ANTI-SOURCE SPACING FOR GLOBAL NOISE REDUCTION

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In general an active sound-cancellation system consists of an acoustic noise source and a collection of controlled sound sources, whose drives are chosen so as to reduce the total noise broadcast. (These latter sources will be called anti-sound source).

There are a number of approaches to deciding on the anti-source drives. Mathematically the simplest is to choose drives that minimise the total noise over a chosen area (to be called the target area). Another appealing choice is to find the drives that maximise the noise reduction for the worst positioned location in the target area (a mini-max approach), this is however calculationally difficult to implement.

Minimising the total power of the broadcast noise does not guarantee that for all positions in the target area noise levels will be reduced. It is perhaps unreasonable to expect that noise everywhere should go down, even in those areas where it was very much below the average level without noise reduction, but a good criterion for global noise reduction is that nowhere is the noise worse than the average level without active sound cancellation.

This paper shows how applying this criterion to a system designed to minimise the total sound level leads to limits on the separation between anti-sources.

If the noise has frequency spectrum $p(\omega)$ and the transfer function between the jth anti-sound and the target area is $A_j(\omega)$ then the optimal drive for the ith anti-source is $<p\bar{A}_j><A_i\bar{A}_j>^{-1}$ where the brackets denote integrated values are taken over the target area, and the residual broadcast power spectrum will be

$$E(\omega) = <p\bar{p}> - <p\bar{A}_j> <A_i\bar{A}_j>^{-1}<A_i\bar{p}>$$

This paper looks at the situation where the noise source is in the open, with the target area in the plane of the source and anti-sources and sufficiently distant to be considered as at infinity.

Consider first a fixed frequency monopole unit noise source at the origin with 2N anti-sources arranged in a ring around it at radius a; the target area will be the whole ring at infinity. By symmetry the optimal anti-source drives will be equal and, if the jth anti-source is at angle θ_j, then the pressure for a distant observer (at coordinates (R,θ)) will be

$$p = \frac{e^{i\omega t}e^{-ikR}}{R}\left(1 + \alpha \sum_{j=1}^{2N} e^{ika\cos(\theta-\theta_j)}\right)$$

where ω is the angular frequency, k the wave-number.

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The optimal drive α is easily calculated, and is given by

$$2N\alpha = \frac{-J_0\,(ka)}{1 + \sum_{j=1}^{2N} \sum_{\substack{\ell=1 \\ \ell\neq j}}^{2N} \frac{J_0(kd_{j\ell})}{2N}}$$

where J_0 is the zeroth order cylindrical Bessel function, and $d_{j\ell}$ the distance between the jth and ℓth anti-source locations. The value of α is not important, but it should be noted that the denominator is positive for all ka. Expanding as a fourier series this can be written as

$$p = (1+2\alpha NJ_0(ka)) + 4\alpha N \sum_{m=1}^{\infty}\left[(-1)^m J_{2m}(ka) \sum_{j=1}^{N} \cos(2m(\theta-\theta_j))\right]$$

(See [1], p361)

Since the anti-sources are evenly spaced the oscillatory part of p must have the same symmetry as the anti-sources, and hence (choosing the origin of the θ coordinate as the location of an anti-source)

$$p = (1+2\alpha NJ_0(ka)) + 4\alpha N \sum_{m=1}^{\infty} (-1)^{mN} J_{2mN}(ka) \cos(2mN\theta)$$

So long as the amplitude of the oscillatory term is smaller than the constant cancelling term there will be sound suppression everywhere; now for fixed ka, if m>ka then $J_m(ka) > 0$, and declines exponentially as m increases, thus so long as $J_0(ka)$ is not near a zero and long as $2N > ka + \{\text{term of order } \log(N)\}$, then there will be sound suppression everywhere.

Now $\quad k = 2\pi/\lambda, \quad 2\pi a = 2Nd \quad$ (approximately)

where λ is the wavelength, and d the anti-source separation.

Substituting we have

$$\lambda\left(1 - \frac{O(\log N)}{2N}\right) > d.$$

The exact bound on λ/d is shown in figure 1.

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The result can be extended to a more general case. Consider a noise source whose pressure field at infinity has a decomposition.

$$P_{source} = \sum_{n=-(N-1)}^{n=N-1} p_n e^{in\theta} \qquad (p_{-n} = \bar{p}_n)$$

surrounded again at a radius a by 2N evenly spaced anti-sources. It may be shown that with optimal cancellation.

$$P_{total} = \sum_{n=-(N-1)}^{N-1} p_n\left[(1 + \alpha_n J_n(ka)) + \alpha_n \sum_{m=1}^{\infty} J_{2mN+n}(ka) i^{(2mN+n)} e^{2mN\theta}\right] e^{in\theta}$$

where the α_n are of opposite sign to $J_n(ka)$. So long as p_0 is not small, the size of the non-cancelling terms will be small so long as $J_{2N}(ka)$ is small compared to $J_0(ka)$, the same criterion as before.

Reference

[1] M Abramowitz and I A Stegun, 'Handbook of Mathematical functions', Dover Press (1964).

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Figure 1 The ratio of the wavelength to inter-source spacing, versus the log of the number of anti-sources.