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THE RESPONSE OF A SIMPLE STRUCTURE TO POINT AND MULTI-POINT EXCITATIONS

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INTRODUCTION

A problem remains in characterising machines as sources of vibration in ship and building structures in that the mobility of the supporting structures have an important influence on the vibrational energy flow through the points or areas of contact between the machine and structure [1]. Attempts to compare the vibrational strength of two machines, say, may be constrained by lack of knowledge of the possibly different structures on or in which they are to be installed. It is also well known that choice of AV mountings must be influenced by the likely resonances to be encountered in the supporting floor.

There remains the need to describe the dynamic response of the supporting structure at low frequencies to point, multi-point and area excitations. In recent work by the authors, an approximate solution was presented for the forced bending vibrations of simple combinations of rectangular plates [2]. The solution is not general in that the edges of the plates are constrained with respect to linear displacement and the generation of in-plane vibration is not allowed. The non-coupled edges are assumed pinned but with a variable complex rotational stiffness which allows consideration of edge constraints ranging continuously from the simply supported to the clamped condition, including the effect of damping. This range is likely to include many real conditions in full scale structures. In addition, it is reasonable to assume that bending vibration is the dominant mode of energy transfer at low frequencies.

THEORETICAL BACKGROUND

In a structure composed of combinations of rectangular plates for which the co-ordinate function vectors can be obtained, the flexural vibration transmission in the global system can be calculated by expressing the displacement amplitude vectors as a linear combination of co-ordinate function vectors. The point and transfer mobilities due to point or multi-point excitation can be derived in explicit form, therefore the response at any position in the global system can be calculated from the mobilities if the exciting forces are known.

Consider the simplest combination of rectangular plates, an L-combination. The governing differential equations for general sinusoidal excitation can be written as:-

$$\begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{pmatrix} \begin{pmatrix} \bar{w}_1(x_1,y_1) \\ \bar{w}_2(x_2,y_2) \end{pmatrix} = \begin{pmatrix} Q_1(x_1,y_1) \\ Q_2(x_2,y_2) \end{pmatrix} \quad \ldots\ldots \quad (1)$$

where $\mathcal{L}_i \equiv D_i^* \nabla_i^4 - \omega^2 \rho_i h_i$,

$\begin{pmatrix} \bar{w}_1(x_1,y_1) \\ \bar{w}_2(x_2,y_2) \end{pmatrix}$ and $\begin{pmatrix} Q_1(x_1,y_1) \\ Q_2(x_2,y_2) \end{pmatrix}$ are respectively the displacement amplitude function and pressure amplitude function vectors. The boundary condition on the coupled and non-coupled edges are similar to

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those given in [2] for the series of T-combinations of plates and are not given here. The co-ordinate function vectors can be organised from the eigen-functions of the L-combination of beams of unit width perpendicular to the coupled edge and the single beams of unit width parallel to the coupled edge. Let $K_{1\ell}$, $K_{2\ell}$, $X_{1\ell}(x_1)$ and $X_{2\ell}(x_2)$ be the ℓth complex eigen wave-numbers and eigen functions of the L-combination of beams. For simplicity, assume:

$$\xi^*_{1a}/D^*_1 = \xi^*_{2a}/D^*_2 \quad \text{and} \quad \xi^*_{1b}/D^*_1 = \xi^*_{2b}/D^*_2$$

where ξ^*/D^* is the ratio of rotational to bending stiffness

The beams on each plate have the same eigen number K_{ym} and eigenfunction $Y_m(y)$.

Therefore the (ℓ, m) th co-ordinate function vector for the combination of plate can be written as:

$$\begin{pmatrix} \bar{W}_{1\ell m}(x_1,y_1) \\ \bar{W}_{2\ell m}(x_2,y_2) \end{pmatrix} = \begin{pmatrix} X_{1\ell}(x_1)Y_m(y_1) \\ X_{2\ell}(x_2)Y_m(y_2) \end{pmatrix} \quad \ldots\ldots (2)$$

$$\ell = 1,\ldots,L\ ;\ m = 1,\ldots,M,$$

satisfying all the boundary conditions on both coupled and non-coupled edges. The approximate solution to the bending vibration can then be expressed as:

$$\begin{pmatrix} \bar{W}_1(x_1,y_1) \\ \bar{W}_2(x_2,y_2) \end{pmatrix} = \sum_{\ell=1}^{L}\sum_{m=1}^{M} A_{\ell m}\begin{pmatrix} X_{1\ell}(x_1)Y_m(y_1) \\ X_{2\ell}(x_2)Y_m(y_2) \end{pmatrix} \quad \ldots\ldots \quad (3)$$

Substituting (3) into equation (1) yields:

$$\begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{pmatrix}\sum_{\ell=1}^{L}\sum_{m=1}^{M} A_{\ell m}\begin{pmatrix} X_{1\ell}(x_1)Y_m(y_1) \\ X_{2\ell}(x_2)Y_m(y_2) \end{pmatrix} = \begin{pmatrix} Q_1(x_1,y_1) \\ Q_2(x_2,y_2) \end{pmatrix} + \begin{pmatrix} \varepsilon_1(x_1,y_1) \\ \varepsilon_2(x_2,y_2) \end{pmatrix} \cdot\cdot (4)$$

where $\begin{pmatrix} \varepsilon_1(x_1,y_1) \\ \varepsilon_2(x_2,y_2) \end{pmatrix}$ is an error function vector.

Similar to the case of a series of T-combinations [2] equation (4) can also be converted into a simple matrix equation for $A_{\ell m}$'s.

$$\begin{pmatrix} T(1,1,1,1) & \cdots & T(1,1,L,M) \\ \vdots & & \vdots \\ T(L,M,1,1) & \cdots & T(L,M,L,M) \end{pmatrix}\begin{pmatrix} A_{11} \\ \vdots \\ A_{LM} \end{pmatrix} = \begin{pmatrix} F_{11} \\ \vdots \\ F_{LM} \end{pmatrix} \quad \ldots\ldots \quad (5)$$

where the generalised force:

$$F_{\ell' m'} = \sum_{i=1}^{2}\int_0^{a_i}\int_0^{b}\tilde{X}_{i\ell'}(x_i)\,\tilde{Y}_{m'}(y_i)\,Q_i(x_i,y_i)\,dx_i\,dy_i$$

$\tilde{X}_{i\ell'}$, $\tilde{Y}_{m'}$ are complex conjugate functions of $X_{i\ell'}$, $Y_{m'}$

$$T(\ell',m',\ell,m) = \sum_{i=1}^{2}\int_0^{a_i}\int_0^{b}\tilde{X}_{i\ell'}\tilde{Y}_{m'}(y_i)\,\mathcal{L}_i X_{i\ell}(x_i)Y_m(y_i)\,dx_i\,dy$$

The detail calculations of $T(\ell',m',\ell,m)$'s are similar to those given in [2]. $A_{\ell m}$'s can be solved from equation (5) and substituting $A_{\ell m}$'s into expression (3) yields the solution to equation (1).

MEASUREMENT

The point and transfer accelerances were predicted and measured for an L-combination and a series T-combination of five plates. The structures were modelled in 2mm aluminium and the joined plates were clamped into a demountable frame of bolted rectangular section mild steel and the whole was mounted, via resilient pads, on a support frame. In addition to a clamped edge a simply supported condition was approximated by cutting a square section notch along three edges of each plate. Variation in the depth and width of the notch allows a range of rotational stiffnesses to be considered. which are expressed simply in terms of plate bending stiffness when incorporated into the computer program. In addition the notch reduces variation in edge constraint likely to occur along a clamping frame bolted at discrete points. Values of plate bending stiffness, to be used in the computer model, were obtained from static and dynamic tests of beam samples. In addition a non systematic parametric survey allowed an estimate of material constraints to be made from visual inspection of a 'best fit' of predicted to measured plate response. The optimum values agreed well with those obtained by the other methods.

The vibration sensors were small accelerometer transducers and the forces were generated by electrodynamic shakers attached to the plates through small force transducers giving an area of contact of 64mm^2 which was considered small enough to constitute a point force at low frequencies [3]. Both steady-state wide band noise and short duration pulses were used, and in both cases, signal averaging was employed to enhance signal noise ratio. Results were obtained by means of a dual channel analyser and the accelerance acc/F was given as a transfer function with frequency resolution 5Hz over a range 0 - 2kHz.

In Figure 1 is shown the measured and predicted transfer accelerance across the junction of a L-combination of simply supported aluminium plates subjected to a point force. The agreement in terms of level is good over the frequency range 0 - 2kHz. The agreement is less good in terms of phase difference (Figure 2) but this was to be expected in a lightly damped system where there is likely to be many discontinuities in measured values at or near $\pm$ 180°. In figure 3 is given the point accelerance of a point excited five plate combination and, again, there is fair agreement between measurement and prediction.

The agreement is less good for transfer accelerances across one or two junctions. In figure 4 is shown the acceleration level difference across one junction and in figure 5 the transfer accelerance across two junctions. Agreement is generally good at low frequencies. The discrepancies are likely to be the result of the high frequency modes which are more sensitive to small variations in the model structure. In addition it is likely that modes of vibration other than bending contribute increasingly with increased frequency. It would appear that the simple systems so far considered are well described by the approximate method of analysis and it should now be possible to estimate the force or forces at any point in the system from measurements of acceleration(s) at any other points.

THE MOBILITY MATRIX

A simple example is a series of T-combinations of rectangular thin plates. Let F_{E_1} ...F_{E_n} be n normal exciting force amplitudes of same frequency at exciting positions E_1 ... E_n, V_{R_1} ... V_{R_n} be the velocity amplitudes at receiving points R_1 ...R_n , and $M_{E_iR_j}$ and $M_{E_iE_j}$ (i,j = 1,...n) be, respectively, the transfer or point mobilities from E_1 ... E_n to R_1 ...R_n and from E_1 ... E_n to E_1 ... E_n; then:

$$\begin{pmatrix} V_{R_1} \\ \vdots \\ V_{R_n} \end{pmatrix} = \begin{pmatrix} M_{E_1R_1} & \cdots & M_{E_nR_1} \\ \vdots & \ddots & \vdots \\ M_{E_1R_n} & \cdots & M_{E_nR_n} \end{pmatrix} \begin{pmatrix} F_{E_1} \\ \vdots \\ F_{E_n} \end{pmatrix} \qquad \cdots\cdots \quad (6)$$

or

$$\begin{pmatrix} F_{E_1} \\ \vdots \\ F_{E_n} \end{pmatrix} = \left(Z_{RE} \right) \begin{pmatrix} V_{R_1} \\ \vdots \\ V_{R_n} \end{pmatrix} \qquad \cdots\cdots \quad (7)$$

where (Z_{RE}) is the impedance matrix from $R_1 \ldots\ldots R_n$ to $E_1 \ldots\ldots E_n$, the inverse of the mobility matrix in equation (6).
The velocity amplitudes at $E_1 \ldots\ldots E_n$ are given by:-

$$\begin{pmatrix} V_{E_1} \\ \vdots \\ V_{E_n} \end{pmatrix} = \begin{pmatrix} M_{E_1E_1} & \cdots & M_{E_nE_1} \\ \vdots & \ddots & \vdots \\ M_{E_1E_n} & \cdots & M_{E_nE_n} \end{pmatrix} \begin{pmatrix} F_{E_1} \\ \vdots \\ F_{E_n} \end{pmatrix} = \begin{pmatrix} M_{E_1E_1} & \cdots & M_{E_nE_1} \\ \vdots & \ddots & \vdots \\ M_{E_1E_n} & \cdots & M_{E_nE_n} \end{pmatrix} (Z_{RE}) \begin{pmatrix} V_{R_1} \\ \vdots \\ V_{R_n} \end{pmatrix} \quad \ldots\ldots \quad (8)$$

and the input power at E_i $(i = 1, \ldots\ldots, n)$ is given by,

$$P_i = \frac{1}{2} \text{REAL} \left[\tilde{V}_{E_i} F_{E_i} \right] \quad \ldots\ldots \quad (9)$$

where $\tilde{V}_{E_i}$ is the complex conjugate of V_{E_i}

It can thus be seen that the force amplitudes and input powers at the exciting points $E_1 \ldots\ldots E_n$ can be calculated from equations (7) (8) (9) by measuring the velocities or accelerations at the receiving points $R_1 \ldots\ldots R_n$.
In figures 6 and 7 are shown estimated and measured forces at two excitation points on one plate of the L-combination. The estimations were obtained from measurements of acceleration at two other points removed from the excitation points; one point on each plate.
The signal to the driver producing force F_2 was in phase with that to the driver producing force F_1 although the phase relationship between F_1 and F_2 is complicated by the strong frequency dependence of the system mobilities at the excitation points.
The agreement between prediction and measurement is promising except in a frequency band cnetred at 650Hz where prediction overestimates the forces.
In order to explain this discrepancy reference must be made to the mobility or, more precisely the accelerance matrix where:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} F_1 + \Delta F_1 \\ F_2 + \Delta F_2 \end{pmatrix} = \begin{pmatrix} a_1 + \Delta a_1 \\ a_2 + \Delta a_2 \end{pmatrix} \quad \ldots\ldots \quad (10)$$

A_{ij}, F_i, a_i are transfer accelerance, force, and acceleration, respectively.
Δa_i is the variation in measured acceleration and F_i is the resultant variation in calculated force.
From (10) is obtained,

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \Delta F_1 \\ \Delta F_2 \end{pmatrix} = \begin{pmatrix} \Delta a_1 \\ \Delta a_2 \end{pmatrix} \quad \ldots\ldots \quad (11)$$

it is seen that when the determinent of the matrix is small then small errors in measured accelerations will produce large errors in calculated forces.
The determinant of the matrix for the case considered is shown in Figure 8 and it can be seen that low values around 650 Hz give the discrepancies indicated in figures 6 and 7.
This problem can be overcome by taking more measurement points than the minimum required and averaging the predicted forces where values are not included where the determinant falls below a minimum value.This work is still in progress; the method appears to work but results are not presented here.

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PRACTICAL APPLICATIONS

So far the description has been for the case of forces operating at known points of contact such as through antivibration mounts at the corners of an engine base. The analytical method will also apply where the engine or machine base forms an area of contact with the supporting structure. Here the contact area can be divided into n area elements and the magnitude from values of acceleration or velocity measured at n or more positions remote from the engine base.

REFERENCES

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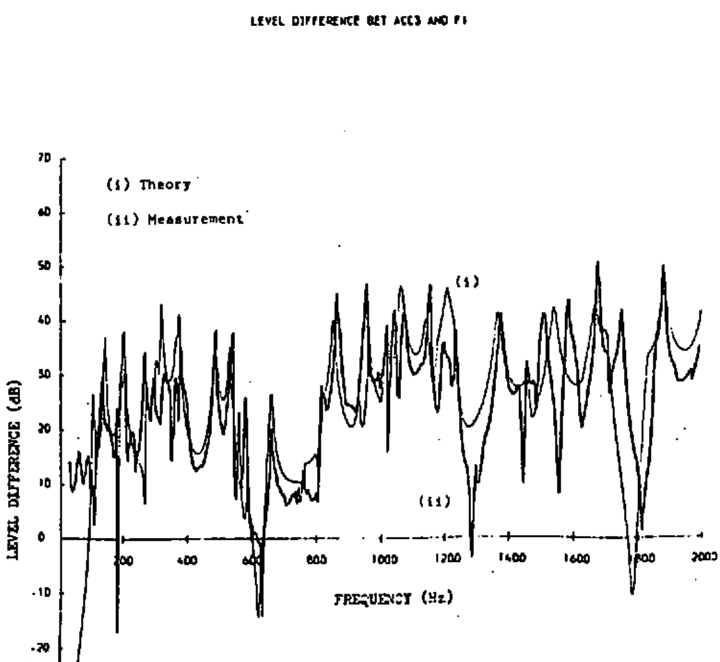

Figure 1. Transfer accelerance across L-combination of plates.

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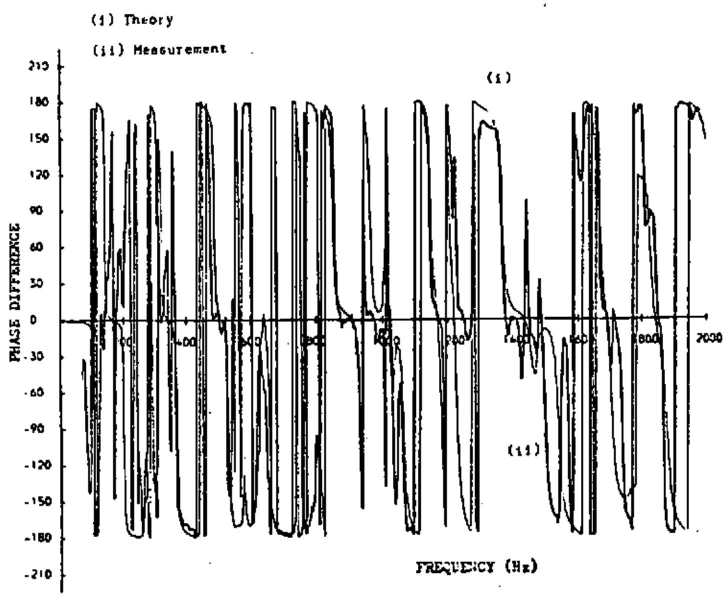

Figure 2. Phase difference across L-combination

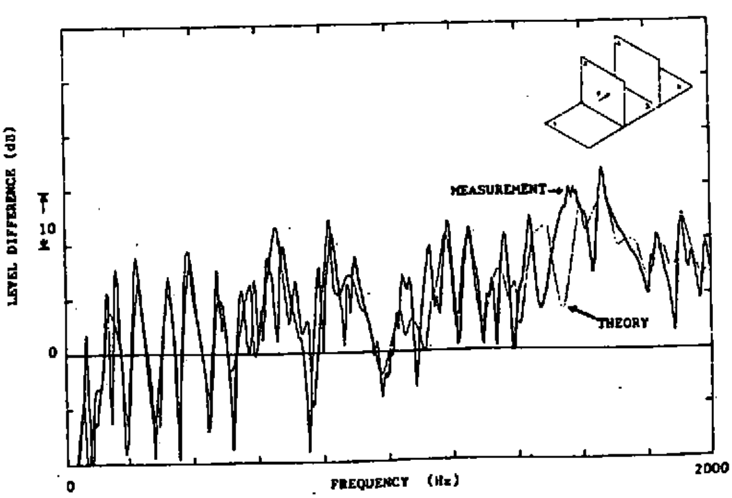

Figure 3. Point accelerance on plate2 of a T-combination of aluminium plates.

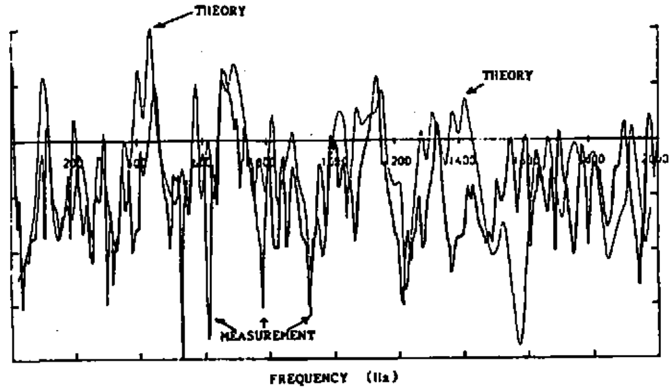

Figure 4. Acceleration level difference between plates 3 and 2

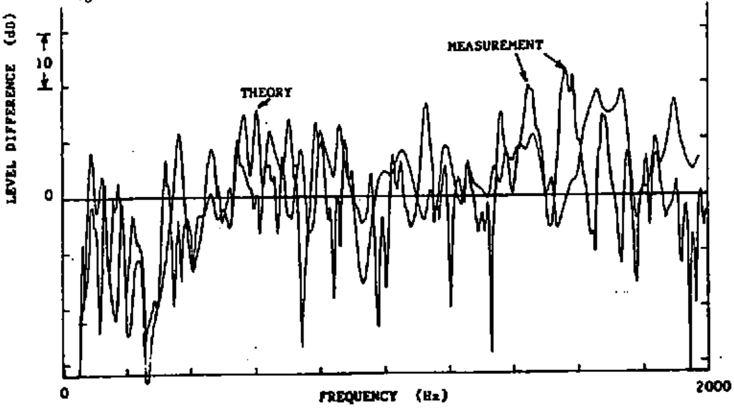

Figure 5. Transfer accelerance between plates 4 and 2.

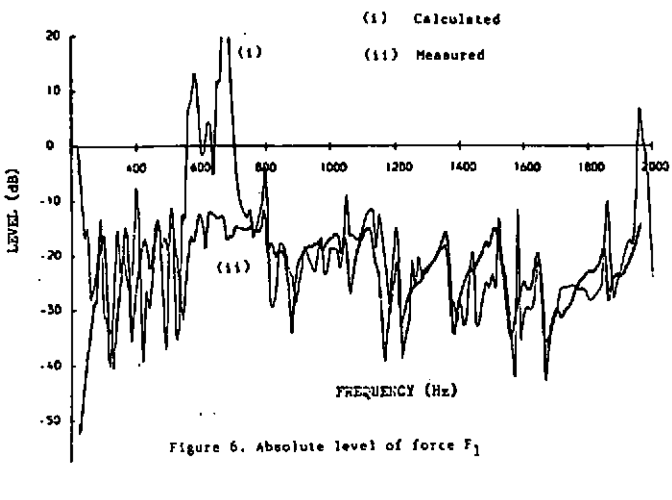

Figure 6. Absolute level of force F_1

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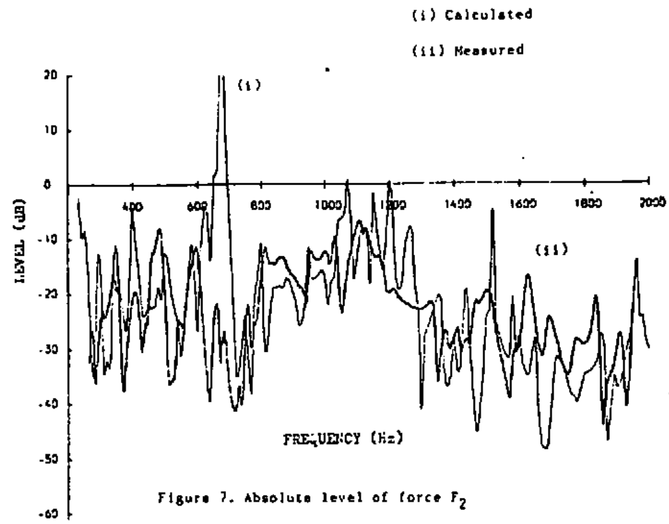

Figure 7. Absolute level of force F_2

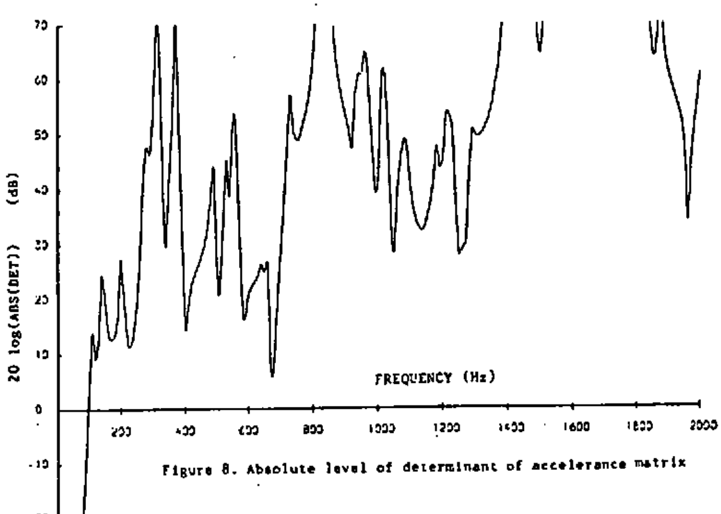

Figure 8. Absolute level of determinant of accelerance matrix