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Acoustic Intensity Fluctuations Caused by Internal Waves in the Ocean

The paper aims to show that the parabolic moment equations can be applied to the random scattering of sound by internal waves in the ocean. The parabolic equation for the fourth moment is described and a simple physical picture is given of how the intensity fluctuations develop. These results are applied to internal waves to provide a possible explanation of observed acoustic intensity spectra.

Fourth moment equation. Acoustic field is E, intensity I is observed. $I = EE^*$. Intensity fluctuations are specified by

$$\sigma_I^2 = \langle I^2 \rangle - \langle I \rangle^2 \qquad (1)$$

Need $m_4 = \langle I^2 \rangle$, the fourth moment of the field. The equation is /1/

$$\frac{\partial m_4}{\partial Z} = i\left\{\frac{\partial^2 m_4}{\partial \xi_1 \partial \xi_2} - \frac{\partial^2 m_4}{\partial \eta_1 \partial \eta_2}\right\} - \gamma\left\{2 + f(\xi_1+\xi_2, \eta_1+\eta_2) + f(\xi_1-\xi_2, \eta_1-\eta_2) - 2f(\xi_1, \eta_1) - 2f(\xi_2, \eta_2)\right\} m_4 \qquad (2)$$

$k = 2\pi/\lambda$, $Z = z/kr_o^2$, $\gamma = k\beta r_o^2$, $f(\xi,\eta) = \int \rho(\xi,\eta,\zeta)d\zeta$ and $\rho(\xi,\eta,\zeta)$ is the 3-D autocorrelation function of the irregularities, β is the power attenuation coefficient, and r_o is the scale size of the irregularities, $\xi_1 = x_1 - x_2$, $\xi_2 = x_2 - x_3$, etc.

Limitations (a) Angles of scatter small, (b) weak scatter in length r_o.

Physical significance of symbols.

Z, distance scaled in terms of a Fresnel length kr_o^2,

β, power in coherent wave attenuated like $\exp\{-\beta z\}$. Also total mean square phase deviation imposed by medium in distance z is

$$\phi_o^2 = \beta z \qquad (3)$$

$$\gamma = \beta z/Z = \frac{\text{amount of scatter in distance } z}{\text{number of Fresnel lengths in } z} \qquad (4)$$

γ is a parameter distinguishing scattering regimes /2/.

Computed solutions of Eq. (1) show that σ_I^2 has peaks in excess of unity when $\gamma \gg 1$, /3/. Analytic solutions confirm this /4/, and show that the wave number ν spectrum of intensity fluctuations behaves like ν^{-n} $(1 \leq n \leq 2)$ in the region $r_o^{-1} < \nu < r_o^{-1}(4\phi_o)$ (4a)

Two mechanisms for intensity fluctuations are at work in Eq. (1). They are (a) Diffraction or Fresnel effect. A small phase modulation of size r_o is converted to intensity modulation in a distance

$z_1 = kr_o^2$, the Fresnel length.

(b) Multiple scatter can produce a deeply phase modulated wave front without effect (a) if γ is large enough. This field then converges to a focus in a distance $z_2 = kr_o^2/4\phi_o$, or in scaled units

$$Z_2 = 1/4\phi_o \tag{5}$$

to produce intensity fluctuations.

Where peaks occur for large γ. Wave propagates to a distance Z_1 where, if γ is large enough, it develops virtually only phase fluctuations of mean square value of, from (3) and (4),

$$\phi_o^2 = \gamma Z_1 \, . \tag{6}$$

The front then focuses at a further distance

$$Z_2 = 1/4\phi_o \tag{7}$$

to produce large intensity fluctuations at a distance (Fig. 2)

$$Z_1 + Z_2 = Z_1 + 1/4\sqrt{\gamma Z_1} \, . \tag{8}$$

Focusing distance is in region where the sum (8) is a minimum, i.e.

$$Z_{fo} \approx 0.75\,\gamma^{-1/3} \, . \tag{9}$$

Agrees with computed and analytical results.

Height of the peaks. In this picture we can treat the wave front as a deeply modulated phase screen at Z_1. Screen theory /5/ gives a value for σ_I^2 at the focus

$$\sigma_I^2 \approx 1 + \frac{2}{\pi e}\log(3/2Z_{fo}) \tag{10}$$

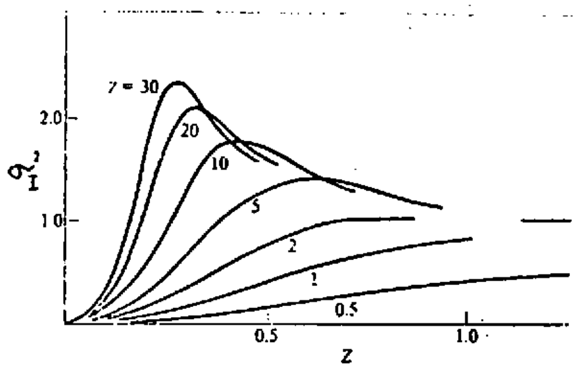

Fig. 1 Curves for σ_I^2 computed from Eq. 1 for different values of γ.

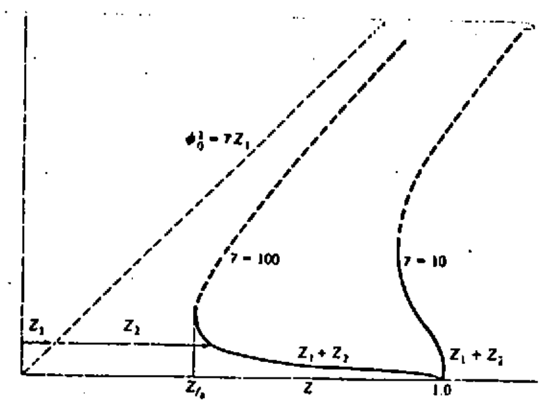

Fig. 2 Build up of phase modulation and position of the focus.

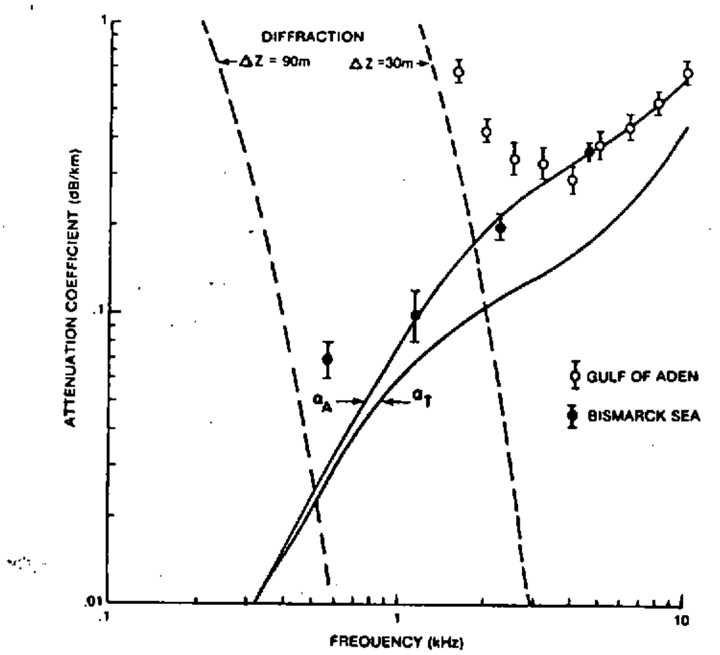

Figure 1 - Surface duct attenuation experiments. Diffraction curves are calculated for duct widths Δz = 90 m (Bismarck Sea) and Δz = 30 m (Gulf of Aden). The curve α_A is the sum of boric acid and magnesium sulfate contributions. The curve α_T is the Thorp formula where only the magnesium sulfate term is corrected for temperature. T = 30°C.

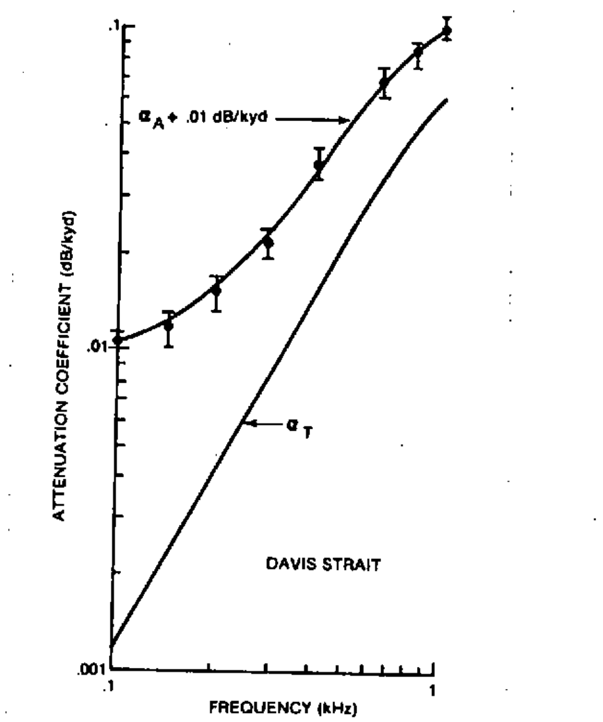

Figure 2 - Attenuation measurements for the Davis Strait. The calculated attenuation is α_A. The additional frequency-independent coefficient .01 dB/kyd is ascribed to internal wave scatter. Thorp's equation is α_T. T = -1.5°C.

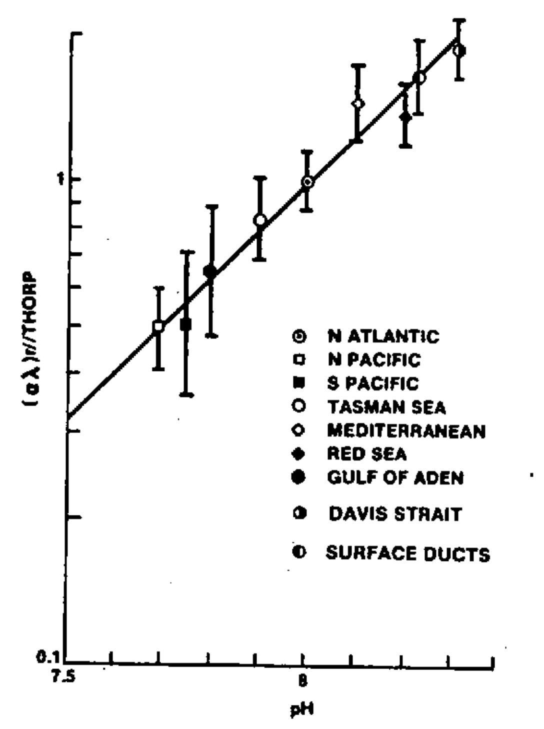

Figure 3 - Experimental values of absorption per wavelength at the relaxation frequency (relative to Thorp's value) vs. pH.