Proceedings of the Institute of Acoustics

ACOUSTIC SCATTERING FROM A BUBBLE PLUME

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1. INTRODUCTION

The main objectives of this paper are to report on some measurements taken of the acoustic backscatter from and transmission loss through a plume of air bubbles and to present comparisons between the backscatter results and theoretical predictions. This work is part of an ongoing study.

The measurements were made over a frequency range of 21 kHz to 50 kHz on several different plumes. The plumes were observed to be approximately circular in cross-section with diameters of the order of 6 to 12 cm. For the plume containing the smallest range of bubble sizes it was calculated, from the results of the measurements of the acoustic transmission loss, that the distribution of the bubble radii, a, approximately covered the range, 60 μm to 170 μm, with a peak in the distribution around 115 μm. The bubble number density, n(a), around this peak was estimated to be about 16×10^9 per m^3. The estimated distribution was found to be in reasonable agreement with the known gas flow rate producing the plume. For the other plumes which contained bubbles with larger bubble radii it was possible to estimate neither the exact size range nor the number density from the transmission loss measurements.

For the case of the plume with the "known" number density distribution it was found that predictions of acoustic backscatter, using single scattering theory, were in rough agreement with observations. However, it is thought that the effect of extinction needs to be properly included to fully explain the results. For the plume with the largest bubble-size range again it was found that theoretical predictions of the backscatter were in agreement with the observations. However, for the plumes with intermediate bubble sizes, theoretical predictions of backscatter when compared with observations were not encouraging.

From this investigation it is proposed that there are three distinct cases which describe the experimental conditions. The first is where the majority of the bubbles in the plume have resonances which coincide with the frequency range of the acoustic measurements (this corresponds to the case of the smallest bubbles). The second is where virtually none of the bubbles in the plume have resonances which coincide with the acoustic frequencies (this is thought to correspond to the case of the plume with the largest size of bubbles). Finally there is the case where only a proportion of the bubbles have resonances which coincide with the acoustic frequencies.

This work was supported by Procurement Executive, Ministry of Defence

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2. THEORETICAL CONSIDERATIONS

2.1 Transmission Loss

The ratio of the coherent transmitted intensity to the incident intensity of an acoustic wave propagating through a slab containing bubbles is given by, for example, Ishimaru [1]:-

$$\exp(-\gamma) = \exp(-\rho<\sigma_t>L),$$

where γ is the 'optical distance',

$\rho = \int_0^\infty n(a)da$ is the total bubble density, n(a) is the number density, ie. the number of bubbles per unit volume in the differential radial range $a \rightarrow a + da$, $<\sigma_t>$ is the average extinction cross-section of the bubbles and L is the thickness of the slab.

The average extinction cross-section becomes:-

$$<\sigma_t> = \int_0^\infty W(a)\sigma_t(a)da,$$

where $W(a) = \frac{n(a)}{\rho}$ is the pdf for the bubble radii.

A graph of extinction cross-section against frequency is sharply peaked at the resonant frequency and so the main contribution to extinction is from a small band of bubbles, Δa_R, with radii centered on the resonant radius, a_R. So, the integral in the expression for the optical distance can be approximated in the following manner (Medwin [2]):-

$$\gamma = L\int_0^\infty n(a)\sigma_t(a)da$$

$$\approx Ln(a_R)\sigma_t(a_R)\Delta a_R$$

$$but \qquad \Delta a_R \approx a_R\delta_R$$

$$\therefore \gamma \approx Ln(a_R)\sigma_t(a_R)a_R\delta_R$$

where δ_R is the damping constant at resonance.

If the transmission loss, expressed in dB at a frequency f_R, measured over a pathlength L is written as TL_R, then the optical distance at this frequency is given by:-

$$\gamma = \frac{TL_R}{4.33}$$

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So the number density, $n(a_R)$ at f_R is given by:-

$$n(a_R) \approx \frac{TL_R}{4.33 L \sigma_t a_R \delta_R}$$

This expression enables the bubble number density to be estimated at all frequencies from the transmission loss measurements. This will be called the Resonant Bubble Theory.

However if only a small proportion of the bubbles are resonant, then there may be a significant contribution to the transmission loss from the extinction produced by the non-resonant bubbles. Under these circumstances it is unlikely that the above expression is applicable.

In the case of entirely non-resonant bubbles the extinction cross-section is approximately equal to the backscattering cross-section, σ_b, [1]. Under these circumstances:-

$$\gamma = \rho < \sigma_t > L \approx \rho < \sigma_b > L$$

This latter expression will also be used for making theoretical predictions. This will be called the Non-resonant Bubble Theory.

2.2 Backscattering Cross-section

The scattering cross-section, σ_s, is defined by, for example [3]:-

$$\sigma_s = \frac{4\pi a^2}{\left[\left(\frac{f_R}{f}\right)^2 - 1\right]^2 + \delta^2}$$

where a is the bubble radius, f_R is the resonant frequency and δ is the damping constant. The geometrical cross-section, σ_g is of course equal to the cross-sectional area, πa^2.

The scattering from a bubble is omnidirectional and so the backscattering cross-section is approximately equal to the scattering cross-section, ie.:-

$$\sigma_b \approx \sigma_s$$

2.3 Single Scattering from a Cylindrical Plume

Consider that the acoustic source and receiver are located at the origin of a general spherical co-ordinate system as shown in Figure 1. The elemental scattering volume, dv, has co-ordinates r, ϕ, θ. Also, dv is positioned using cylindrical polar co-ordinates, ξ, z, ψ, relative to the axis of the vertical bubble plume.

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In general the bubble and number densities will be functions of position within the plume, but it is most likely that they will exhibit cylindrical symmetry and so they will only be functions of $\xi \,\&\, z$, ie.:-

$$\rho(\xi, z) \quad \& \quad n(a, \xi, z)$$

Therefore:-

$$\rho(\xi, z) = \int_0^\infty n(a, \xi, z)\,da$$

The average backsattering cross-section for scatterers within the volume dv then becomes:-

$$\langle\sigma_b\rangle = \int_0^\infty \frac{n(a, \xi, z)}{\rho(\xi, z)} \sigma_b(a)\,da$$

Let the acoustic intensity at dv for a particular frequency be:-

$$I_r = \frac{D(\theta, \phi) I_0}{r^2}$$

where I_0 is the axial intensity of the source at unit range and $D(\theta, \phi)$ is the source directivity.

Absorption has been neglected since it is negligible for the conditions of the experiments described later. Also extinction has been neglected because a single scattering approach is being assumed. Therefore the total backscattered intensity is given by:-

$I_{bs} \approx \frac{I_0}{4\pi} \rho \langle\sigma_b\rangle \int_v \frac{D(\theta, \phi)}{r^4} dv$,where $\rho\langle\sigma_b\rangle$ has been assumed to be constant over the plume. The volume integral in this above expression is the reverberant volume V_{rev}. In addition to the effects of directivity and spherical spreading the volume is also dependent upon the pulse length, τ, through the factor $\frac{c\tau}{2}$. Therefore an approximate expression for the backscattered intensity under the assumption of single scattering is given by:-

$$I_{bs} \approx \frac{I_0}{4\pi} \rho \langle\sigma_b\rangle V_{rev}.$$

In the experiments the measured backscattered signal is compared to the echo that would be received from a perfect planar reflector (in the absence of the plume), which is located at a distance, r, equivalent to the centre of the plume. The echo intensity from this reflector is simply, $\frac{I_0}{4r^2}$. A normalised echo level, NEL, expressed in dB can be defined as:-

$$NEL = 10 \log_{10}\left(\frac{\rho\langle\sigma_b\rangle V_{rev}}{\pi r^2}\right)$$

This is the expression that is used to compare theory with experiment. For the Resonant Theory the product of the bubble density and averaged backscattering cross-section is calculated by integrating, over the bubble radial range, the backscattering cross-section weighted by the number density.

For the Non-resonant Theory the product of the bubble density and averaged backscattering cross-section is determined directly from the transmission loss.

3. ACOUSTIC MEASUREMENTS

3.1 Experimental Conditions

The measurements were made in a tank of water of dimensions 10mx4mx3m. Several bubble generators were tried. The one finally used in the measurements comprised a single orifice controlled by a needle valve and supplied with air. The orifice was surrounded by an annular orifice through which water could be pumped at varying rates. The air flow was held constant at a rate of 30 cc/min indicated on a "Rotameter" type flowmeter calibrated for air at NTP. The air pressure was approximately 300 mbar above the assumed atmospheric pressure of 1000 mbar and the mass flow rate of the air was estimated to be 0.75 mg/s. The water flow was varied from zero to 3.0 l/min. The effect of the faster flow rate was to break up the air stream into smaller bubbles.

Photographs of the plumes were taken and although the resolution was not satisfactory to enable the bubbles to be sized it was possible to establish that a symmetrical bubble plume was being generated and that the lower flow rates produced larger bubbles. From these photographs crude estimates of the "widths" of the plumes were made.

The transducers used for the acoustic measurements comprised two identical sandwich elements resonant at 30 kHz. The housings were covered with closed-cell rubber to reduce the effect of unwanted reverberation. The bandwidth was sufficient to enable measurements to be made from 21 kHz to 50 kHz in 1 kHz steps. The front face diameter of an element was 31 mm, approximately half a wavelength at 21 kHz. Unexpectedly it was observed that the beamwidths of the transducer changed very little over the bandwidth.

The source transducer was driven by gated sinusoidal signals of 0.8 ms gate duration, with an interval of 180 ms between successive transmissions. The received signals were passed to a switched gain amplifier, through a low-pass filter of varying cut-off frequency, dependent on the frequency being transmitted, and thence to a signal analyser. This measured the r.m.s. voltage from many samples taken during a time window 0.4 ms long, set so as to select that portion of the signal which was least affected by transients and unwanted reverberation. The transmission level and gain settings were set for each frequency during a calibration run without the bubble plume present. The voltages of 1000 transmissions were analysed by a microcomputer to calculate the mean and standard deviation.

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3.2 Transmission Loss

Transmission loss measurements were made horizontally, through the rising plume of bubbles, between the two transducers spaced 2.05 m apart at a depth of 1.36 m. Care was taken to ensure that the plume was accurately aligned in the direct path between the transducers and approximately midway between them. At each frequency the attenuation of the transmission relative to that in the absence of the plume was measured. The standard deviation, expressed as a percentage of the mean signal, was mostly less than 1% for the transmission in the absence of bubbles. It was thus concluded that the background noise and reverberation were negligible. The results for a number of different flow rates are plotted in Figure 2.

For a water flow rate of 0.5 l/min the attenuation shows no clear cut dependence upon the frequency. At 1.0 l/min there is a suggestion that the attenuation increases as the frequency is reduced and at 1.5 l/min this trend is more marked. At 3.0 l/min there is a distinct peak in the attenuation around 23 kHz, which suggests that there is a maximum in the bubble number density at a bubble radius whose bubble resonance coincides with this frequency. Generally a close correlation was observed between the standard deviation and the transmission loss.

3.3 Backscattering Measurements

The geometry and signal processing for the backscattering measurements were the same as with the transmission measurements except that the receiving transducer was placed next to the source. However, the backscattered signal was very much lower than the attenuated, directly transmitted signal and it was necessary to make some corrections for the background reverberation, particularly at low flow rates and at low frequencies. With the bubble plume switched off the pair of transducers was also aimed vertically upwards to look at the reflection from the surface of the water. This provided the reference signal for the normalisation of the backscattered signal. The measured normalised backscattered echo levels are plotted in Figure 3. The background reverberation level with the plume switched off is also shown for comparison. The results at frequencies below about 26 kHz generally show considerable variations from frequency to frequency.

4. COMPARISONS WITH PREDICTIONS

4.1 Resonant Theory

The observed peak in the transmission loss at a 3.0 l/min flow rate suggests that the bubble resonances largely coincide with the frequencies used in the measurements. So, it was proposed that it should be possible to predict the bubble number density from the transmission loss plot, using the Resonant Theory outlined in section 2.1. By this method the number density plot of Figure 4 was obtained. In calculating this curve a plume diameter of 12 cm was used which was estimated from the photographs. From Figure 4 it may be seen that the bubble radii cover the range from about 60 μm to about 170 μm. This distribution seems reasonable and gives results consistent with the total known air-flow rate.

Using this calculated number density the normalised backscatter echo level (NEL) was calculated. The results are compared with the measurements for the 3.0 l/min case in Figure 5(a). In general the levels are of a similar order, but the trend with frequency looks to be somewhat different. A more accurate theory would be first-order multiple scattering in which the waves have attenuation because of the extinction. This would reduce the backscatter at lower frequencies because the extinction is greater and so the trend with frequency would be more consistent with the measurements. However because of the considerable ammount of extinction at these lower frequencies a full multiple-scattering treatment may be necessary to completely explain the results.

4.2 Non-resonant Theory

If the Resonant Theory is used for the 1.0 and 1.5 l/min cases the resulting bubble densities are not consistent with what is known about the size of bubbles. This suggests that non-resonant bubbles make a significant contribution to the transmission loss in these cases. If the bubbles were all non-resonant then the average backscattering cross-section and hence NEL can be directly estimated from the transmission loss results using the Non-resonant Theory of section 2.1. This has been done for these two cases and the results are compared with the measurements in Figures 5 (b) and 5(c). The general trend of these predicted curves with frequency generally agree, but the overall levels do not. The difference reduces as the flow rate reduces, but these differences cannot be explained by extinction.

Using the Non-resonant Theory for the 0.5 l/min case, Figure 5(d), the predictions generally compare well with measurements, although there are considerable variations from frequency to frequency. The background reverberation (noise) is also shown.

5. CONCLUSIONS

When the resonances of the bubbles in the plume coincide with the acoustic frequencies it is possible to estimate the bubble number density from transmission loss measurements. From this density and the backscattering cross-section it is then possible to predict the backscattering from the plume. It is thought that a multiple scattering approach is necessary to fully explain the results.

In the other extreme when very few, if any, of the bubbles in the plume have resonances which coincide with the acoustic frequencies, then it is possible to estimate the product of the bubble density and averaged backscattering cross-section directly from the transmission loss. Again encouraging agreement was obtained between theory and experiment.

In the intermediate range of bubble sizes neither the Resonant nor the Non-resonant Theories give good predictions and some alternative hybrid theory is necessary.

7. REFERENCES

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[3] C. S. CLAY & H. MEDWIN,'Acoustical Oceanography', John Wiley, New York, 1977.

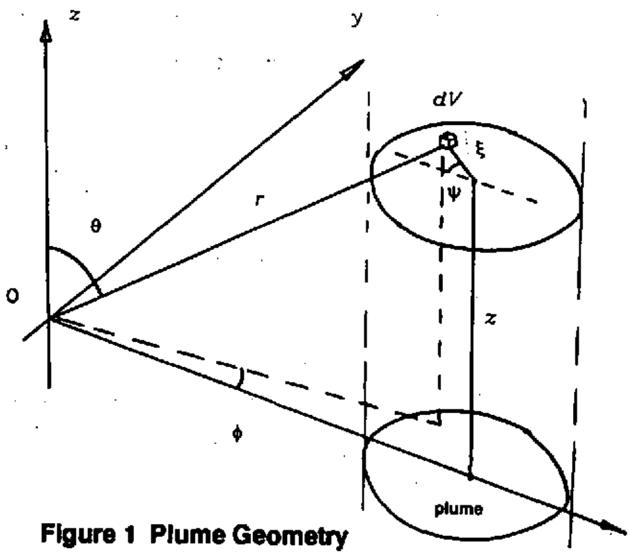

Figure 1 Plume Geometry

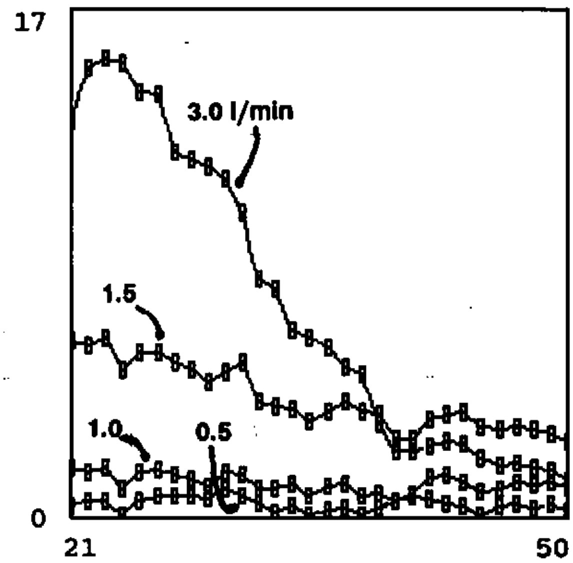

Figure 2 Transmission Loss, TL, (dB) v's Frequency, f, (kHz)

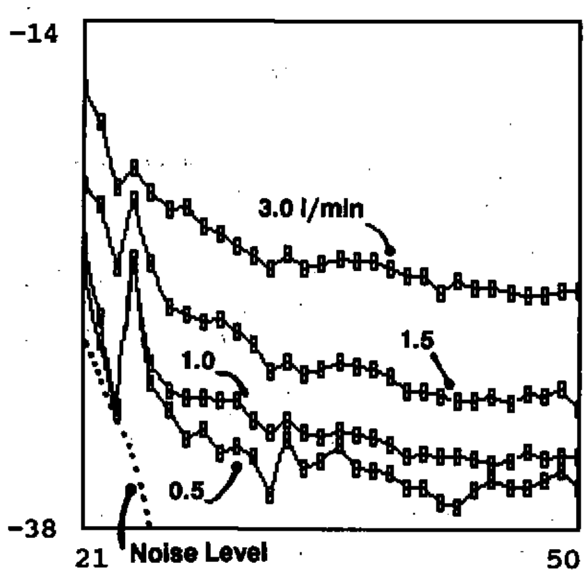

Figure 3 Normalised Backscattering Echo, NEL, (dB) v's Frequency, f, (kHz)

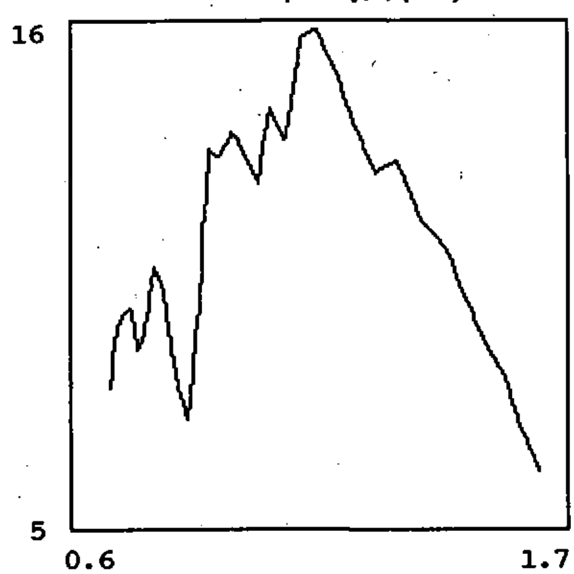

Figure 4 Bubble Number Density, n(a), (10^9) v's Bubble Radius, a, ($10^{-4}m$)

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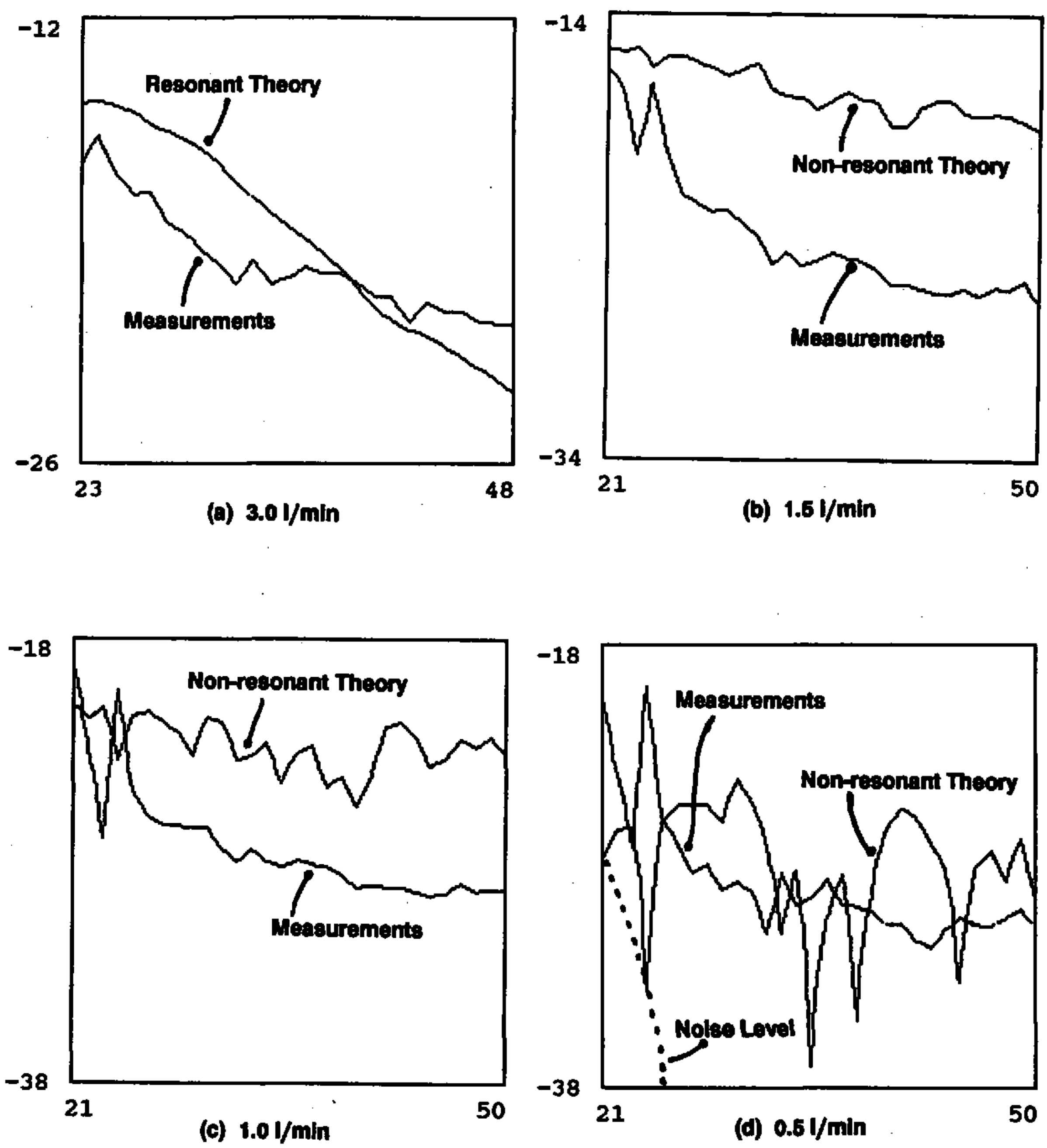

Figure 5 NEL (dB) v's f (kHz)