Proceedings of The Institute of Acoustics

A STRUCTURAL DYNAMICIST LOOKS AT STATISTICAL ENERGY ANALYSIS

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1. INTRODUCTION

It is a pleasure to welcome to this Annual Conference of the Institute of Acoustics specialists in the field of structural dynamics who do not naturally think of Acoustics as their basic discipline. I have chosen to speak about the development and use of a technique for dynamic analysis which has grown out of traditional acoustics roots. In so doing I hope that I can show the contribution which the field of acoustics can make to structural dynamics and also show the acousticians how one of their techniques can be developed in the hands of a structural designer. At the outset it must be said that those of us with a background in structural or mechanical engineering do not take easily to the gross approximation implied by using dB's and the wholesale, even wanton, averaging of everything in sight which takes place in the statistical energy analysis - SEA for short. However, many of us have consoled ourselves by the thought that much of what we try to do cannot have much greater accuracy than is commonplace in acoustics and now accept these averaging techniques with better grace ! Indeed in many cases we are deluding ourselves (and others less versed in dynamic analysis) if we imagine that in many dynamics problems we can give answers to the sort of accuracy expected in static designs.

In this address I shall briefly outline the method of analysis used in SEA and then go on to describe some recent work which has been done to try to evaluate the parameters required for typical structures. I shall illustrate this with reference to studies of spacecraft dynamics.

2. THE STATISTICAL ENERGY ANALYSIS METHOD

The requirement for such a method arose when attempts were made to estimate the vibration levels in structures such as aircraft and spacecraft excited by broad band random excitation caused by jets or boundary layer pressure fluctuations. Many many modes of vibration are excited in the frequency range up to about 3 or 4 kHz. Component stresses are greatest in the lower modes of vibration and so the spectral density of stress does not contain many modal responses above 600 or 700 Hz but acceleration levels on equipment remain high up to the 3 kHz range. A mode by mode estimation is not practicable in such a situation and so an alternative was sought.

The SEA method was developed by Lyon [1] and others to provide this alternative. It evolved from some of the ideas used in room acoustics. The theoretical analysis of sound fields in rooms uses statistical models to describe the sound field, the structural response and transmission characteristics of walls etc. and gives a statistical description of the response behaviour. In the application to structural vibration problems the independent dynamic variable used is the vibrational energy. The aim of the analysis is to estimate the distribution of vibrational energy among the various elements of the coupled system. This is done by equating the power flowing into the system at the input point, to the power flowing from one subsystem to another plus the energy dissipated. Because of the statistical descriptions of the forces and system parameters, the equations involve average energies and power flows. These are expressed as averages over time and also over the modes of vibration having natural frequencies in a band of frequencies which is wide enough to include at least five modes. Because there will usually be some frequency depdendance in the dynamic interactions the frequency bands should be chosen so as to highlight any such trends. The acousticians have established analysis procedures based on third octave bands but for many structural applications a 100 Hz band width might be more suitable.

From random vibration theory it can be shown that in a frequency band wide enough to contain several modes of vibration the velocity response of a uniform plate or shell to a broad band random force is given approximately by:

$$\{<\overline{v^2(t)}>\} = \frac{n(f)\ \{\overline{F^2(t)}\}}{8\ \pi\ f\ \eta\ M^2} \tag{1}$$

This shows that the mean square response of a given component such as a plate or shell is directly proportional to its modal density n(f) and inversely proportional to its average loss factor η .

The average power flow into the component is $\overline{F(t)\ v(t)}$ which can be written as $\overline{F^2(t)}$ Re(Y). Equating this to the average power dissipated, $M<\overline{v^2(t)}>\eta\ 2\pi f$, and substituting from the equation above we get

$$n(f) = 4\ M < Re(Y) > \tag{2}$$

Thus the frequency band average modal density is directly proportional to the spatial average of the frequency band average point mobility.

In a typical structure we must now consider the power flowing between coupled components. For simplicity in representing this in the energy balance equation the power flowing through the coupling link or joint is thought of as another energy loss mechanism analagous to the internal damping and acoustic radiation. To put it in the same form it is written as

$$P_{12} = \eta_{12}\ 2\pi f\ \bar{E}_1$$

The power flowing from component 1 to 2 is equated to the coupling loss factor η_{12} multiplied by the average energy of component 1 and the frequency.

An energy balance for the whole system can now be written down and the mean square velocity levels of each major component estimated.

3. EVALUATION OF MODAL DENSITY, LOSS FACTOR AND COUPLING LOSS FACTOR

Theoretical results are available for the modal density of some structural components such as uniform beams plates and shells and there are one or two results for coupling loss factor. Equations (1) and (2) form the basis for an experimental determination of modal density and loss factor of spacecraft components [2]. The Address will include examples of a range of spacecraft structural components such as uniform plates and cylinders, stiffened cylinders, honeycomb and corrugated plates and corrugated cylinders. Some typical examples are given here.

Figure 1 shows the estimated modal density of a plain cylindrical shell. The semi-empirical result of Szechenyi [3] is shown for comparison. The ring

frequency is also marked. Reasonable agreement between theory and experiment has been achieved. Figure 2 gives the estimated modal density of the honeycomb platform of the MAROTS satellite. In this case the experimental result is much higher than the theory. The majority of the difference arises because the total mass of the platform (used in equation (2)) is 10 kg whereas the mass of the honeycomb alone is only about half that value. The additional mass is attributable to the many attachments and inserts for equipment mounts. Figure 3 shows the modal density of the MAROTS corrugated cylinder. The equivalent ring frequency in this case is about 1600 Hz and the high frequency assymptote is calculated to be about 0.27. The coupling loss between the honeycomb platform and the corrugated cone is shown in figure 4.

Further examples will be given in the lecture.

4. REFERENCES

1. R.H. Lyon 1975 Statistical Energy Analysis of Dynamical Systems: Theory and Applications. MIT Press.

2. B.L. Clarkson, R.J. Pope, Experimental Determination of Modal Densities and Loss Factors of Flat Plates and Cylinders. (Submitted to J.S.V.).

3. E. Szechenyi 1971 Modal Densities and Radiation Efficiencies of Unstiffened Cylinders using Statistical Methods. J.S.V. 19(1), 65-82.

5. ACKNOWLEDGEMENTS

The experimental results given in this paper were obtained in a programme of work sponsored by the European Space Agency.

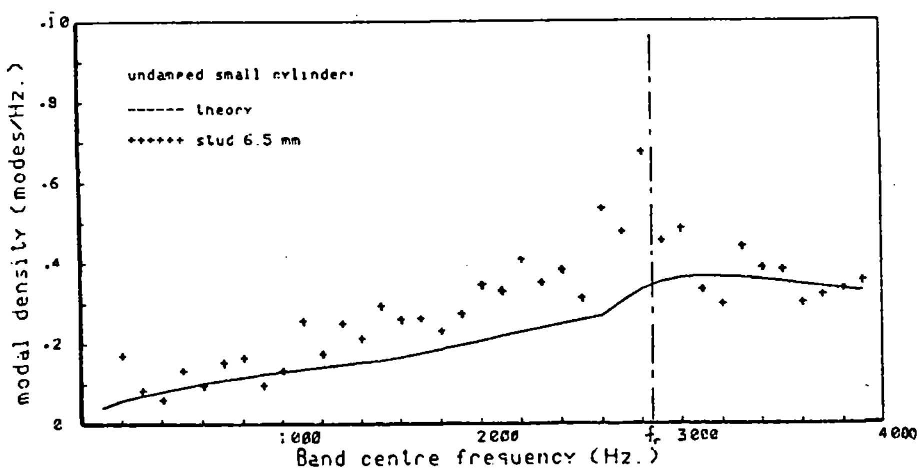

Fig. 1 Modal Density of Uniform Cylinder

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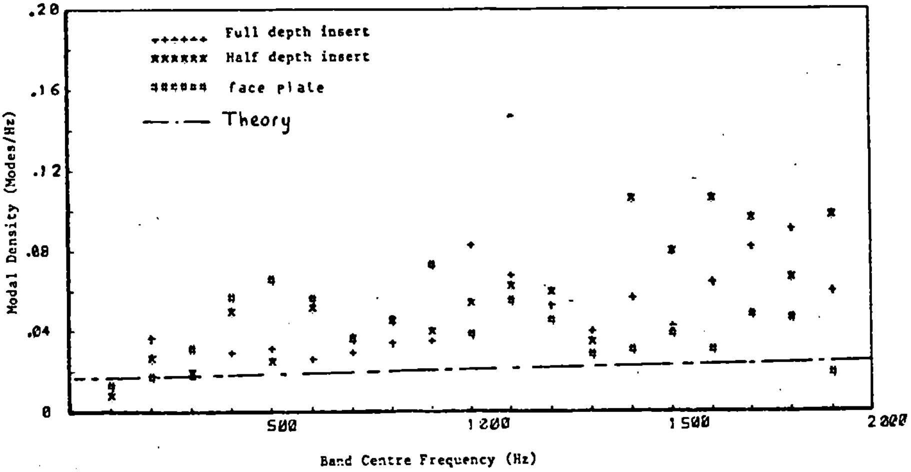

Fig. 2 Modal Density of MAROTS Honeycomb Platform

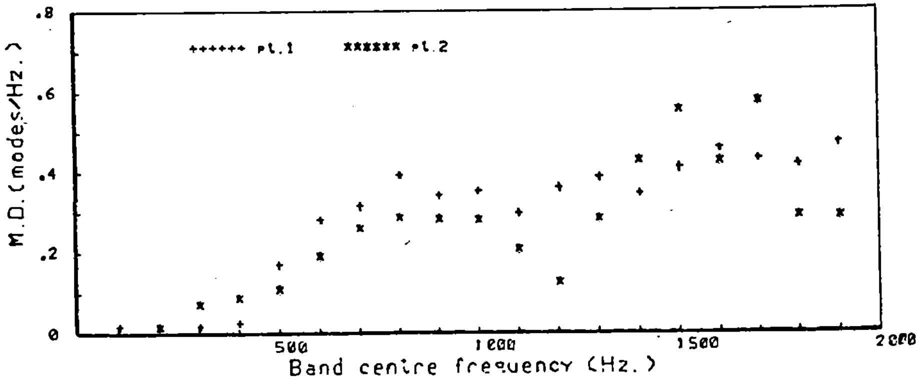

Fig. 3 Modal Density of MAROTS Corrugated Cylinder

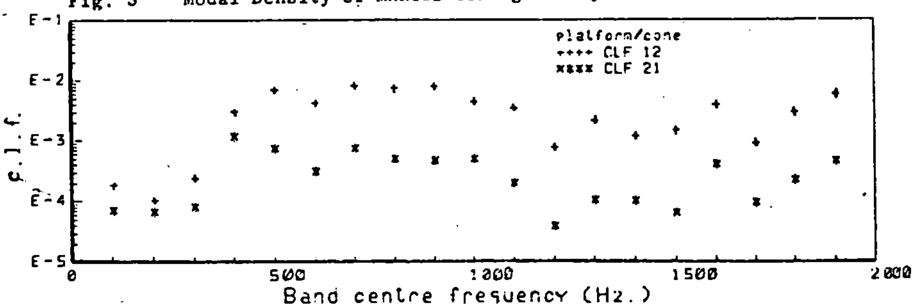

Fig. 4 Coupling Loss Factor for MAROTS Platform to Cone Junction with Cylinder attached