Proceedings of The Institute of Acoustics

SOUND RADIATION FROM INFINITE PERIODICALLY-STIFFENED FLUID-LOADED PLATES

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Introduction

The response and consequent sound radiation from stiffened plates is of interest in the study of aircraft and ship structures. Amongst previous work is the space-harmonic approach of Mead et al [1,2]. In these papers the harmonic amplitudes were found to satisfy an infinite set of simultaneous equations which were truncated and inverted to find an approximate solution. The transform method presented in this paper is essentially identical to this approach, except that the harmonic amplitudes are found explicitly, simplifying calculation considerably.

Formulation and transform solution

Consider the displacement w of an infinite plate with flexural rigidity D and mass per unit area m excited by a convected harmonic pressure $P_o \exp i(\omega t - k_x x - k_z z)$. The plate, lying in the y=0 plane, is periodically stiffened by identical line stiffeners attached along the lines $x=n\ell$ (n integral) and loaded by a fluid of density ρ occupying the half space $y \geq 0$. Assuming that the t and z dependence is $\exp i(\omega t - k_z z)$ and suppressing this dependence, then the displacement satisfies

$$D\left(\frac{\partial^4 w}{\partial x^4} - 2k_z^2 \frac{\partial^2 w}{\partial x^2} + k_z^4 w\right) - m\omega^2 w = P_o e^{-ik_x x} - p(x,0) - \sum_{n=-\infty}^{\infty} F_n \delta(x-n\ell) + \sum_{n=-\infty}^{\infty} M_n \frac{\partial \delta(x-n\ell)}{\partial x} \tag{1}$$

where p(x,0) is the acoustic pressure in the fluid at y=0 and F_n and M_n are the force and moment exerted on the plate by the n'th stiffener. The acoustic pressure satisfies

$$\nabla^2 p + \frac{\omega^2}{c_o^2} p = 0, \quad \left.\frac{\partial p}{\partial y}\right|_{y=0} = \omega^2 \rho w \tag{2}$$

Taking the Fourier transform of (2) with respect to x leads to

$$\tilde{p}(k) = -\frac{\omega^2 \rho \ell}{\mu} \tilde{w}(k), \quad \mu = \sqrt{(k\ell)^2 + (k_z \ell)^2 - \omega^2 \ell^2 / c_o^2} \tag{3}$$

To satisfy the radiation conditions we must take $Re(\mu) \geq 0$ and $Im(\mu) \geq 0$ if $Re(\mu)=0$.

The transform of (1) together with (3) gives

$$\{D(k^2+k_z^2)^2 - m\omega^2 - \frac{\omega^2 \rho}{\mu}\}\tilde{w}(k) = 2\pi P_o \delta(k-k_x) - \sum_{n=-\infty}^{\infty} F_n e^{ikn\ell} - \sum_{n=-\infty}^{\infty} iM_n k e^{ikn\ell} \tag{4}$$

Since the stiffeners are identical we can assume that

$$F_n = F_o e^{-ink_x \ell}, \quad M_n = M_o e^{-ink_x \ell}$$

Using the Poisson sum formula the first infinite sum then becomes

$$\sum_{n=-\infty}^{\infty} F_n e^{ikn\ell} = F_o \sum_{n=-\infty}^{\infty} e^{-in\ell(k_x - k)} = 2\pi F_o \sum_{n=-\infty}^{\infty} \delta(k\ell - (k_x \ell + 2n\pi)) \tag{5}$$

with a similar expression for the second sum.

Taking the inverse transform of equation (5) gives in dimensionless form

$$w(x)/\ell = \bar{P}_o\beta_o e^{-ik_x x} - \bar{F}_o \sum_{n=-\infty}^{\infty} \beta_n e^{-i(k_x+2n\pi/\ell)x} - i\,\bar{M}_o \sum_{n=-\infty}^{\infty} \Gamma_n e^{-i(k_x+2n\pi/\ell)x} \tag{6}$$

where $\bar{P}_o = P_o\ell^3/D$, $\bar{F}_o = F_o\ell^2/D$, $\bar{M}_o = M_o\ell/D$, $\Omega^4 = m\omega^2\ell^4/D$, $\Gamma_n = (2n\pi+k_x\ell)\beta_n$,

$\mu_n = \sqrt{(k_x\ell+2n\pi)^2+(k_z\ell)^2-\omega^2\ell^2/c_o^2}$ and $\beta_n^{-1} = ((k_x\ell+2n\pi)^2-\Omega^4-\omega^2\rho\ell^5/\mu_n D$.

The acoustic pressure in the fluid is given by

$$p/\omega^2\rho\ell^2 = -\bar{P}_o\beta_o e^{-\mu_o y/\ell} e^{-ik_x x}/\mu_o + \bar{F}_o \sum_{n=-\infty}^{\infty} \beta_n e^{-\mu_n y/\ell} e^{-i(k_x+2n\pi/\ell)x}/\mu_n$$

$$+ i\,\bar{M}_o \sum_{n=-\infty}^{\infty} \Gamma_n e^{-\mu_n y/\ell} e^{-i(k_x+2n\pi/\ell)x}/\mu_n \tag{7}$$

Boundary conditions between the plate and the stiffeners

If we consider beam-type stiffeners with an axis of symmetry perpendicular to the plate, then the boundary conditions which ensure continuity between the plate and the stiffener at x=0 can be written as

$$\bar{F}_o = K_T\, w(0)/\ell\ , \quad \bar{M}_o = K_R\, (\partial w/\partial x)_{x=0} \tag{8}$$

Generally both K_R and K_T are functions of k_z and ω. If we substitute equations (8) into equation (6), then the resulting equations can be solved for $\bar{F}_o$ and $\bar{M}_o$ giving

$$\bar{F}_o = (\frac{1}{K_R} + S_2 - k_x\ell S_1)\,\beta_o\,\bar{P}_o/\Delta\ , \quad i\,\bar{M}_o = (\frac{kx\ell}{K_T} + k_x\ell S_o - S_1)\,\beta_o\,\bar{P}_o/\Delta\ ,$$

$$\Delta = (\frac{1}{K_T} + S_o)\,(\frac{1}{K_R} + S_2) - S_1^2 \tag{9}$$

where $S_o = \sum_{n=-\infty}^{\infty} \beta_n$, $S_1 = \sum_{n=-\infty}^{\infty} \Gamma_n$, $S_2 = \sum_{n=-\infty}^{\infty} (2n\pi+k_x\ell)\,\Gamma_n$. If fluid loading is neglected, the infinite sums can be evaluated by the Poisson sum formula and reduce to trigonometric and hyperbolic functions.

Acoustic radiation

Equations (6), (7) and (9) together give the solutions for w(x) and P(x,y,z). Due to the presence of the periodic stiffeners the wave-number spectrum of the response contains harmonics of wave-number $k_x+2n\pi/\ell$ (n integral). The amplitudes of these harmonics are given by

$$W_n = \ell\{\,\bar{P}_o\,\beta_o\,\delta_{on} - \bar{F}_o\,\beta_n - i\,\bar{M}_o\,\Gamma_n\}$$

Similarly the acoustic pressure contains harmonics whose amplitudes are $P_n = \omega^2\rho\ell\, W_n/\mu_n$. These harmonics contribute to the decaying near-field if μ_n is real, and radiate if μ_n is imaginary, that is if

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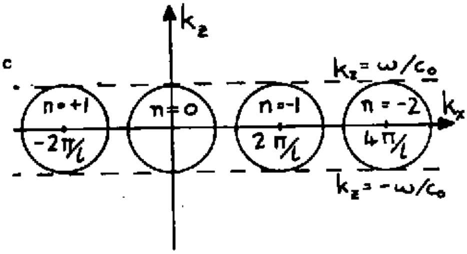
$(k_x+2n\pi/\ell)^2 + k_z^2 < \omega^2 / c_o^2$. The existence of radiation therefore depends on the excitation parameters k_x, k_z, ω, and we can divide the k_x, k_z plane into three regions.

Figure 1.



1). Acoustically fast excitation, the disc $k_x^2+k_z^2<\omega^2/c_o^2$. The primary (n=0) component always radiates, and other components may.

2). Acoustically slow excitation, with $k_z^2>\omega^2/c_o^2$. No radiation.

3). Acoustically slow excitation, with $k_z^2<\omega^2/c_o^2$. If we also have $\omega^2/c_o^2 - ((m+1)\pi/\ell)^2<k_z^2<\omega^2/c_o^2-(m\pi/\ell)^2$ with m integral and ≥ 0, then as k_x increases there will be alternating regions where m+1 and m harmonic radiate.

Figure 2.

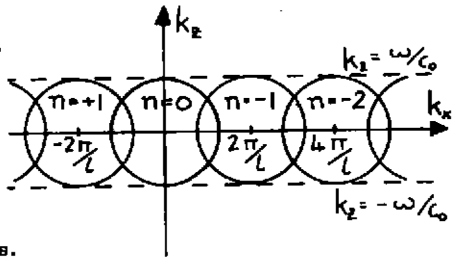


Figure 1. shows the case where $\omega^2/c_o^2<\pi^2/\ell^2$ (i.e. $\ell<$ half an acoustic wavelength. In this case throughout the strip $k_z^2<\omega^2/c_o^2$ there are alternating regions of no radiation and radiation from one harmonic. If the point (k_x,k_z) lies in the circle centred at $2n\pi/\ell$ then the -n'th harmonic radiates.

Figure 2. shows the case where $\pi^2/\ell^2<\omega^2/c_o^2< (2\pi)^2/\ell^2$. Again if $\omega^2/c_o^2-\pi^2/\ell^2<k_z^2$ there are either one or no radiating harmonics, but if $k_z^2<\omega^2/c_o^2-\pi^2/\ell^2$, then there is always one radiating harmonic, and sometimes two.

The total sound power radiated per unit area of the plate is equal to the sum of the powers radiated by those harmonic which are acoustically fast. Thus the power radiated

$$\Pi = \sum_{\text{radiators}} \frac{1}{2} \rho_o \omega^3 \; |W_n|^2 / \sqrt{\omega^2/c_o - (k_x + 2n\pi/\ell)^2 - (k_z\ell)^2} \quad (10)$$

Radiation from a point excited plate

We can find the response and radiation due to general excitation from the solution given above for harmonic excitation. Denoting W_n now as the harmonic amplitude for unit convected pressure, then the response $R(x,z)$ to a time-harmonic excitation $\mathcal{F}(x,z)$ is given by

$$R(x,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int \sum_{n=-\infty}^{\infty} W_n(k_x,k_z)\, e^{-i2n\pi x/\ell}\, \tilde{\mathcal{F}}(k_x,k_z) e^{-i(k_x x+k_z z)}\, dk_x\, dk_z \quad (11)$$

where $W_n(k_x,k_z) = \ell(\beta_o\delta_{on} - \bar{F}_o \beta_n - i \bar{M}_o \Gamma_n)$ and $\tilde{\mathcal{F}}(k_x,k_z)$ is the Fourier transform of $\mathcal{F}(x,z)$. The corresponding acoustic pressure is

$$p(x,y,z) = -\frac{\omega^2\rho\ell}{(2\pi)^2}\int_{-\infty}^{\infty}\int \sum_{n=-\infty}^{\infty} \frac{W_n(k_x,k_z)}{\mu_n(k_x,k_z)} e^{-i2n\pi x/\ell} e^{-\mu_n y x/\ell} \tilde{\mathcal{F}}(k_x,k_z)\, e^{-i(k_x+k_z z)} dk_x dk_z \qquad (12)$$

If we substitute $k_x'=k_x+2n\pi/\ell$ in the n'th term of the sum, then since $\mu_n(k_x'-2n\pi/\ell, k_z) = \mu_o(k_x',k_z)$, and $\bar{W}_n(k_x-2n\pi/\ell,k_z) = W_{-n}(k_x,k_z)$ then

$$p(x,y,z) = -\frac{\omega^2\rho\ell}{(2\pi)^2}\int_{-\infty}^{\infty}\int \sum_{n=-\infty}^{\infty} \frac{W_n(k_x,k_z)}{\mu_o(k_x,k_z)} \tilde{\mathcal{F}}(k_x+2n\pi/\ell,k_z)\, e^{-\mu_o y/\ell} e^{-i(k_x x+k_z z)} dk_x dk_z \qquad (13)$$

Only the integral over the disc $k_x^2+k_z^2 < \omega^2/c_o^2$, where μ_o is imaginary, contributes to the radiated pressure in the far-field, the remainder of the integral giving the decaying near-field.

For a point force F applied at $(x_o,0)$, the far-field pressure in spherical co-ordinates R,θ,ϕ is, by the method of stationary phase:

$$P(R,\theta,\phi) = -\rho\omega^2\mathcal{F}\ell\, e^{ik_x x_o}\left|\beta_o - \sum_{n=-\infty}^{\infty}(\bar{F}_o\beta_n + i\bar{M}_o\Gamma_n)e^{i\ell n\pi x_o/\ell}\right| \frac{e^{-iR\omega/c_o}}{2\pi R} \qquad (14)$$

where $\mathcal{F} = F\ell^3/D$. The first term in the brackets gives the radiation from an unstiffened plate, the other terms giving the additional radiation induced by the stiffeners.

<u>The evaluation of the infinite sums when fluid loading is neglected</u>

When fluid loading can be neglected, then the infinite sums are:

$$T_o(x,k_x,k_z) = \sum_{n=-\infty}^{\infty} \beta_n\, e^{-i(k_x+2n\pi/\ell)x} \qquad (15)$$

$$= \frac{1}{4\Omega^2}\left[\frac{\sin\lambda_1(1-x/\ell)+e^{-ik_x\ell}\sin\lambda_1 x/\ell}{\lambda_1(\cos\lambda_1-\cos k_x\ell)} - \frac{\sinh\lambda_2(1-x/\ell)+e^{-ik_x\ell}\sinh\lambda_2 x/\ell}{\lambda_2(\cosh\lambda_2-\cos k_x\ell)}\right]$$

where $\lambda_1^2 = \Omega^2-(k_z\ell)^2$, $\lambda_2^2 = \Omega^2+(k_z\ell)^2$ and $0 \le x < \ell$.

$$T_1(x,k_x,k_z) = \sum_{n=-\infty}^{\infty} \Gamma_n\, e^{-i(k_x+2n\pi/\ell)x} = i\ell\,\frac{\partial T_o}{\partial x}$$

$S_o = T_o(0,k_x,k_z)$, $S_1 = T_1(0,k_x,k_z)$, $S_2 = i\ell\,(\partial T_1/\partial x)_{x=0}$

<u>References</u>

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2. D.J. MEAD and A.K. MALLIK 1978 Journal of Sound and Vibration 61, 315-326. An approximate theory for the sound radiated from a periodic line-supported plate.