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SPATIAL CORRELATION IN THE BACKSCATTERED ACOUSTIC FIELD FOR NORMALLY INCIDENT SOUND ON A ROUGH SURFACE

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1. INTRODUCTION

When acoustic waves are incident upon a randomly irregular boundary, such as the sea bed, the resultant backscattered field exhibits spatial correlation. For the case of normal incidence the existence of such correlations in the horizontal direction has been exploited in correlation logs designed to measure the speeds of vessels at sea, Dickey & Edward [1]. The normal incident, widebeam geometry gives them advatages over the alternative Doppler logs which use narrow beams in grazing incidence. The present authors have been engaged in the development of a correlation log, Atkins & Smith [2]. As a part of this research a study has been made of the form of the horizontal spatial correlation of the backscattered field for normal incidence and its dependence upon the nature of the sea bed. The purpose of this present paper is to discuss this spatial correlation in terms of the transmission of a spherically spreading wave through a random phase screen.

2. THEORETICAL BACKGROUND

The problem of specular reflection of sound from a planar, perfectly reflecting surface is often treated in terms of image sources, whereby the reflected field is effectively computed by considering the boundary as a transparent screen. If the incident sound is resolved into an angular spectrum of plane waves then the corresponding angular spectrum of the transmitted, ie. reflected, wave is determined by treating the screen as a filter of angular spectra.

When the perfectly reflecting surface is rough a similar notion applies except that additional phase shifts are introduced because of the variation of the surface heights around the mean plane. This leads to the idea of treating the backscattering of sound from a randomly rough surface in terms of its transmission through a random phase screen, eg. Tamoikin & Fraiman [3] and Jakeman & McWhirter [4].

In this present paper the derivation of the spectral characteristics of the phase screen in terms of the statistics of the bottom heights and slopes is outlined. The results are then used to examine the correlation function of the backscattered sound and its dependence on the bottom. The derivation is made using the Kirchhoff-Helmholtz equation and from this the phase screen interpretation clearly arises.

2.1 Scattering Geometry

Fig.1 shows the geometry. T, R_1, R_2 and r, r_1, r_2 define respectively the positions of the source and two receivers relative to an elemental surface area, ds, on the boundary. R_1 and R_2 are separated a distance g, parallel to the x-axis. These transmission and reception points are located a distance z vertically above the x-y plane, which coincides with the mean plane of the randomly rough boundary. h is the local height of ds above

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the x-y plane and is considered to be a random function of positions x and y. $\hat{n}$ is the unit vector normal to ds. The co-ordinates r_0, ϕ and θ define the position in the x-y plane of dxdy, the projected area of ds.

2.2 Kirchhoff-Helmholtz Scattering Integral

Making use of the tangental-plane Kirchhoff approximation and assuming that the local reflection coefficient, Γ, of the boundary is described by an average value independent of incidence angle, eg. Clay & Medwin [5], then the scattering integral gives pressures p_1 and p_2 at positions R_1 and R_2 respectively:-

$$p_{1,2} = \frac{\Gamma}{4\pi}\int_s \frac{\partial}{\partial n}\left(p_i \frac{e^{-jkr_{1,2}}}{r_{1,2}}\right)ds$$

where the incident pressure is given by:-

$$p_i = \frac{p_0 D_t(\phi,\theta)}{r}e^{-jkr}$$

where p_0 is a constant source term and $D_t(\phi,\theta)$ is the transmit directivity function.

Following Boyd & Deavenport [6] by differentiating with respect to the normal, but retaining the effects of the bottom slopes, the scattering integral becomes:-

$$p_{1,2} = \frac{-jk\Gamma p_0}{2\pi}\int_s \frac{D_t e^{-jk\Lambda_{1,2}}}{r_0^2}\left(\frac{\partial h}{\partial x}\sin\phi\cos\theta + \frac{\partial h}{\partial y}\sin\phi\sin\theta + \cos\phi\right)dxdy$$

where $\Lambda_{1,2} = 2r_0 - 2h\cos\phi \mp \frac{g}{2}\sin\phi\cos\theta$.

2.3 Spatial Correlation of the Pressure

A spatial crosscorrelation between the pressures at the two receiver positions is made by taking an ensemble average:-

$$\langle p_1 p_2^* \rangle$$

This is the average of the total field, but for the case of significant scattering this will approximate to the average of the incoherent portion of the field since the average of the coherent component will be small in comparison.

Forming this crosscorrelation and transforming to relative and centre of mass co-ordinates, eg. Chernov [7], then :-

$$\langle p_1 p_2^* \rangle = \frac{k^2\Gamma^2 p_0^2}{4\pi^2}\iiiint_{-\infty}^{\infty} \frac{DH}{R_0^4} e^{\frac{-j2k(\xi x + \eta y)}{R_0}} e^{\frac{jkg\xi}{R_0}} \times$$

$$\times\left[\frac{z^2}{R_0^2} + S\left(\frac{\xi^2 - \frac{x^2}{4}}{R_0^2} + \frac{\eta^2 - \frac{y^2}{4}}{R_0^2}\right)\right]d\xi\, d\eta\, dx\, dy$$

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where $R_0 = \sqrt{z^2 + \xi^2 + \eta^2}$

$D \approx |D_t|^2$

$H = \langle e^{j2k(h'\cos\phi' - h''\cos\phi'')} \rangle$

$S = \left\langle \frac{\partial h'}{\partial x'} \frac{\partial h''}{\partial x''} \right\rangle = \left\langle \frac{\partial h'}{\partial y'} \frac{\partial h''}{\partial y''} \right\rangle$

H is the characteristic function of the joint pdf of the surface heigts ,h, at the two positions, x',y' and x'',y'' and S is the spatial correlation of the surface slopes. It is to be assumed that H and S are functions only of the relative co-ordinates, x = x' - x'' and y = y' - y''. H and S are also assumed to be isotropic and homogeneous.

It may be readily shown that the above integral reduces to the specular reflection result:-

$$\langle p p^* \rangle = \frac{p_0^2}{4z^2}$$

as expected for the specular case of H = 1 and S = 0.

Two further co-ordinate transforms can be made which enables some integrations to be completed. Firstly the centre of mass co-ordinates, ξ and η are transformed to polars, ϕ and θ, according to Fig. 2. This simplifies the integration if circular symmetry for the source is assumed. The second transformation is to convert x and y to σ and ψ according to Fig. 2. This again simplifies the integrations because of the isotropic assumption about the surface statistics.

The integral finally reduces to, Smith & Atkins [8]:-

$$\langle p_1 p_2^* \rangle = \frac{k^2 \Gamma^2 p_0^2}{z^2} \int_0^1 D[(1-\gamma^2)\chi_h + \gamma^2 \chi_{hs}] \gamma J_0(k\gamma g) d\gamma$$

where $\gamma = \sin\phi$

J_0 is the Bessel function of zero order

χ_h, χ_{hs} are effectively angular power spectra obtained from Hankel Transforms of the surface correlation statistics H and S. These are defined:-

$$\chi_h = \int_0^\infty H \sigma J_0(2k\gamma\sigma) d\sigma$$

$$\chi_{hs} = \int_0^\infty H S \sigma J_0(2k\gamma\sigma) d\sigma$$

A result of a similar form to this has been obtained by Ishimaru [9] for the case of an omnidirectional source without the bottom slopes being taken into account.

It may be shown that the integral in this final form gives the specular reflection result:-

$$\langle p p^* \rangle = \frac{\Gamma^2 p_0^2}{4z^2}$$

when $H \Rightarrow 1$, $S \Rightarrow 0$.

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2.4 Grazing Incidence
Another result which can be deduced from this integral is the grazing incident case, when $\gamma \Rightarrow 1$. Then for an omnidirectional source with a point scatter model D = 1 and $\chi_{hs} = 1$ and the horizontal spatial correlation becomes proportional to $J_0(kg)$; a result obtained by Ol'shevskii [10].

3. DEEP PHASE SCREEN AS A SPECTRAL FILTER

The final integral in the previous section shows that the spatial correlation of the backscattered field is given by the inverse Hankel Transform of an angular power spectrum. This power spectrum is the product of two spectra, D and P where D is the power spectrum of the source and P is a power spectrum associated with the random bottom. P, from the integral, is given by:-

$$P = (1 - \gamma^2)\chi_h + \gamma^2 \chi_{hs}$$

Thus the random bottom can be regarded as a random phase screen, refer to Fig. 3, which 'filters' the source spectrum with its transmission spectrum P.

3.1 Effect of Source Spectrum
An immediate and interesting conclusion is that if the sceen's spectrum P is much 'broader' than the source's spectrum D, then the 'output transmitted' spectrum, associated with the backscatter, is given by that of the source. Under these circumstances when the inverse transform of the source spectrum is taken then the spatial correlation is equal to the correlation of the source shading. Thus the field correlation is determined entirely by the source in this case and contains little information about the bottom.

The same conclusion for grazing incidence has also been reached by Jackson & Moravan [11].

In the other extreme, when the source is omnidirectional such that D = 1, then the field correlation will be determined entirely by the bottom statistics.

3.2 Spectrum of Rough Surface
The spectrum P associated with the bottom has two components, sketched in Fig. 4. The first, Fig. 4(a), is χ_h weighted by $(1 - \gamma^2)$, which arises from the the bottom height statistics. The weighting function biases the influence of this component spectrum towards the normal direction, which is physically reasonable, since this represents the shallow slope approximation. The second component, Fig. 4(b), is χ_{hs} weighted by γ^2, which arises from the slope statistics. The weighting function in this case biases the spectrum away from the normal direction. Again this is physically reasonable, since the 'average' effect of the slopes will be to deflect the sound away from the incident direction. The effect of this latter component on the field correlation will be to introduce oscillations.

4. COMPARISON WITH EXPERIMENTAL RESULTS

For the case discussed in section 3.1, where the source spectrum is dominant, the conclusion is that the field spatial correlation is equal to the correlation of the source shading. Now, consider a circular source of radius, a, which has a uniform surface velocity; the shading is correspondingly uniform. Therefore the spatial correlation will be of the form:-

$$\left|\left(1-\frac{g}{2a}\right)\right|$$

where g is the separation. This becomes zero when g = 2a, ie. when the separation is equal to the array diameter.

The theoretical development outlined in this paper has used circular symmetry for convenience and the Hankel Transform has resulted. However, before this symmetry was assumed the integral was in the form of a two dimensional Fourier Transform, so it is to be anticipated that similar conclusions should result with sources with other geometries. So it is to be expected that the field will become largely uncorrelated for separations equal to the source dimension.

Some results of correlation measurements of speckle in ultrasonic imaging by Wagner et al [12] support this conclusion, as does some work by the present authors [13] on spatial correlation in laser speckle.

Some spatial correlation curves measured by Martin [14] for bottoms with three different roughnesses are shown in Fig. 5. The projector diameter was 12.5mm and the wavelength was 3mm. The curves approach the minimum correlation for separations equal to the source diameter. There is only a small dependence of the curves on the gravel sizes, which were all greater than a wavelength. The oscillations in the correlations could be accounted for by the influence of the bottom slopes, but there is insufficient data on the bottoms used to enable this to be proved.

5. CONCLUSIONS

The main conclusion is that the horizontal spatial correlation of the backscattered field from a rough surface may be evaluated by treating the surface as a random phase screen. The screen acts as an angular spectral filter such that the 'transmitted', ie. backscattered field, has an angular spectrum which is the product of the incident and screen spectra. The correlation is then given by the inverse transform of this 'transmited' spectrum. It has been shown that the phase screen's spectrum has two components; one arising from the bottom heights and the other from the bottom slopes.

For the case of a very broad screen spectrum compared to the source spectrum the spatial correlation has been shown to be equal to the correlation of the source shading.

6. REFERENCES

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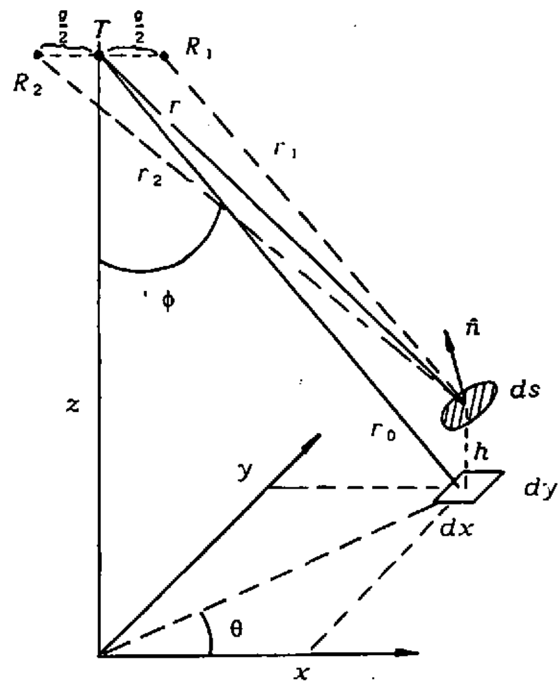

Figure 1

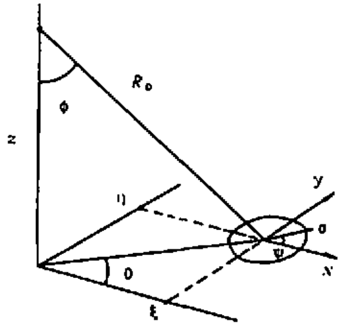

Figure 2

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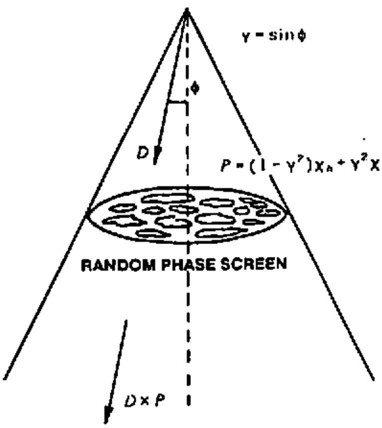

Figure 3

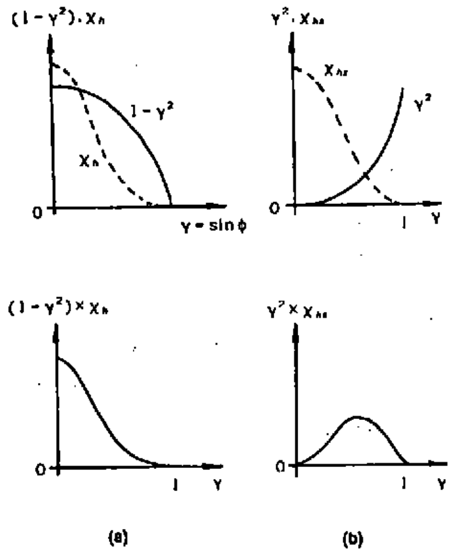

Figure 4

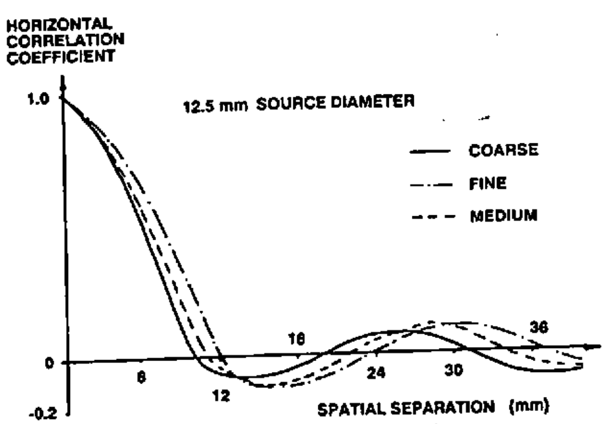

Figure 5