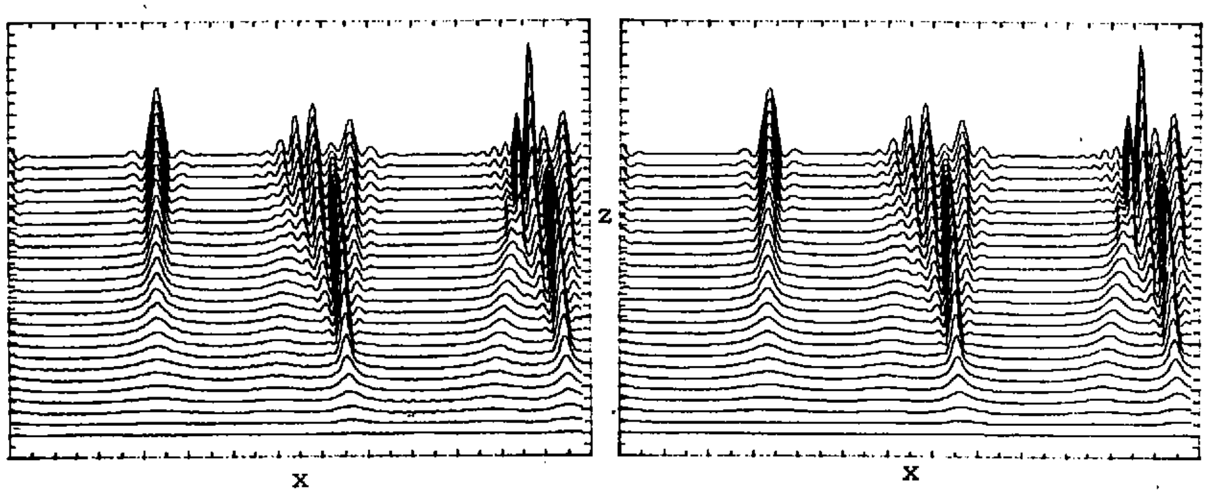
Table 1: Reflected energies E of [3] compared with the present results for a sinusoidal surface with $\hat{h}$ = .1588 and $\hat{k}$ = 12.4. FS = full solution, KA = Kirchhoff approximation.

It is clear from the value of NQ used in Table 1 that a very long surface (about three hundred surface wavelengths) is needed to adequately predict local intensity patterns. This is due to the periodic nature of the surface with disturbances caused by individual surface features constructively interfering. We expect that for an aperiodic surface, effects from ranges of, say, several correlation lengths will interfere destructively and thus not contribute greatly to the near field. This reduction in the amount of surface required is of great benefit since (10) indicates we must invert a matrix equal in size to the number of points taken on the surface.

The major results shown in this section used a 256 point Gaussianly distributed surface with a Gaussian correlation function and 32

Fig. 1: The surface used in Figs 2, 3 with $h = \frac{1}{2}$.

points per correlation length (see Fig. 1). Different correlation functions will be examined in future work. As a preliminary investigation, we compare the present numerical results with the Kirchhoff approximation (KA), made by neglecting the kernel term Q_{mn} in (10) (so avoiding a matrix inversion). The KA is generally considered to be a high-frequency, small slope approximation, though exact interpretations vary (see, for example, Wirgin [8]). It would appear from the scalings of the previous section, that the KA should be valid for $\hat{k} \gg 1$ and $\hat{h} \ll 1$. Figures 2,3 are waterfall plots of intensity for the surface of Fig. 1 and various depths such that $z > \zeta_{max}$. Fig. 2 shows a comparison of the full numerical solution and the KA for $\hat{k} = 50$, $\hat{h} = 0.1$ and $\theta = 90^\circ$ (normal incidence). The agreement is excellent as is the energy balance in each case. Note that $\hat{k}\hat{h} = kh$ is not small. It can be shown in the limits $\hat{k} \gg 1$ and



E = 1.00000　　　　E = 1.00001

Fig. 2: Plots of intensity versus range for successive equally spaced values of depths ($\Delta z = .002$). The graph on the left shows the full solution and on the right KA. The parameter values used are $\hat{h} = .1$, $\hat{k} = 50$, $\theta = 90^\circ$ and $z_{min} = .002$.

$\hat{h} << 1$, the problem of surface scatter is equivalent to the problem of transmission through a phase-changing screen which is known to exhibit focusing effects precisely as in Fig. 2 (Mercier [9]). Equivalence of the two problems will be demonstrated elsewhere.

In Fig. 3(a), (b) we relax the conditions on $\hat{k},\hat{h}$ giving $\hat{k} = 15$, $\hat{h} = {}^1/_3$ and $\theta = 90^\circ$ and 45° respectively. In this parameter regime we have lost much of the focusing seen in Fig. 2. At this greater surface roughness, on the other hand, multiple interactions with the surface give rise to intensity fluctuations nearer the surface. Energy conservation indicates that the KA is still quite good for normal incidence (Fig. 3a) but, as expected, is a poorer approximation for oblique incidence (Fig. 3b), although the intensity plots are remarkably similar. We expect any method of solution, in which a fixed length of surface is considered, to deteriorate as we decrease the angle of incidence since specular and near-specular reflections from other parts of the surface contribute more and more to the scattered intensity in the field of view. It is not clear as yet why the plots at non-normal incidence are "smoother" than those at normal incidence.

In conclusion it has been demonstrated that numerical solutions can be obtained for the fully elliptic problem of scattering from a randomly modulated surface. It is interesting to note that the Kirchhoff approximation appears to give more accurate answers for the surface considered here, than is the case for a sinusoidal surface.

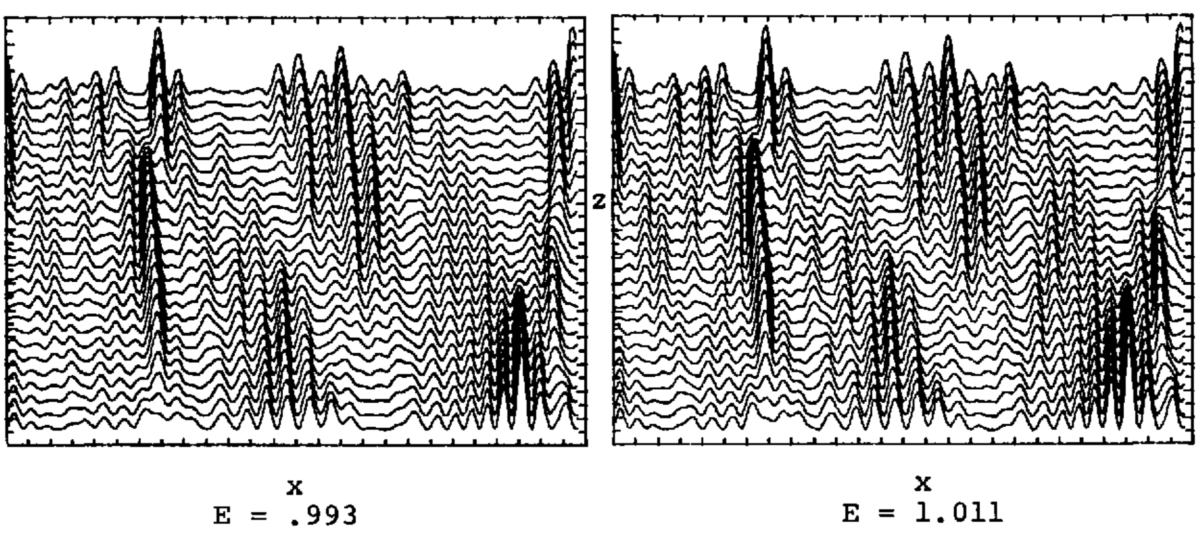


Fig. 3a.

A NUMERICAL APPROACH FOR ROUGH SCATTERING

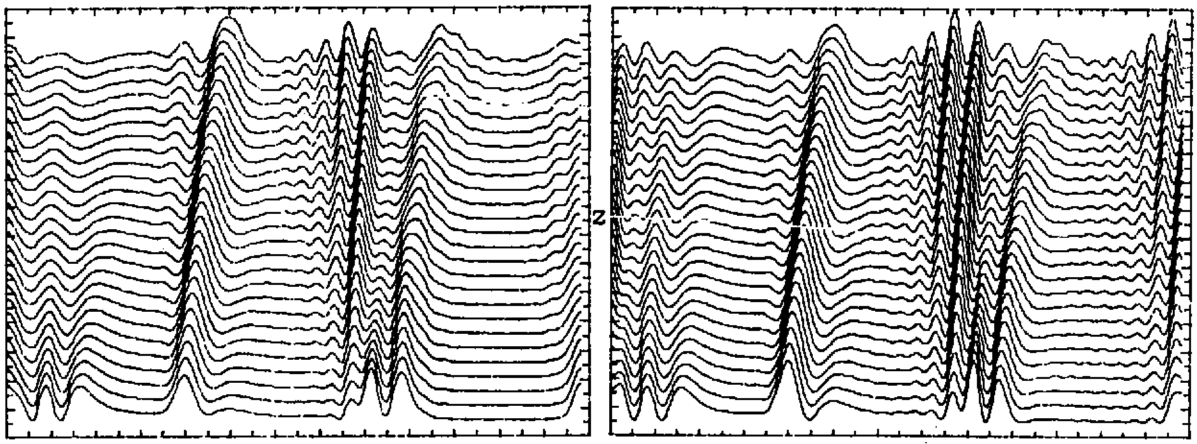


Fig. 3: As for Fig. 2 but with $\hat{h} = {}^1/_3$, $\hat{k} = 15$, $z_{min} = .0267$ and (a) $\theta = 90^\circ$, (b) $\theta = 45^\circ$.

This work was partly supported by the Royal Australian Naval Research Laboratory of the Defence Science and Technology Organization.

REFERENCES

[1] S. Ito, 'Analysis of Scalar wave scattering from slightly rough surfaces: A multiple scattering theory', Radio Sci., Vol. 20, 1-12, (1985).

[2] R.L. Holford, 'Scattering of sound waves at a periodic, pressure-release surface: An exact solution', J. Acoust. Soc. Am., Vol. 70, no.4, 1116-1128, (1981).

[3] D.F. McCammon and S.T. McDaniel, 'Application of a new theoretical treatment to an old problem; sinusoidal pressure release boundary scattering', J. Acoust. Soc. Am. Vol. 78, no.1, 149-156, (1985).

[4] O.M. Phillips, 'The Dynamics of the Upper Ocean', 2nd ed., Cambridge University Press, 1977.

[5] C. Macaskill and T.E. Ewart, 'Computer Simulation of two-dimensional random wave propagation', IMA J. App. Math., Vol. 33, 1-15, (1984).

[6] W.C. Meecham, 'On the use of the Kirchhoff approximation for the solution of reflection problems', J. Rat. Mech. Anal., Vol. 5, no.2, (1956).

[7] J.L. Uretsky, 'The scattering of plane waves from periodic surfaces', Ann. Phys., Vol. 33, 400-427, (1965).

[8] A. Wirgin, 'Scattering from sinusoidal gratings: an evaluation of the Kirchhoff approximation', J. Opt. Soc. Am., Vol. 73, no.8, (1983).

[9] R.P. Mercier, 'Diffraction by a screen causing large random phase fluctuations', Proc. Camb. Phil. Soc., Vol. 58, 382-400, (1962).