24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

AN IMPROVED INVERSE-PHASED ARRAY METHOD FOR ESTIMATING FREE-FIELD ENGINE CORE NOISE SPECTRA FROM MEASUREMENTS IN REVERBERANT TEST CELLS

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Core noise is a significant contributor to aircraft engine noise in the rear arc, especially at low engine powers including the Approach condition. The significance of this source is likely to become even greater in future engines with low-emissions combustors. In order to extract core noise from measured engine noise spectra, phased array data can be acquired on outdoor, static engine test beds and processed with an inverse method, which separates out the contribution of the individual, directional broadband noise sources as a function of far-field angle and frequency. One such method, called AFINDS, requires a parameterized model of the far-field coherence for each axially distributed jet exhaust noise source and for each transversely distributed 'source' at engine duct inlet/exhaust planes, that is the fan and core noise. Using full-scale engine data acquired on an outdoor test bed, AFINDS has yielded plausible source PWL spectra and directivity shapes for each source type, which do not change dramatically with frequency, as expected for broadband sources. One outstanding problem with the AFINDS method is the coincidence of the core noise source 'position' at the engine nozzle and the initial jet mixing noise region. To overcome this, AFINDS has been extended to incorporate a Wiener-Hopf solution for the coherence and directivity of the core noise radiation, which enables a more robust jet/core source separation to be achieved. Another challenge is related the need to acquire data on the noise of new combustor designs in a cost-efficient manner from *indoor measurements taken in reverberant* engine test cells. AFINDS has been recently developed to allow for the presence of a reverberant field and results for a core noise PWL spectrum from an indoor and outdoor engine test is shown to compare favourably.
Keywords: Core and combustion noise, phased arrays

1. Introduction

Core noise is a significant contributor to aircraft engine noise in the rear arc, especially at low engine powers including the Approach condition. The significance of this source is likely to become even greater in future engines with low-emissions combustors.

In order to extract core noise from measured engine noise spectra, phased array data can be acquired under free-field conditions on outdoor, static engine test beds and processed with an inverse method, which separates out the contribution of the individual, directional broadband noise sources as a function of far-field angle and frequency. One such method, called AFINDS, requires a parameterized model of the far-field coherence for each axially distributed jet exhaust noise source and for each transversely distributed 'source' at engine duct inlet/exhaust planes, that is the fan and core noise.

Using full-scale engine data acquired on an outdoor test bed, AFINDS has yielded plausible source PWL spectra and directivity shapes for each source type, which do not change dramatically with frequency, as expected for broadband sources[1].

One outstanding problem with the AFINDS method is the coincidence of the core noise source ‘position’ at or near the engine nozzle and the initial jet mixing noise region. To overcome this, AFINDS has been extended (AFINDS-GX) to incorporate a Wiener-Hopf solution [2] for the coherence and directivity of the core noise radiation, which enables a more robust jet/core source separation to be achieved.

Another challenge is related to the need to acquire data on the noise of new engine combustor designs in a cost-efficient manner from *indoor measurements taken in reverberant* engine test cells. AFINDS has been recently developed to allow for the presence of a reverberant field and results from indoor and outdoor engine tests are shown to compare favourably. Another inverse method, SODIX, has also made good progress in this direction[3].

2. Summary of basic AFINDS method

AFINDS is non-linear least squares (NLS) inverse method, which has been developed from earlier work [4] and which requires a model of components of the far-field cross-spectral matrix (CSM) for each source type. Each source model is characterised by a small number of parameters: the jet noise source has a relative intensity, an axial origin, a centroid position, a shape parameter and four parameters to model the directivity making eight in all. Likewise the duct termination plane source models described below have a similar count with the duct radius replacing the centroid parameter. For a typical short cowl engine there are three internal sources and one or two jet noise sources, so the total number of ‘unknown’ parameters is 40 to 50.

Based on past experience, provided some of the key parameters - in particular the source positions (which must be approximately known) - are given sensible initial values and are suitably constrained, a non-linear least squares process such as the Matlab function 'nlinfit' is able to converge to the global minimum in about five or six iterations. The main advantages of the process is the relatively small number of unknowns and a jet noise source model that can be used in the geometric near-field.

The original application of NLS to the source breakdown problem, combined with far-field coherence source models, is outlined in Tester & Fisher[4] and Strange *et al.* [5] but it has been extended to include source directivity and improved internal source coherence models. The latter are required in place of the point source models when the microphone angles are significantly different from 90˚.

The NLS function $E(\omega)$ minimised is the modulus squared of the difference between the model of the far-field CSD, $C_{pp}(\omega,\theta_i,\theta_j)$, and the measured CSD, $\tilde{C}_{pp}(\omega,\theta_i,\theta_j)$, summed over all the microphone positions in the array θ_j and all the selected microphone reference angles, θ_i, at each frequency, ω. To this we add the modulus squared of the difference between the model PSD and the measured values. Here θ_j is the polar angle of each microphone relative to the engine intake centreline.

$$E(\omega) = \sum_{i,j} \| C_{pp}(\omega,\theta_i,\theta_j) - \tilde{C}_{pp}(\omega,\theta_i,\theta_j) \|^2 + \sum_{i} \| S_{pp}(\omega,\theta_i) - \tilde{S}_{pp}(\omega,\theta_i) \|^2 \tag{1}$$

Here the other coordinates of the microphone position, the polar radius and azimuthal angle (R_i,ϕ_i), have been omitted for clarity. The CSD is assumed to be equal to the sum of the source CSD’s of the individual engine sources, $C_{pp}^{(n)}(\omega,\theta_i,\theta_j)$, for example the inlet fan, the aft fan, the core and the jet mixing noise ($n = 1,4$), which is valid provided the sources are mutually incoherent. This is a reasonable assumption for broadband noise sources. It is not always true for tonal sources (e.g. the inlet fan tone may be coherent with aft fan) but this work is largely aimed at broadband noise sources. Thus:

$$C_{pp}(\omega,\theta_i,\theta_j) = \sum_n C_{pp}^{(n)}(\omega,\theta_i,\theta_j) \tag{2}$$

It follows that the spectral densities ($\theta_i = \theta_j$) are also added incoherently.

The source breakdown problem to be solved is therefore: to determine the far-field PSD of each source, $S_{pp}^{(n)}(\omega,\theta_i)$ given the measured CSD (which includes the measured PSD), by using a model for each individual source and minimising the error with a non-linear least squares algorithm. We can express the CSD in terms of the PSD and the far-field source coherence, $\gamma_{pp}^{(n)}(\omega,\theta_i,\theta_j)$, that is by definition:

$$C_{pp}^{(n)}(\omega,\theta_i,\theta_j) = \sqrt{S_{pp}^{(n)}(\omega,\theta_i)S_{pp}^{(n)}(\omega,\theta_j)}\gamma^{(n)}(\omega,\theta_i,\theta_j) \tag{3}$$

A directivity function is defined for each individual source, $D_{pp}^{(n)}(\omega,\theta_i)$, such that $D_{pp}^{(n)}(\omega,\theta_i = 90°) = 1$, which is meant to represent the combined effect of the different types of directivity, namely multipole, convective amplification and flow-acoustic interactions, that is

$$S_{pp}^{(n)}(\omega,\theta_i) = A^{(n)}(\omega)D_{pp}^{(n)}(\omega,\theta_i)\Big/\left(R_i^{(n)}\right)^2 \tag{3}$$

Here $A^{(n)}(\omega)$ is the source intensity and the directivity $D_{pp}^{(n)}(\omega,\theta_i)$ is expressed as a function of the directivity angle θ_i, which is the angle between the (upstream) engine centreline and the line joining the source position on the centreline and the microphone position, with $R_i^{(n)}$ being the distance between the two.

As the multipole, convective amplification and flow-acoustic interactions are not known or fully understood for engine noise sources, this directivity function is treated as an unknown and is evaluated as part of the NLS process, as described below. Substituting Eq. (3) into Eq. (2) and summing over all the sources gives:

$$C_{pp}(\omega,\theta_i,\theta_j) = \sum_n A^{(n)}(\omega)\sqrt{D^{(n)}(\omega,\theta_i)D^{(n)}(\omega,\theta_j)}\Big/\left(R_i^{(n)}R_j^{(n)}\right)\gamma^{(n)}(\omega,\theta_i,\theta_j) \tag{4}$$

Eq. (4) is then substituted into Eq. (1) with suitable normalisation applied to ensure approximately equal weighting between the two sums.

The NLS process thus requires a model of the far-field coherence of each source, the measured CSD matrix (CSM) including the measured PSD, with appropriate initial values of all the model parameters including source directivities. Provided convergence is achieved, the process provides an estimate of all the model source parameters (source intensity, source position etc) along with the directivity of each source.

The modelling of a single stream axial jet noise source distribution was developed by Glegg[6] and is used in the current work. Application to dual-stream model jets or coaxial jet exhaust flow of a turbofan engine normally requires two sources of this type[5] but the engine data used in sections 3 is and 4 have been taken on a long cowl engine and a single jet noise source is sufficient for present purposes, for which its source strength spectral density is given as

$$\hat{Q}_\omega(y) \propto y^{m-1}\exp\left(-my/y_c\right);\ y \ge 0,\ m \ge 2 \tag{5}$$

where y is the axial coordinate. This is normalised to ensure that its integral over $0 \le y \le \infty$ is unity. Here y_c is the jet noise centroid and m is the shape parameter ($m \ge 2$). The corresponding far-field coherence of radiation from this distributed source, between two points on a polar arc at angles θ_i, θ_j is given by:

$$\gamma(\omega, \theta_i, \theta_j) = \{1 + \frac{jky_c}{m}(\cos\theta_i - \cos\theta_j)\}^{-m} \quad (6)$$

where $k = \omega / c_0$ .This assumes that the source is compact but a non-compact model has been developed for special applications[7]. It is noted that the coherence is a *non-linear function* of the source position and the shape parameter, which is one of the reasons we have to adopt a non-linear least squares process to determine their values, along with the source intensity.

However the coherence model given by Eq. (6) is limited by the far-field assumption. Typical measurement distances are of the same order as the length of the noise producing region of the jet plume. To overcome this Pack and Strange[8] proposed using an axial array of point sources, but with a source spectral density variation still given by Eq. (5), spaced typically half a wavelength apart. This allows the field to be computed at any distance from the jet axis, i.e. in the geometric near-field, provided the microphone is in the acoustic far-field.

The effects of 'off-axis sources' on the far-field coherence have been studied previously by Glegg[9] and also Harper-Bourne[10], within the context of a jet noise *volume* source distribution. Here a similar approach is taken to the problem of radiation from a surface source distribution over a duct termination plane, including the effects of the duct itself.

The approximate models are based on simulated phased array cross-spectral data computed with an analytical Wiener-Hopf solution for sound radiation from a semi-infinite duct (with or without an 'exhaust' mean flow) developed by Gabard[11] in the form of a code called GXMunt and have been reported previously[12]. The analytic models are summarised here. Based on a flanged duct approximation the far-field coherence for a statistically uniform compact monopole source distribution was shown to be closely approximated by:

$$\gamma(\omega, \theta_i, \theta_j) = 2J_1(ka(\sin\theta_i - \sin\theta_j)) / ka(\sin\theta_i - \sin\theta_j) \quad (7)$$

where a is the duct radius. Here it is assumed the two far-field points are located at the same azimuthal angle, which is generally true for the linear phased array data used in the present study. This expression is in striking contrast to that of a point source, which has unit coherence, (for a point source at the origin). However, as this approximate result is based on a flanged duct model it necessarily restricts its applicability to the forward hemisphere of the field radiated from the duct.

Another approximation may be appropriate for the rearward hemisphere and that is based on an assumption that the field in the rear arc is the largely the result of scattering of sound from the rim of the duct. This was shown to be[12]:

$$\gamma(\omega, \theta_i, \theta_j) = J_0(ka(\sin\theta_i - \sin\theta_j)) \quad (8)$$

Unfortunately application of the former in the rear arc and the latter in the forward arc of the core noise radiation results in a discontinuity at 90° and can cause problems with the converged solutions.

3. Extensions to AFINDS method to include improved core noise model and measurements in reverberant field

One outstanding problem with the AFINDS method is the coincidence of the core noise 'position' at or near the engine nozzle and the initial jet mixing noise region. To overcome this, AFINDS has been extended to incorporate the full Wiener-Hopf solution for the coherence and directivity of the core noise radiation, which enables a more robust jet/core source separation to be achieved. As noted above, the Wiener-Hopf solution is provided by the code referred to as GXMunt at the ISVR *plus* an assumption about the modal distribution, which will be addressed in the next section. The GXMunt unflanged duct radiation model includes the effects of a single stream jet in terms of refraction and convection by the mean flow and also the diffraction of sound around the nozzle lip. Thus the model requires the jet Mach number and jet static temperature (assumed to be uniform inside the 'plug' jet flow), as well as the duct radius and frequency. This exact numerical GXMunt solution also overcomes the problem of the discontinuity at 90°.

Thus the two expressions used previously for the core noise coherence are replaced by a single expression covering the whole radiation field in the form:

$$\gamma(\omega,\theta_i,\theta_j) = \sum_{m,n} A^{mn} s_p^{mn}(\theta_i)\, s_p^{mn}*(\theta_j) \Big/ \sqrt{S_{pp}(\theta_i)}\sqrt{S_{pp}(\theta_j)} \qquad (9)$$

(omitting the frequency dependence on the RHS), $s_p^{mn}(\theta)$ is the GXMunt far-field complex pressure amplitude radiated at polar angle θ, by unit amplitude in-duct mode *(m,n)* and $S_{pp}(\theta)$ is the PSD directivity given by summing incoherently the GXMunt modal solutions. The modal intensity A^{mn} is an unknown but by comparing the directivity of various modal models with the empirically derived directivity embedded in the SAE ARP876 method, it has been found that an equal energy per mode model is the most appropriate, see Fig. 1. The GXMunt/equal energy per mode predictions vary slowly with frequency but over the frequency range of interest, there is good agreement with the SAE directivity (which is frequency-independent) and hence this model has been adopted for the purposed of the present study.

Using this extended version of AFINDS, called AFINDS-GX, measurements acquired with DLR phased array on the open air test bed at Stennis in 2011 with a BR700 type engine, see Figs. 2 & 3, have been re-analysed to obtain source breakdown results with an improved robustness and speed of analysis, despite the need to compute the GXMunt solutions for each cut-on mode at each frequency; a typical source breakdown at 400 Hz is shown in Fig. 4.

The other issue addressed is the problem of extracting the required information from phased array measurements taken in the reverberant field of an in-door engine test cell. One approach is to image the source and then manipulate the CSM, e.g. by diagonal removal, to minimise the effect of unwanted noise on the CSM.

Here we have taken a different approach and introduced an omni-directional 'virtual' source into the AFINDS model to represent the reverberant field, which, if perfectly diffuse, has the two-point coherence given by:

$$\gamma(\omega,\underline{r}_i,\underline{r}_j) = \sin c(k\left|\underline{r}_i - \underline{r}_j\right|) \qquad (9)$$

where the microphone position is now denoted by vector $\underline{r}_i$ and $k = \omega / c_0$. The intensity of this reverberation 'source' is now another unknown along with the real source intensities, which are all determined by the AFINDS process. Source breakdown results using the DLR phased array in the engine test cell at Dahlewitz are presented and compared with the Stennis results in the next section.

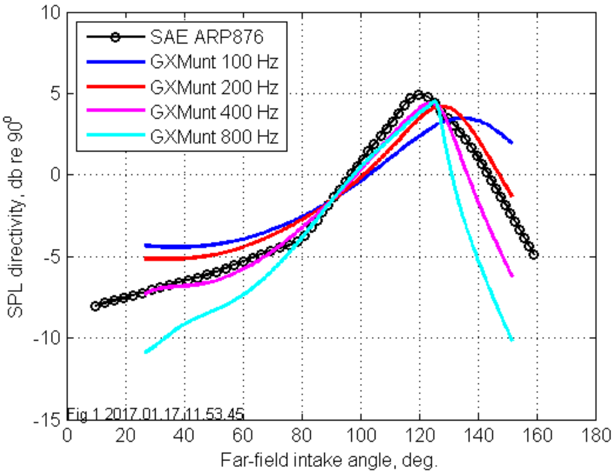

Figure 1: AFINDS-GX ‘equal energy’ core noise source directivities at selected frequencies v. SAE ARP 876

Figure 2 Stennis: BR700 engine and DLR linear phased array.

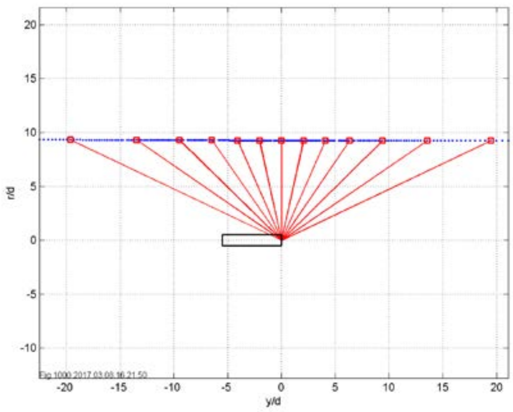

Figure 3 Stennis: DLR linear phased array (blue), 235 microphones, showing selected reference microphones (red).

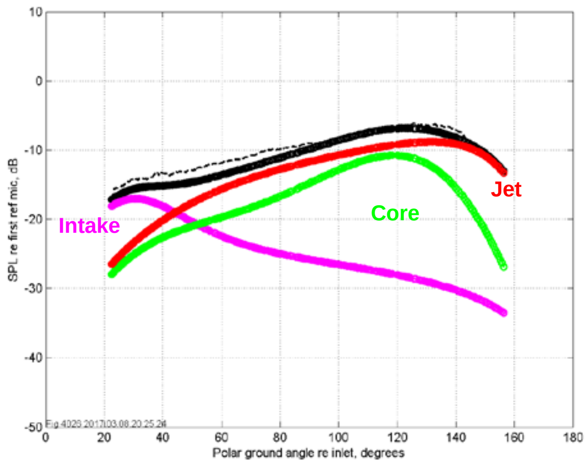

Figure 4 Stennis: measured SPL along phased array and AFINDS source breakdown at 400 Hz

4. Application of AFINDS-GX to engine test cell phased array measurements taken in a reverberant field, compared to free-field

Initial tests with AFINDS-GX on the Stennis and Dahlewitz BR700 data, see Fig. 5, the latter using a sub-array of the DLR linear array as shown in Fig. 6, have been completed at a low power condition. Fig. 7 illustrates the AFINDS-GX fit for the Dahlewitz data at 400Hz and Fig. 8 compares directivity levels for both the Stennis and Dahlewitz data over the range 100-800Hz. Fig. 9 shows the AFINDS spectral breakdown for the Dahlewitz data at 90°, with the reverberant field spectrum 7 to 10 dB above that of the core noise. Finally, the Stennis and Dahlewitz core PWL spectra, computed from the AFINDS-GX breakdown are compared in Fig. 10, agreeing to within 2 dB, except at the lowest frequencies.

Figure 5 Dahlewitz test cell: BR700 series engine with DLR linear array (right corner)

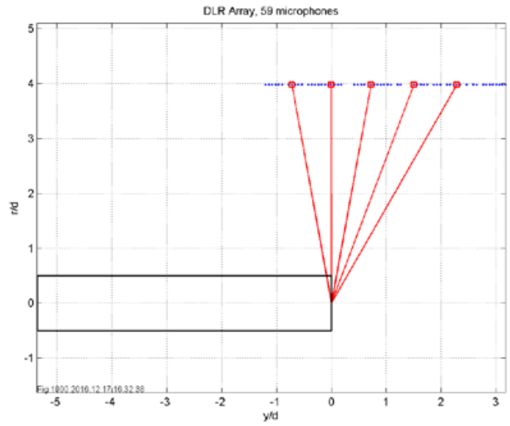

Figure 6 Dahlewitz test cell: sub-array used by AFINDS, showing the five reference microphones.

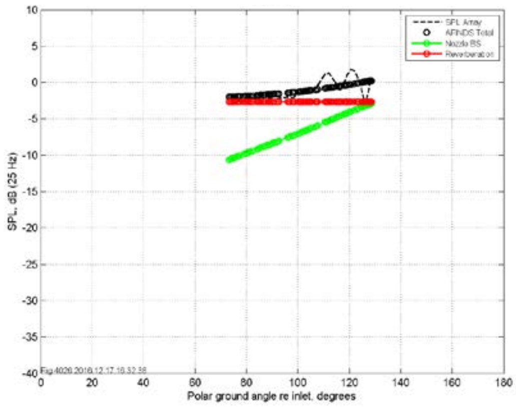

Figure 7 AFINDS-GX NBS core and reverberant source breakdown at 400Hz from Dahlewitz test cell BR700 sub-array data; low power condition.

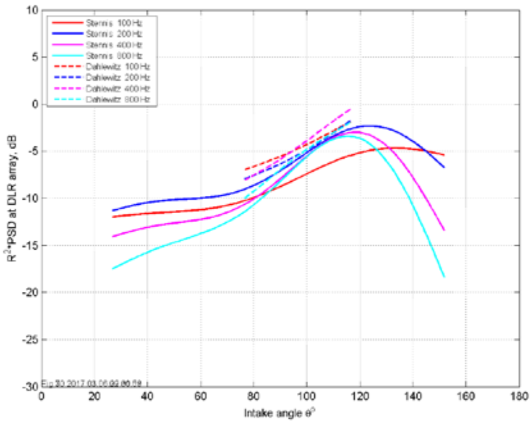

Figure 8 AFINDS-GX core source directivities for Stennis & Dahlewitz BR700 (100, 200, 400 & 800Hz); low power condition.

5. Conclusions

A free-field source breakdown method (AFINDS) has been extended to meet the engine noise challenges of (1) spatial over-lapping of the apparent core noise and jet mixing positions along the centreline of the engine and (2) processing phased array measurements taken in a *reverberant* engine test cell environment. The former has been achieved by utilising a numerical Wiener-Hopf solution for the coherence and directivity of the core noise radiation, which enables a more robust jet/core source separation to be achieved. The latter appears to have been achieved through a novel technique of including the reverberant sound field in the source model, the level of which is determined by AFINDS. Preliminary core noise PWL results at a low engine power condition, from an indoor and an outdoor engine test, are shown to compare favourably.

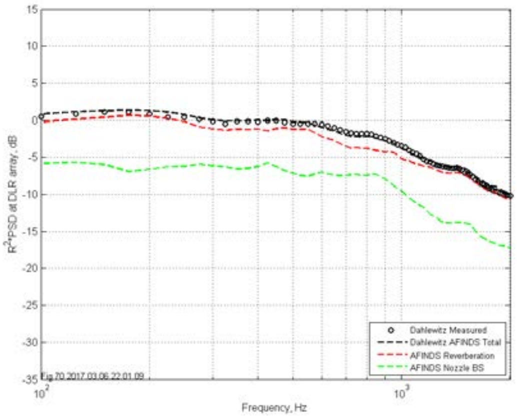

Figure 9 AFINDS-GX SPL breakdown at 90°, core spectrum (green), reverberation spectrum (red); Dahlewitz BR700, low power condition.

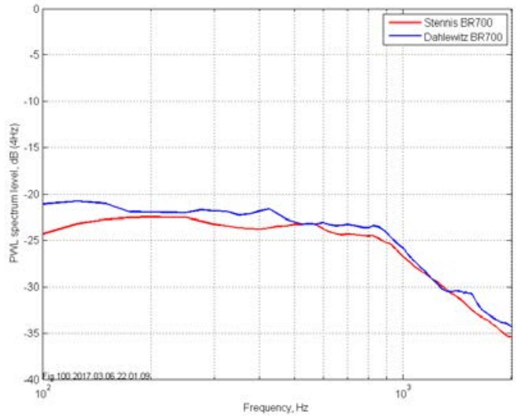

Figure 10 AFINDS-GX PWL core spectrum from Stennis and Dahlewitz BR700 data using Stennis array aperture; low power condition.

Acknowledgements

This work has been supported in the past by the Aeroacoustics Research Consortium and currently by Rolls-Royce plc and the ACAPELLA project. The author would like to acknowledge helpful discussions with Christoph Richter & Stefan Funke of Rolls-Royce Germany and Kevin Britchford of Rolls-Royce, Derby.

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