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EFFECT OF INTERNAL WAVES ON THE DIRECTIONAL PATTERN OF A TOWED ARRAY

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1. INTRODUCTION

The gain and directional pattern of a towed array are degraded by distortion of the signal propagating through internal waves in the ocean. The Garrett-Munk spectrum for a saturated internal wave field is used to derive a form for the acoustic coherence in the horizontal, and hence to find the directional pattern of a towed array. Simple approximations are given that allow useful array lengths to be estimated and directional performance to be examined.

2. BEAM PATTERN OF A HORIZONTAL ARRAY

An acoustic source of wave number k is at the origin of a Cartesian set of axes x, y, z, with z directed downwards, in an ocean containing a saturated internal wave field whose vertical and horizontal scales are L_V and L_H respectively. Let

$$Z = z/L_V \ , \ Y = y/L_H \ , \ X = x/kL_V^2 \quad . \tag{1}$$

The array of length L_a having closely and equally spaced hydrophones lies in the y direction, intersected at midpoint by the x axis at a scaled range x from the origin. Let $V(Y, Z, X)$ be the acoustic field at the array. The complex amplitude of the array output due to the component of acoustic field making an angle θ with the normal to the array in the horizontal plane is given by

$$A(\theta, Z, X) = \sum_j W(Y_j)V(Y_j, Z, X)\exp\{-i\nu_1 Y_j\}$$

where

$$\nu_1 = kL_H \sin\theta \quad , \tag{2}$$

Y_j is the position of the j-th element of the array and $W(Y_j)$ is the shading function of the array. This will be taken to be uniform

$$\begin{aligned} W(Y) &= 1 \ , \ |Y_j - Y_0| < \ell_a|2 \\ &= 0 \ , \ |Y_j - Y_0| > \ell_a|2 \quad ; \end{aligned} \tag{3}$$

Here Y_0 is the position of the array centre. The array has a physical length L_a and a scaled length $\ell_a = L_a/L_H$.

The ensemble average power output of the array in this case is [1,2]

$$I(\theta, Z, X) = 2\int_0^{\ell_a} (\ell_a - \bar{\eta}) m_2(0, \bar{\eta}, X) \cos\{\nu_1 \bar{\eta}\} d\bar{\eta} \quad , \tag{4}$$

where $m_2(\bar{\xi}, \bar{\eta}, X)$ is the second moment of V and

$$\begin{aligned} \bar{\zeta} &= Z_1 - Z_2 \quad , \\ \bar{\eta} &= Y_1 - Y_2 \quad , \end{aligned} \tag{5}$$

In the case of a point source [2]

$$m_2(\bar{\zeta}, \bar{\eta}, X) = \exp\left\{ -\Gamma X \int_0^1 [1 - f_0(\bar{\zeta}t, \bar{\eta}t)] dt \right\} \quad , \tag{6}$$

where Γ is the scattering strength parameter of the medium

$$\Gamma = k^3 \mu^2 L_H L_V^2 \quad , \tag{7}$$

μ^2 is the mean square value of fractional sound speed variations with autocorrelation function $\rho(\zeta, \eta, \xi)$, and

$$\begin{aligned} f(\bar{\zeta}, \bar{\eta}) &= \int_{-\infty}^{\infty} \rho(\bar{\zeta}, \bar{\eta}, \bar{\xi}) d\bar{\xi} \quad , \\ f_0(\bar{\zeta}, \bar{\eta}) &= f(\bar{\zeta}, \bar{\eta}) / f(0, 0) \quad . \end{aligned} \tag{8}$$

3. THE AUTOCORRELATION FUNCTION f_0 FOR INTERNAL WAVES

The quantity f_0 can be calculated for a saturated internal wave field whose normalised spectrum of vertical spatial frequencies β is given by [3,4]

$$\begin{aligned} H(\beta) &= [\pi - \mathrm{arctg}(1/j_*)]^{-1} \frac{\beta_*}{(\beta_*^2 + \beta^2)} \qquad (\beta > \beta_0) \\ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad (\beta < \beta_0) \end{aligned} \tag{9}$$

where

$$\beta_* = L_V^{-1} \quad , \tag{10}$$

is the turn-over spatial frequency and β_0 is the lower cut-off spatial frequency

$$j_* = \beta_*/\beta_0 \quad . \tag{11}$$

The coherence m_2 in (4) involves the integrated form of f_0

$$F(\bar{\eta},0) = \int_0^1 f_0(\bar{\eta}t,0)dt \quad . \tag{12}$$

In the case of the spectrum (9) this becomes [3,4]

$$F(\bar{\eta},0) = \frac{4}{\bar{\eta}\ln(1+j_*^2)} \int_{1/j_*}^{\infty} \left[\int_0^{\infty} \frac{\sin(xy\bar{\eta})}{(1+y^2)^2} dy\right] \frac{dx}{(1+x^2)x^2} \quad , \tag{13}$$

which is shown in Figure 1 for $j_* = 3, 5, 7$ by the full line.

A number of useful analytical approximations can be made to simplify (13). A good fit is given by the exponential

$$F(\bar{\eta},U) \sim \exp\{-|\bar{\eta}/\bar{\eta}_0|\} \quad , \tag{14}$$

where

$$\bar{\eta}_0 = 1 + 0.8j_* \quad . \tag{15}$$

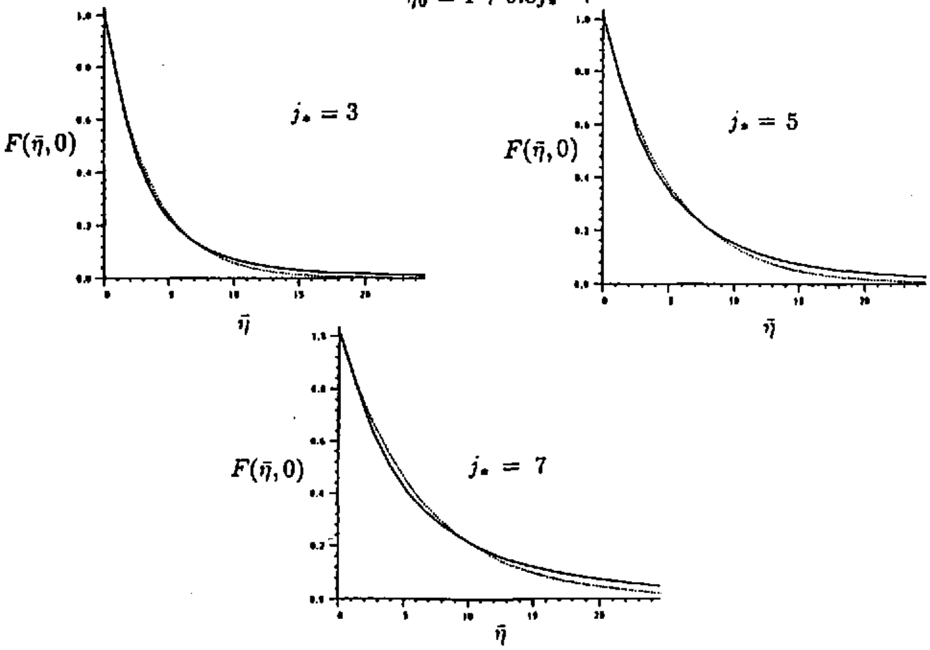

Fig. 1: The projected and integrated horizontal autocorrelation function of internal waves $F(\bar{\eta},0)$, (13). The broken line is the exponential approximation (14).

The approximate form is shown in Figure 1 by the broken line. The agreement is quite adequate for all practical, and indeed most theoretical purposes. The approximation has been checked for values of $j_* \leq 10$.

4. RESPONSE OF THE ARRAY

From (14) and (4) the average directional pattern of the array is

$$I(\theta, Z, X) = 2\int_0^{\ell_a} (\ell_a - \eta)\exp\{-\Gamma X(1 - e^{-\bar{\eta}/\bar{\eta}_0})\}\cos(\nu_1 \eta)dy \quad , \tag{16}$$

which is the Fourier transform of the product of two functions [1]. The first of these is the autocorrelation function of the array, a triangle of half-base ℓ_a. The second is the horizontal coherence of the acoustic field. When $\Gamma X \geq 2$ this latter becomes, with a good degree of approximation, $\exp\{-\Gamma X\bar{\eta}/\bar{\eta}_0\}$.

When the field coherence is very much wider than the array autocorrelation ℓ_a the array pattern I is given by the Fourier transform of this autocorrelation and thus has a main lobe whose width is

$$\theta_a \approx \lambda/2L_a \quad . \tag{17}$$

As the medium has more effect and the field coherence becomes narrower I is given by the Fourier transform of the coherence and its width increases. In this case

$$I(\theta, Z, X) \approx I_0[1 + (\theta/\theta_s)^2]^{-1} \quad , \tag{18}$$

with

$$\begin{aligned} I_0 &= 2(1 + 0.8j_*)/\Gamma X \quad , \\ \theta_s &= k\mu^2 x/(1 + 0.8j_*) \quad . \end{aligned} \tag{19}$$

If the scattering by internal waves is not to have a marked effect on the width of I it can be seen from the above that we require

$$L_a \ll L_a(\text{crit}) = (1 + 0.8j_*)/k^2\mu^2 x \quad . \tag{20}$$

This can be regarded as a critical array length. Extending the array beyond this limit will not lead to any improvement of directivity.

<u>Example</u>

Consider an array in the upper thermocline, where we can take $\mu^2 \sim 5 \times 10^{-8}$, operating at 500 Hz at a range of 50 km from the source. Then $\Gamma X = 25$ and the critical array length is 0.14 L_H. If we take L_H to be about 2.5 km the critical length is equal to 350 m.

Figure 2 shows I in the case when the array has this critical length. The full line indicates the result when the internal wave field is present and the broken line that when there are no internal waves. Figure 3 shows similar curves for I when the array length is increased to 2100 m.

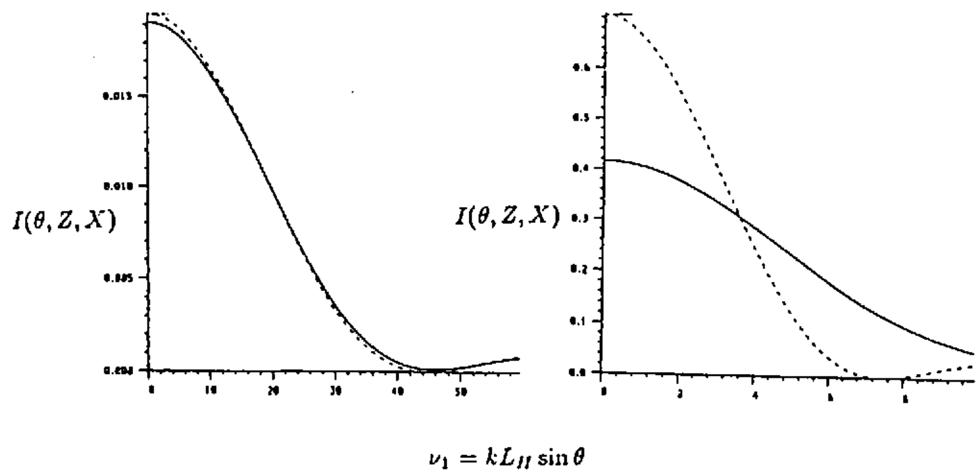

Fig. 2: (a) The directional pattern of the array in the example $L_a = 350$ m. The broken line shows the result in the absence of internal waves. (b) The same as in Fig. 2 but with $L_a = 2100$ m.

CONCLUSION

The simple expression for the limiting useful length of a horizontal array in an internal wave field (20) can be used to make quick estimates of this important quantity. More accurate curves of the directional pattern can be easily computed from (16) when these are needed.

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