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THE EFFECTS OF ACCELEROMETER INERTIA ON MEASUREMENTS OF BEAM VIBRATION

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1. INTRODUCTION

In any attempt to measure vibration with an accelerometer, the mass of the accelerometer will inevitably affect the dynamics of the structure on which it is mounted. The vibration levels and natural frequencies of the mass loaded structure will therefore differ from those of the unloaded one. It is, of course, desirable that these differences are acceptably small. This paper considers the case where the accelerometer is mounted on a part of a system which vibrates as a beam in bending. Estimates are provided for the average and maximum differences involved which are seen to depend on the mass and moment of inertia of the accelerometer and also on its location on the structure.

The transducer introduces errors as indicated in figure 1. The overall levels are changed from a to a^* while the natural frequencies are reduced from ω_n to ω_n^*. A simple experimental technique that may be used to estimate these errors is to introduce an additional mass at the response point. A new measurement is taken and by comparing the loaded and doubly-loaded measurements, estimates of the perturbing effects can be found. While this method may be viable for vibration-testing purposes error estimates can only be obtained after measurements have been taken and cannot therefore give prior indications of the errors involved. Additionally, it is not applicable if the excitations cannot be reproduced.

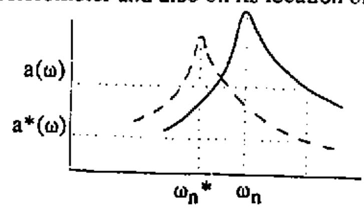

Fig. 1 Response against frequency: —— unloaded; ----- loaded.

A second experimental technique, which explicitly corrects for the added mass, is that of mass cancellation [1]. It, however, requires knowledge of the point inertance α (acceleration per unit force) at the response point. If the measured acceleration is a^* then the accelerometer introduces an additional force $-ma^*$ at its mounting point. Hence, the acceleration in the absence of the accelerometer is $a = (1 + m\alpha)a^*$. For the special case of vibration testing where the structure is excited at a point and the acceleration is measured at this point then the point inertance is given in terms of the measured inertance α^* by $1/\alpha = 1/\alpha^* - m$. In practice, this can be achieved by either post-processing the measured data or by subtracting a signal proportional to the measured acceleration from the measured force, thus producing a signal proportional to the net force applied to the structure [1]. The method requires additional information concerning the system's point inertance which may not be available. Furthermore, if the rotational inertia of the accelerometer is important, particularly if measurements are being taken on thin beams or panels or using a larger accelerometer or an impedance head, then information concerning the displacement or rotation per unit force or moment is necessary. This information may not be available.

Theoretical estimates for the magnitudes of the changes can be found if the effective mass m_e at the transducer location can be estimated [2]. Typically, the acceleration level and natural frequencies decrease by amounts of the order of m/m_e and $m/2m_e$ respectively. For a lumped mass system the effective mass will be greater or equal to the mass of the element on which the accelerometer is mounted. If, however, the accelerometer is mounted on a continuous element, such as the flexible beams considered in this paper, then the effective mass is less easy to determine. It may be estimated from the input inertance of an equivalent infinite structure. Typically this will be of the same order as the mass of the continuous system in half a wavelength. Again, it is not clear how rotational inertia should be taken into account.

This paper considers the mass loading effects of a transducer mounted on a part of a system which vibrates as a beam in bending. Both the mass and moment of inertia of the transducer are considered. The main results from [3] are reviewed and some experimental results presented. It is found that the rotational inertia and the location of the transducer have substantial effects on the changes in natural frequencies while the changes in level depend primarily on the mass and location of the transducer.

2. THE PERTURBING EFFECTS OF TRANSDUCER INERTIA

The vibrations of the beam can be regarded as consisting of propagating and near-field wave components travelling in both directions. Their amplitudes at the transducer location are indicated in Figure 2. The transducer scatters incident waves and the scattered waves have amplitudes that are determined by reflection and transmission matrices [4,5]. The effects of transducer inertia appear in these matrices as two parameters,

$$\varepsilon = mk/4\sigma \ ; \ \varepsilon_R = Jk^3/4\sigma = \alpha\varepsilon^3 \qquad (1)$$

where m and J are the transducer mass and moment of inertia and σ and k are the mass per unit length and wavenumber of the beam. Thus ε and ε_R represent the effects of translational and rotational inertia respectively. The constant $\alpha = (4\sigma e/m)^2$ where e is the radius of gyration of the transducer. Note that as frequency increases k increases as do the mass loading parameters. Furthermore rotational effects become relatively more important at higher frequencies, as one would expect.

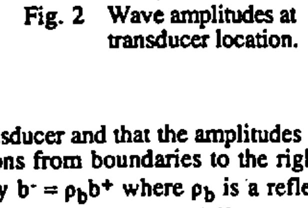

Fig. 2 Wave amplitudes at transducer location.

If it is assumed that no excitations act to the right of the transducer and that the amplitudes at the transducer of near-field components generated by reflections from boundaries to the right are negligible then the propagating components are related by $b^- = \rho_b b^+$ where ρ_b is a reflection coefficient. On the other side of the transducer $a^- = \rho_a a^+$ where ρ_a and ρ_b differ due to the perturbing presence of the transducer. This change in reflection coefficient changes the natural frequencies and vibration amplitudes as follows.

2.1 Natural Frequencies

For a finite structure in the absence of damping the reflection coefficient ρ_b represents a pure phase change. The transducer introduces an additional phase lag

$$\psi = 2 \tan^{-1} \frac{\varepsilon + \varepsilon_R + \mu \cos \Phi_R}{1 + \mu + \mu \sin \Phi_R} \tag{2}$$

where $\mu = \varepsilon - \varepsilon_R - 2\varepsilon\,\varepsilon_R$ (3)

The wavenumber at each natural frequency is thus reduced by $\psi/2l$ where l is the flexural path length and the consequent change in the n'th natural frequency is

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{4M}\frac{\psi}{\varepsilon} \tag{4}$$

where $M = \sigma l$ is the total mass of the flexural element (i.e. total mass neglecting added point masses). The phase angle Φ_R is generally an unknown random variable uniformly probable between 0 and 2π. The change in natural frequency has a mean value

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{2M}\frac{\psi_o}{\varepsilon} \quad ; \quad \psi_o = \tan^{-1}\frac{\varepsilon + \varepsilon_R}{1 + \mu} \tag{5}$$

and fluctuates between the limiting values

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{2M}\left(\frac{\psi_o}{\varepsilon} \pm \frac{\psi_d}{\varepsilon}\right) \quad ;$$

$$\psi_d = \tan^{-1} \frac{\mu}{\varepsilon - \varepsilon_R + 1} \tag{6}$$

Consider first for simplicity the case where rotational effects are neglected (i.e. $\varepsilon_R = 0$). Now

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{2M}\frac{1}{\varepsilon}\tan^{-1}\frac{\varepsilon\,(1 + \cos \Phi_R)}{1 + \varepsilon\,(1 + \sin \Phi_R)} \tag{7}$$

The natural frequency is decreased. The mean of the decrease is

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{2M}\frac{1}{\varepsilon}\tan^{-1}\frac{\varepsilon}{1 + \varepsilon} \tag{8}$$

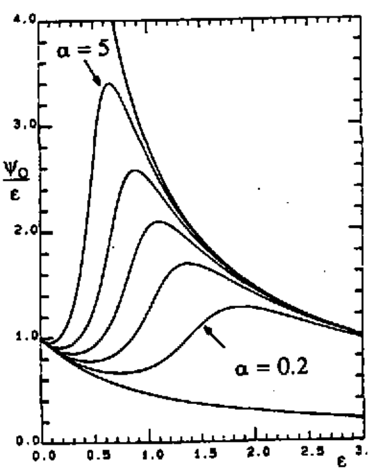

Fig. 3 Mean decrease in natural frequency: $\delta\omega_n/\omega_n = (-m/2M)\,\psi_o/\varepsilon$; α = 0, 0.2, 0.5, 1, 2, 5, ∞.

which tends to 0 as ε increases. If the accelerometer is located at a node, then $\cos \Phi_R = -1$ and $\delta\omega_n = 0$. As would be expected the accelerometer mass has no effect. The largest changes occur if the accelerometer is located close to an antinode and are equal to twice the mean.

Returning now to the case where rotational effects are not ignored, the mean decrease in natural frequency is indicated in Figure 3 which shows the variation of ψ_o/ε with ε for a range of values of α, where $\varepsilon_R = \alpha\varepsilon^3$. Firstly, it should be noted that since ψ_o is positive the added inertia always decreases the natural frequencies. For small ε rotational effects are small and $\delta\omega_n$ is given by equation (8). However, for large ε rotational effects dominate and ψ_o/ε asymptotes towards the curve for $\alpha \to \infty$ which is such that

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{2M\varepsilon}\left(\frac{\pi}{2} + \tan^{-1}(1 + 2\varepsilon)\right) \le \lambda/l \quad (9)$$

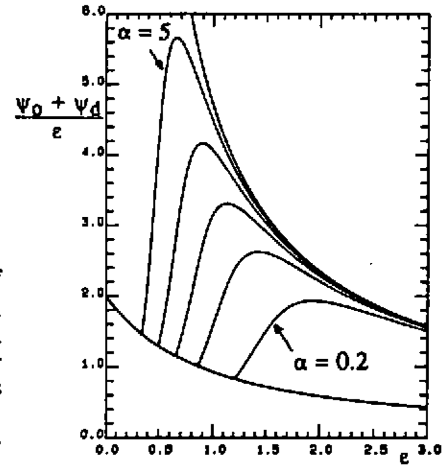

Fig. 4 Maximum decrease in natural frequency: $\delta\omega_n/\omega_n = (-m/2M)(\psi_o + \psi_d)/\varepsilon$; $\alpha = 0, 0.2, 0.5, 1, 2, 5, \infty$.

where λ is the wavelength in the beam.

Transition between these two regions occurs approximately when $\varepsilon_{tr} = (2\alpha)^{-\frac{1}{3}}$ at which value $\varepsilon_R = \frac{1}{2}$ and $\mu \doteq -\frac{1}{2}$. The peaks of ψ_o in Figure 3 occur approximately where $\varepsilon = \alpha^{-\frac{1}{3}}$ at which value $\mu = -1$. Rotational effects begin to be significant at about half this value, namely $\varepsilon \approx 1/2\alpha^{\frac{1}{3}}$.

Figure 4 shows $(\psi_o + \psi_d)/\varepsilon$ for a range of α and thus indicates the maximum decrease in natural frequency that could occur. Once again since $\psi_d \le \psi_o$ natural frequencies decrease and $\psi_d + \psi_o \le 2\psi_o$. At low frequencies the maximum change in natural frequency is approximately twice the mean while at high frequencies, where rotational effects dominate, the maximum change asymptotes to

$$\frac{\delta\omega_n}{\omega_n} = -\frac{m}{M}\frac{3\pi}{4\varepsilon} \le 3\lambda/2l \quad (10)$$

These results may be summarised to put simple bounds on the maximum decrease in natural frequency. Firstly, at low frequency, where $\varepsilon < 1/2\alpha^{\frac{1}{3}}$ rotational effects are unimportant and

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$$\left|\frac{\delta\omega_n}{\omega_n}\right| \leq \frac{m}{M}\,\frac{1}{1+\varepsilon} \tag{11}$$

For high frequencies such that $\varepsilon > 1/\alpha^{\frac{1}{3}}$ rotational effects dominate and

$$\left|\frac{\delta\omega_n}{\omega_n}\right| \leq \frac{3\lambda}{2l} \tag{12}$$

Between these limits, around ε_{tr}, both rotational and translational effects are relevant.

2.2 Forced Response
If forces act to the left of the transducer in Figure 2 then the transducer inertia affects the vibration levels, the changes depending on whether reflections from boundaries are important.

2.2.1 Non-Resonant Structures. These are structures which are large enough and have sufficiently high damping so that ρ_b is negligible. The ratio of the responses of the unloaded and unloaded structures is

$$\frac{w^*}{w} = \frac{1}{1+(1+i)\,\varepsilon} \tag{13}$$

The rotational inertia of the transducer does not affect the displacement, but it does affect the slope. As frequency increases ε increases so the loaded response decreases.

2.2.2 Resonant Structures. Since the natural frequencies in the unloaded and loaded cases differ, it is appropriate to compare the responses at different frequencies, separated according to equation (4), so as to compare responses at similar points on the resonance curves of the two systems. In that case, ignoring rotational inertia which has no effect in the non-resonant case, the responses are related by

$$\frac{w^*}{w} = \frac{1}{1+(1+i)\,\varepsilon + i\varepsilon\rho_b} \tag{14}$$

For light loading, this fluctuates about a mean given by equation (13) and approximately between the limits of 1 and $1/(1+2\varepsilon)$.

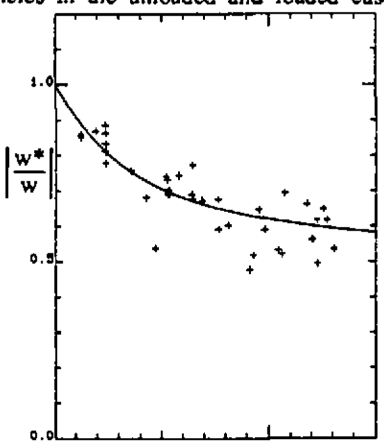

Fig. 5 Change in response level: + measured non-resonant response; —— theory.

3. EXPERIMENTAL RESULTS

3.1 Procedure
An experiment was performed on a freely suspended aluminium beam of cross-section

32 mm x 2.9 mm for which M = 0.274 kg, EI = 3.26 Nm^2, σ = 0.273kg/m. It was excited 0.302 m from one end by a Ling model V201 exciter whose input signal was a transient generated by a Wavetek model 75 arbitrary waveform generator. A Bruel and Kjaer accelerometer type 4367 (mass 13.2 g) was used to measure the acceleration at the response point. The response was measured at 5 different points, 10 mm apart, the first point being 0.702 m from the end.

Measurements were taken to give results over a frequency range up to 5 kHz. Firstly the transfer accelerance (acceleration per unit force) was measured. Then an additional mass of 13.2 g was attached at the response point and a new measurement taken. The effect can be regarded as comparing the behaviour of a uniform beam with perturbing masses of m and 2m, with m = 13.2 g. For this beam and accelerometer $\varepsilon_R \approx 0.758\varepsilon^3$ and the frequency range corresponds to values of ε and ε_R up to about 1.2.

3.2 Change in Level

The theoretical change in response level corresponds to the ratio of the doubly loaded and singly loaded responses. In Figure 5 non-resonant levels are compared. These have been measured at frequencies between resonances where the response amplitude varies very slowly with frequency. The agreement between the theoretical and experimental results can be seen to be good.

Figure 6 shows level changes measured at resonances, together with the theoretical mean and maximum changes. Much more scatter in the results is evident due primarily to the dependence of the level change on accelerometer position. However, the results generally lie within the predicted limits, differences being attributable partly to damping but also to rotational effects.

The resonance around ε = 0.6, for example, indicates that the doubly-loaded level is larger than the singly-loaded one. This is due to the fact that this mode is only weakly excited so that

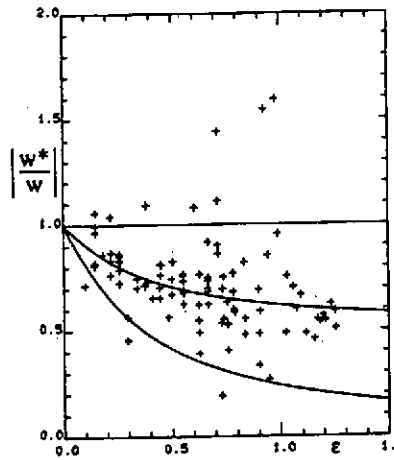

Fig. 6 Change in response level: + measured resonant response; —— theoretical mean and small ε limits.

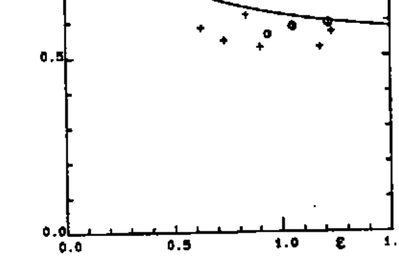

Fig. 7 Space-averaged change in response level: + resonant; ∘ off-resonant; —— theory

translational effects are small whereas rotational ones are substantially larger. Two other causes of increased scatter are any changes in modal damping the accelerometer may introduce and the poorer resolution that can be obtained experimentally at lightly damped resonances.

In Figure 7 an attempt has been made to "average out" the effects of the accelerometer location by averaging the level changes of Figures 5 and 6 over the different measurement positions where results from four or more points were available. The theoretical and experimental mean results can be seen to agree well.

3.3 Change in Natural Frequency

The changes in natural frequencies are shown in Figure 8. The theoretical curves give the mean, maximum and minimum changes between the singly and doubly loaded cases. Scatter of results is evident due to the dependence on the accelerometer location. However, the experimental results generally lie within the theoretical limits. Clear low and high frequency ranges, where rotational effects are negligible and dominant respectively, can clearly be seen. The transition between these takes place around $\varepsilon_{tr}=(2\alpha)^{-\frac{1}{3}} \approx 0.87$, at frequencies around 2.8kHz.

In Figure 9 the effects of accelerometer location are "averaged out" by averaging the experimental changes in natural frequency where results from four or more measurement points were available. These averaged results can be seen to agree well.

At higher frequencies, the experimental decreases are consistently smaller than the theoretical ones. This is thought to be due to two factors. Firstly, damping has been ignored in the derivation of the theoretical results. Secondly, at high frequencies, where rotary inertia becomes most important, the larger decreases in natural frequency tend to occur when the accelerometer is mounted at a position of large slope. These positions necessarily have small or zero

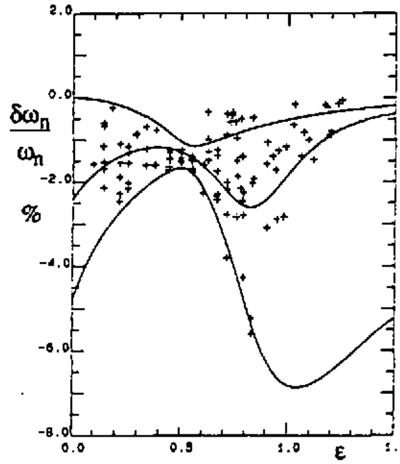

Fig. 8 Change in natural frequency (%): + measured; ——— theoretical mean, minimum and maximum.

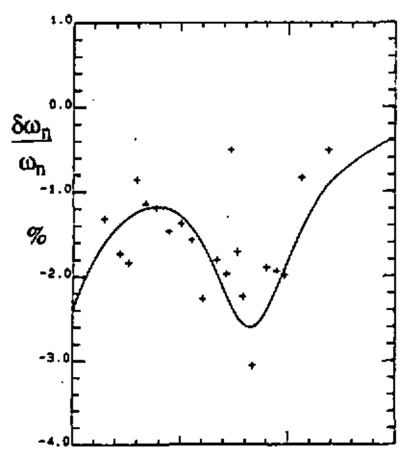

Fig. 9 Space-averaged change in natural frequency (%): + measured; ——— theory.

displacements and therefore are difficult to detect by measuring the acceleration at the accelerometer location.

4. CONCLUDING REMARKS

In the previous sections the perturbing effects of the transducer were seen to depend on the mass and moment of inertia through the parameters ε and ε_R. For finite systems the location of the transducer is important. The location was described in terms of a phase angle Φ_R which in practice is a random variable. The perturbing effects for finite structures were therefore described by a mean value, independent of location, and a maximum value achieved for certain specific locations.

The acceleration levels are reduced according to equations (13) and (14). The rotational parameter ε_R does not appear in these expressions but it does significantly affect the reduction in natural frequencies. The mean and maximum decreases are given by equations (5) and (6). In general terms at low frequencies ($\varepsilon < 1/2\alpha^{-\frac{1}{3}}$) rotational effects are not important and the reduction is less than the ratio of transducer and beam masses, m/M. At higher frequencies, when rotational effects dominate, the reduction is governed by the ratio of the flexural wavelength to the length of the beam and is less than $3\lambda/2l$. In practical situations rotational effects may well be important, especially for larger accelerometers or impedance heads.

One of the assumptions made was that near-field interaction terms are negligible. These should have negligible effects so long as the length of the flexural element is, say, half a wavelength, and may well be valid for much shorter beams. In the analysis of finite structures damping was ignored although it could be included through the reflection coefficients, ρ_b. Small levels of damping have negligible effects on natural frequencies, but they may have larger effects on damping factors of individual modes.

As far as the flexural vibration of plates and panels is concerned the results above do not in general hold exactly but do give an indication of the magnitude of the changes in level and natural frequency involved. It is appropriate here to substitute $4\rho_s/k$, where ρ_s is the surface density, for the beam mass per unit length σ.

5. ACKNOWLEDGEMENTS

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