The parametric end-fire array in a random medium

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Introduction

A factor which it was thought may possibly limit the exploitation of the parametric end-fire array in practical sonar and echo-sounding system is the random amplitude and phase fluctuations resulting from the passage of a wave in a randomly inhomogeneous medium. Because the parametric array can extend a considerable distance into the medium, typically 100m or more, then any reduction in coherence of the array's virtual sources resulting from these random fluctuations may adversely effect the radiated difference-frequency wave.

The fluctuations on the difference-frequency wave may be considered to arise from two separate mechanisms. The two interacting waves each have amplitude and phase fluctuations imposed upon them as they propagate through the random medium. As these waves propagate they interact because of the inherent nonlinearity and hence the interaction or difference-frequency sources will also have, in some modified form, these fluctuations imposed upon them. In addition to these source-induced fluctuations the difference-frequency wave is subject to fluctuations as it propagates through the random medium from the source position to the far-field observation point. A theoretical study of these effects has been made [1] in order to provide an estimate of their importance. To assess the deterioration in performance resulting from these random effects is, in general a complex problem and hence the approach adopted in [1] was to deduce the fluctuation level of the difference-frequency wave and compare this with the fluctuation level on a wave of the same frequency which was transmitted by conventional techniques and was traversing the same random medium. In particular the coefficient of variation of the fluctuations on the difference-frequency wave for a collimated plane-wave model was compared with the coefficient of variation of the fluctuations that would have been obtained if the difference-frequency wave was transmitted directly. In this way a comparative assessment between 'conventional' and 'non-linear' systems was obtained.

The purpose of this paper is to discuss in more detail the results obtained in [1].

Theoretical results

The equation, derived in [1] for the ratio of the coefficient of variation, V_n for the non-linear case applicable to a collimated plane-wave model and the coefficient of variation, V_l for direct radiation of the difference-frequency wave is given by

$$\frac{V_n}{V_l} \approx \left\{1+\frac{1}{\alpha R}\left[\left(\frac{\theta}{\alpha a}\right)^4-1\right]\right\}^{\frac{1}{2}}$$

where $\alpha \approx \alpha_1+\alpha_2$, α_1 and α_2 are the absorption coefficients of the two interacting waves, R is the axial observation point, θ is the 3dB beamwidth of the difference-frequency wave for collimated plane-wave interaction and a is the spatial correlation length for the medium inhomogeneities assuming a gaussian correlation coefficient. This expression was derived under a number of assumptions; these are listed below:

1. Single scattering was assumed throughout and also the scattering was assumed to be large scale ie. $ka \gg 1$.

where k is the respective wavenumber. Large-scale scattering implies directive scattering and hence the plane-wave values for the mean square amplitude and phase fluctuations may only be used in the collimated plane-wave model provided 1/ka is of the order of the radius of the collimated beam; where 1 is the parametric-array effective truncation length and k is the wavenumber of the respective interacting plane wave.

2. The observation point was assumed to be in the far-field of the parametric array, ie. $R\alpha \gg 1$.

3. Complete longitudinal and transverse correlation of the amplitude and phase fluctuations of the two interacting waves over the interaction volume was assumed. For complete transverse correlation the area of the collimated beam must be less than or of the same order as the area of a 'typical' inhomogeneity.

4. There was assumed to be complete frequency correlation of the fluctuations of the two interacting waves (frequency correlation in this context has been discussed elsewhere, see for example [2]).

5. The transit time of an acoustic wave $1/C_o$ along the interaction volume was assumed to be short compared to the correlation time of the amplitude and phase fluctuations of the interacting waves, i.e. a quasi-static assumption.

These assumptions were discussed in [1] and it was argued that the collimated plane-wave model studied was not a completely unrealistic model for a number of important practical cases in which the parametric array could be exploited.

Although a number of deductions may be made about the magnitude of the ratio V_n/V_1 as given in the above equation it is of more interest to express this ratio in terms of the difference frequency directly. Thus by using the equation [4], $\alpha = \theta^2 K/8$, the above equation may be rewritten:-

$$\frac{V_n}{V_l} \approx \left\{1+\frac{1}{\alpha R}\left[\left(\frac{109}{\Delta f\theta_o}\right)^4-1\right]\right\}^{\frac{1}{2}},$$

where Δf is the difference frequency expressed in kHz and θ_o is the difference-frequency 3dB beamwidth expressed in degrees.

One of the conditions for the validity of this equation is $R\alpha \gg 1$. Thus when evaluating V_n/V_l it is convenient to define a particular value for $R\alpha$. This value must be large but at the same time must satisfy the single-scattering criterion [3], $R \ll 1/\overline{\mu^2}K^2a$, where K is the difference-frequency wavenumber and $\overline{\mu^2}$ is the mean square value of the fluctuations of the refractive index of the random medium. An alternative definition for the validity of single scattering is to assume that $V_l < 0.5$,

where

$$V_l = \left(\sqrt{\pi}\,\overline{\mu^2}\,K^2 a R\right)^{\frac{1}{2}}$$

Using the values of $R\alpha = 20$, $a \sim 1.0\,m$ and $\overline{\mu^2} \sim 5\times10^{-9}$ [3], the above equations become:-

$$\frac{V_n}{V_l} \approx \left[0.95+\frac{1}{20}\left(\frac{109}{\Delta f\theta_o}\right)^4\right]^{\frac{1}{2}} \qquad \ldots\ldots\ldots(1)$$

and

$$V_l \approx \frac{14\sqrt{\Delta f}}{\theta_o}\ \% \qquad \ldots\ldots\ldots(2)$$

These two equations thus form the basis for discussions on the parametric array in a random medium.

Discussion and Conclusions

Figure 1 shows curves of V_n/V_l plotted against difference frequency with difference-frequency beamwidth as parameter, as calculated from equation (1).

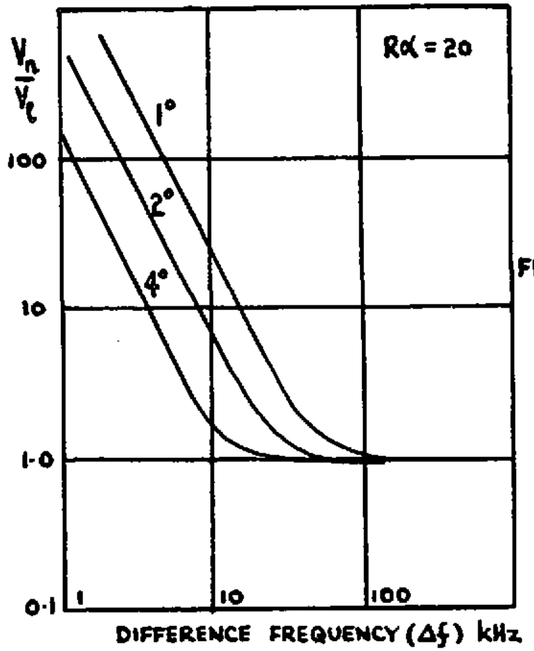
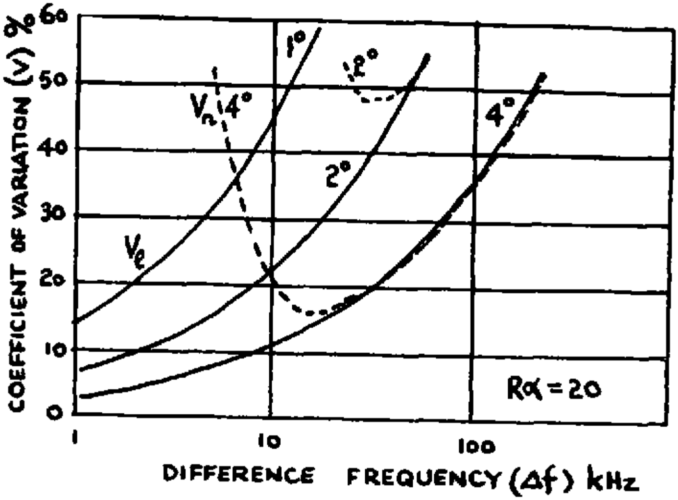


FIGURE 1

It may be seen that for each curve there is a sharp transition region where V_n/V_1 increases rapidly from unity as Δf decreases. This arises because a reduction in difference frequency, for a constant beamwidth, requires an increase in the effective length of the parametric array and thus the source-induced fluctuations on the difference-frequency wave increases. It may also be seen that the source-induced fluctuations quickly predominate and increase rapidly, according to a $(\Delta f)^{-2}$ law, with decreasing difference frequency. If the 3dB beamwidth increases the effective length of the array decreases and so the transition region occurs at lower difference frequencies. These curves however, give no indication of the region over which single scattering is valid since R has been maintained constant. In order to examine the region over which the assumption of single scattering is valid curves of V_1 have been plotted in figure 2 from equation (2) for $R\alpha = 20$.

FIGURE 2



The coefficient of variation V_n may be deduced from figures 1 and 2 as a function of difference frequency and are also included in figure 2. It may be observed that for this value of R only the 4°-beamwidth curve is strictly valid over most of the difference-frequency range examined. The 2°-beamwidth curve violates the single scattering condition over most of the range and the 1°-beamwidth curve violates this condition over all the range. This implies that for the narrower beamwidth conditions R=20/α is too large. It is rather interesting that in terms of an absolute value for the level of the fluctuations in a non-linear system an optimum range of difference frequencies may be chosen for a particular specified beamwidth.

Within the range of validity of the study made it would seem that the non-linear system is only likely to have a significantly poorer performance in a random medium than a conventional system when the 3dB beamwidths are very narrow i.e. the order of 1°.

Unfortunately it is some of these narrow-beam systems that are of special interest when the parametric array is being exploited. Under these circumstances therefore the larger the value of difference frequency that can be accomodated the better. Also for these very narrow-beam situations the range of distances of R over which the equation for the ratio V_n/V_1 is valid becomes small since R must satisfy simultaneously the conditions $R\alpha \gg 1$ and $R \ll 1/\overline{\mu^2} K^2 a$.

Further quantitative analysis would not seem to be profitable at this stage until experimental investigations have been made.

References

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