24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

NOISE ATTENUATION PERFORMANCE OF MULTIPLE HELMHOLTZ RESONATOR ARRAYS SYSTEM

Chenzhi Cai, Cheuk Ming Mak and Yi Jing Chu

The Hong Kong Polytechnic University, Department of Building Services Engineering, Hung Hom, Kowloon, Hong Kong, China
email: chenzhi.cai@connect.polyu.hk

Xu Wang

Tongji University, Institute of Acoustics, No. 1239 Siping Road, Shanghai 200092, China

This paper focuses on improving the noise attenuation performance of a ducted Helmholtz resonator (HR) arrays system. The noise attenuation performance achieved by the HR system is fairly depended on the number of HRs, which is limited by the available space for HR's installation. Two cases are investigated: the periodic ducted HR system and the multiple HR arrays system. The multiple HR arrays system is based on the periodic system by adding identical resonators on the cross-section. Several identical resonators mounted on the same cross-section is considered as one unit in the system, the resonance frequency of the unit is the same as a single resonator. The transfer matrix method and the Bragg theory are used to investigate the wave propagation in the multiple HR array system and the periodic ducted HR system theoretically. The theoretical prediction results fit well with the Finite Element Method (FEM) simulation results. The results show that different arrangement of the HRs mounted on the duct has no effect on the $\overline{TL}$ once the number of HRs and the periodic distance are both the same. The proposed multiple HR array system has potential application in noise control with space limitation.

Keywords: Helmholtz resonator, noise control, multiple arrays, finite element methods

1. Introduction

A ventilation ductwork system is an essential components of modern buildings that maintains comfortable indoor air environment, for instance, air quality, air temperature and air humidity [1,2]. However, it is common to encounter an accompanied duct-borne noise problem in these ventilation systems, which is mainly produced by flow noise and fan operation noise. The flow noise is generated by in-duct elements such as dampers, bends transition pieces, duct corners or even attenuators [3]. The fan operation noise is related to the power and the structure of the fan itself [4]. The unpleasant noise accompanied with the fresh air from the opening of the ventilation ductwork system in a room is a direct source of annoyance that seriously influence people's living and working quality [5]. Besides, the noise may also have negative impacts on residents in other rooms by transmitting from one room to the nearby rooms through the building structures as windows, door, floor and wall [6]. It is therefore significant to reduce duct-borne noise, especially the low-frequency and broadband noise in ventilation ductwork system. Although many successful applications and algorithms has been achieved in active noise control, serious challenges still exist in engineering applications [7-9]. The traditional dissipative silencers are not effective at low-frequency and may result in a significant static pressure drop in the ductwork system [10]. Passive reactive silencer, the Helmholtz resonator (hereafter HR), is commonly used in air duct noise control applications due to its characteristics of affordable, tuneable, stable. However, the HR can only provide a good noise attenuation performance at its resonance frequency within a relative narrow band. The resonance frequency of a HR is only determined by the geometries of the cavity and the neck. It is therefore easy to design a HR with a desired resonance frequency [11,12].

Since a single HR has a narrow noise attenuation band, growing number of applications adopt sets of HRs to obtain a relative wider noise attenuation band. Many researchers and engineers around the world have devoted their attention to broaden the noise attenuation band. A lot of achievements have been made and are documented in numerous pieces of literature. Sugimoto and Horioka [13] investigated the sound waves propagation in a tunnel with periodic ducted HRs and exhibited the peculiar dispersion characteristics marked as stopbands and passbands. Bradley [14] conducted an experimental to measure the Bloch wave number and the relative component wave amplitude. The measured results compared well with the theoretical predictions. Wang and Mak [15,16] presented theoretical methods of noise attenuation band prediction and examined the effect of disorder in the geometries of HRs or in periodic distance on the noise attenuation band. Cai and Mak [17] shown a noise control zone compromising the attenuation bandwidth or peak amplitude of a periodic ducted HR system. Coulon et al. [18] investigated the role of distance between HRs on the transmission loss of the whole HR array system and proposed an optimization approach for wide band noise attenuation.

This paper focuses on improving the noise attenuation performance of a ducted HR arrays system. Owing to the coupling of Bragg reflection and HR's resonance, it is found that a periodic ducted HR system can provide much broader noise attenuation band. However, the noise attenuation band achieved by the HR system is fairly depended on the number of HRs, which is limited by the available space for HR's installation. In order to improve the noise attenuation performance of ducted HR system, HRs can be increased in number by using multiple HR arrays. Two cases are investigated: the periodic ducted HR system and the multiple HR arrays system. The multiple HR arrays system is based on the periodic system by adding identical resonators on the cross-section. Several identical resonators mounted on the same cross-section is considered as one unit in the system, the resonance frequency of the unit is the same as a single resonator. The transfer matrix method and the Bragg theory are used to investigate the wave propagation in the multiple HR array system and the periodic ducted HR system theoretically. The theoretical predictions are validated by Finite Element Method (FEM) simulation and show a good agreement with the FEM simulation results.

2. Theoretical analysis of side-branch Helmholtz resonators

2.1 A single side-branch HR

The HR is traditionally considered as an equivalent mass-spring. The sound fields inside the HR are clearly multidimensional due to the area discontinuities at the neck-cavity interface. The multidimensional approach could provide a more accurate prediction of HR' acoustic impedance than the traditional equivalent lumped approach. However, the main purpose here is to investigate the acoustic performance of the ducted HR system. It is therefore that the traditional lumped model with appropriate end-correction length is adopted and is given as [11]: $Z_r = j\rho_0 l_n'(\omega^2 - \omega_0^2)/S_n\omega$ (ρ_0 is air density, l_n' and S_n are the neck's effective length and area respectively, ω_0 and ω are the resonant circular frequency and circular frequency respectively). ω_0 is determined by its geometries as $c_0\sqrt{S_n/l_n'V_c}$ (V_c is the cavity volume and c_0 is the speed of sound in the air). Once the acoustic impedance of a HR has been achieved, the transmission loss of a single-branch HR mounted on the duct with cross-sectional area S_d can be expressed as: $TL = 20\log_{10}\left|(1+\rho_0 c_0/2S_d Z_r)\right|$.

2.2 Several identical HRs mounted on the same cross-section

The HR is well known as a narrow band silencer with high transmission loss peak. Several identical HRs mounted on the same cross-section can broaden the noise attenuation band without the change of HR's resonance frequency [19,20]. Fig. 1 demonstrates N ($N=4$ for example) identical HRs ducted on the same cross-section.

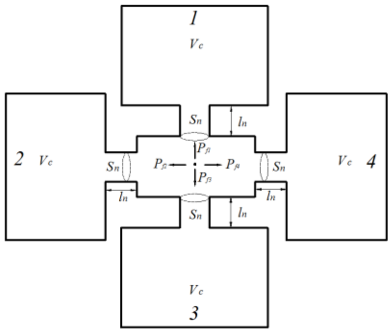

Figure 1: Four identical Helmholtz resonators ducted on the same cross-section.

The characteristics of sound wave propagation before and after side-branch HRs could be described as sound pressures and volume velocities: p_{in}, u_{in} and p_{out}, u_{out} respectively. According to the continuities conditions of sound pressure and volume velocity at the duct-neck interface, the relation of sound wave propagation through the side-branch HRs could be expressed as:

$$p_{in} = p_{out} = p_{fi}, \; S_d u_{in} = S_d u_{out} + \sum_{i}^{N} p_{fi} / Z_r \tag{1,2}$$

where i=1,2,3,4 represents each individual HR. Eq. (1) and Eq. (2) could be rewritten in the transfer matrix form as:

$$\begin{bmatrix} p_{in} \\ \rho_0 c_0 u_{in} \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{\rho_0 c_0}{S_d} \dfrac{N}{Z_r} & 1 \end{pmatrix} \begin{bmatrix} p_{out} \\ \rho_0 c_0 u_{out} \end{bmatrix} \tag{3}$$

By considering several identical HRs as an equivalent "one HR", the acoustic impedance of the "one HR" could be expressed as: $Z_{er} = Z_r / N$ according to Eq. (3). It should be noted that the equivalent "one HR" has the same resonance frequency as the single HR. Then, the transmission loss of the side-branch HRs can be expressed as:

$$TL = 20\log_{10}\left(\frac{1}{2}\left|2 + \frac{\rho_0 c_0}{S_d}\frac{1}{Z_{er}}\right|\right) = 20\log_{10}\left(\frac{1}{2}\left|2 + N\frac{\rho_0 c_0}{S_d}\frac{1}{Z_r}\right|\right) \tag{4}$$

3. Theoretical analysis of a periodic ducted HR system

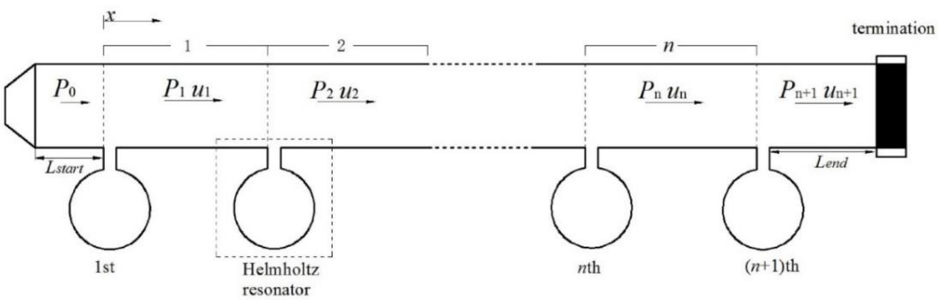

Figure 2: A periodic ducted Helmholtz resonator system.

An array of lined HRs ducted periodically is demonstrated in Fig. 2. A periodic unit is composed of a HR and a connection tube. The connection tube length is considered as the periodic distance by assuming the diameter of the HR's neck is negligible compared with the connection tube length. Only planar wave is assumed to propagate in duct. The sound pressure is a combination of positive-x and negative-x directions. The sound pressure and particle velocity could be described as:

$$p_n(x) = I_n e^{-jk(x-x_n-\omega t)} + R_n e^{jk(x-x_n+\omega t)} \tag{5}$$

$$u_n(x) = \frac{I_n}{S_d Z_d} e^{-jk(x-x_n-\omega t)} - \frac{R_n}{S_d Z_d} e^{jk(x-x_n+\omega t)} \tag{6}$$

where $e^{j\omega t}$ is a time-harmonic disturbance, k is the number of waves, $x_n = (n-1)d$ represents the local coordinates, d is the periodic distance, Z_d is the acoustic impedance of the duct, and I_n and R_n represent respective complex wave amplitudes. The continuity conditions at $x = nd$ boil down to:

$$\begin{bmatrix} I_{n+1} \\ R_{n+1} \end{bmatrix} = \begin{bmatrix} (1-\frac{Z_d}{2Z_r})\exp(-jkd) & -\frac{Z_d}{2Z_r}\exp(jkd) \\ \frac{Z_d}{2Z_r}\exp(-jkd) & (1+\frac{Z_d}{2Z_r})\exp(jkd) \end{bmatrix} \begin{bmatrix} I_n \\ R_n \end{bmatrix} = \mathbf{T} \begin{bmatrix} I_n \\ R_n \end{bmatrix} \tag{7}$$

$\mathbf{T}$ is the transfer matrix. The characteristics of sound in an arbitrary cell could be achieved successively according to Eq. (7) once the initial sound pressure is given. When each periodic unit has several identical HRs mounted on the same cross-section with equal number of HRs instead of just one HR, the acoustic impedance of a HR Z_r in Eq. (7) could be replaced by the acoustic impedance of the equivalent "one HR" Z_{er}. Eq. (7) could be rewritten in the form of Bloch wave theory as:

$$\begin{bmatrix} I_{n+1} \\ R_{n+1} \end{bmatrix} = \lambda \begin{bmatrix} I_n \\ R_n \end{bmatrix} = \mathbf{T} \begin{bmatrix} I_n \\ R_n \end{bmatrix} \tag{8}$$

where λ is set to be $\exp(-jqd)$, and q is the Bloch wave number and is allowed to be a complex value. The analysis of periodic structure then boils down to the eigenvalue and its corresponding eigenvector problem. The eigenvalue λ has two solutions: λ_1 and λ_2 with corresponding eigenvectors $[v_{I1}, v_{R1}]^{\mathbf{T}}$ and $[v_{I2}, v_{R2}]^{\mathbf{T}}$ respectively. It is therefore that characteristics of sound wave in the duct could be described in eigenvector form as:

$$\begin{bmatrix} I_{n+1} \\ R_{n+1} \end{bmatrix} = \mathbf{T} \begin{bmatrix} I_n \\ R_n \end{bmatrix} = \mathbf{T}^2 \begin{bmatrix} I_{n-1} \\ R_{n-1} \end{bmatrix} = \ldots = \mathbf{T}^n \begin{bmatrix} I_1 \\ R_1 \end{bmatrix} = A_0 \lambda_1^n \begin{bmatrix} v_{I1} \\ v_{R1} \end{bmatrix} + B_0 \lambda_2^n \begin{bmatrix} v_{I2} \\ v_{R2} \end{bmatrix} \tag{9}$$

where A_0 and B_0 are complex constants determined by boundary conditions. Combining the end boundary conditions with reflection coefficient at $x=(n-1)d+L_{end}$ and the initial condition with given A_0 and B_0 at $x = -L_{start}$, the average transmission loss of per HR in the whole system can be expressed as:

$$\overline{TL} = \frac{20}{N(n+1)} \log_{10} \left| \frac{I_0}{I_{n+1}} \right| = \frac{20}{N(n+1)} \log_{10} \left| \frac{A_0 \lambda_1^{-1} v_{I1} + B_0 \lambda_2^{-1} v_{I2}}{A_0 \lambda_1^{n-1} v_{I1} + B_0 \lambda_2^{n-1} v_{I2}} \right| \tag{10}$$

$B_0 = 0$ is required when the duct ends with an anechoic termination. Eq. (10) could be simplifies as $-20\log_{10}|\lambda_1|$. The solution of λ is a function of wave frequency, periodic distance and the geometric dimensions of the system. Generally, the HR's resonance and the Bragg reflection are two formation mechanisms of the noise attenuation band. An appropriate periodic distance (integral multiple of $d = \lambda_0/2$) could help to obtain a wider noise attenuation band at the resonance frequency of the HR. Practically, $d = \lambda_0/2$ is often adopted for a broader noise attenuation band due to the coupling of the HR's resonance and the first Bragg reflection [14,15].

4. Theoretical analysis of a multiple HR arrays system

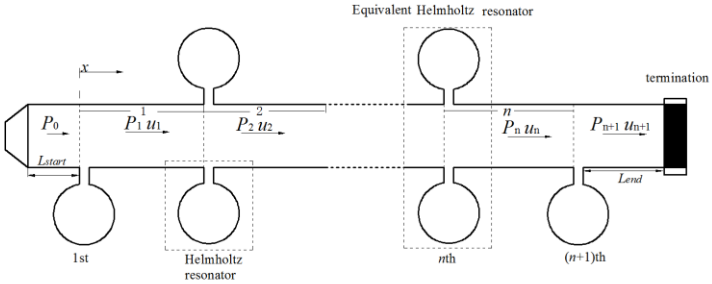

Figure 3: A multiple HR arrays system.

A periodic ducted HR system can provide much broader noise attenuation band owing to the coupling of the HR's resonance and the Bragg reflection. However, the noise attenuation capacity of each HR remains unchanged in spite of HR's number or the periodic distance [17]. It indicates that the transmission loss achievable by a periodic system is fairly depended on the HR's number, which is restricted to the available space in longitudinal and transverse direction of the duct. However, the available space in the transverse direction is sometimes not sufficient for a long distance along the longitudinal direction. It is therefore that the multiple HR arrays system is considered here, as illustrated in Fig. 3. The number of HRs mounted on the same cross-section depends on the available space. Each unit of the multiple system could comprise different number of HRs. As discussed above, several identical HRs mounted on the same cross-section of the duct could be considered as an equivalent "one HR" with acoustic impedance of Z_{er}. The number of HRs in each unit is variable according to the available space, the transfer matrix between each two nearby unit should be specified as $\mathbf{T}_n$ respectively. By introducing the equivalent "one HR" into Eq. (7), the transfer matrix $\mathbf{T}_n$ between the *nth* and *n+1th* unit could be given as:

$$\mathbf{T}_n = \begin{bmatrix} (1-N\dfrac{Z_d}{2Z_r})\exp(-jkd) & -N\dfrac{Z_d}{2Z_r}\exp(jkd) \\ N\dfrac{Z_d}{2Z_r}\exp(-jkd) & (1+N\dfrac{Z_d}{2Z_r})\exp(jkd) \end{bmatrix} \tag{11}$$

When the complex wave amplitudes are expressed in the form of a state vector as $\mathbf{a}_{n+1} = [I_{n+1} \quad R_{n+1}]^T$ (superscript T means transposition), the characteristics of sound wave in the duct could be described as: $\mathbf{a}_{n+1} = \mathbf{T}_n \mathbf{a}_n$. It follows immediately that: $\mathbf{a}_{n+1}\mathbf{a}_{n+1}^{T\,*} = \mathbf{T}_n(\mathbf{a}_n\mathbf{a}_n^{T*})\mathbf{T}_n^{T*}$ (the superscript * means conjugation). The matrix equation is then could also be described in vector form as: $\mathbf{e}_{n+1} = \mathbf{A}_n\mathbf{e}_n$ ($\mathbf{e}_{n+1}$ and $\mathbf{e}_n$ are: $\left[I_nI_n^* \quad I_nR_n^* \quad R_nI_n^* \quad R_nR_n^*\right]^T$ and $\left[I_{n+1}I_{n+1}^* \quad I_{n+1}R_{n+1}^* \quad R_{n+1}I_{n+1}^* \quad R_{n+1}R_{n+1}^*\right]^T$ respectively). When the transfer matrix $\mathbf{T}_n$ takes the form of reflection and transmission coefficients t_n and r_n into expression according to its original definition, it can be given as [21]:

$$\mathbf{T}_n = \begin{bmatrix} e^{-jkL_n} & 0 \\ 0 & e^{jkL_n} \end{bmatrix} \begin{bmatrix} 1/t_n^* & -(r_n/t_n)^* \\ -(r_n/t_n) & 1/t_n \end{bmatrix} \tag{12}$$

The matrix $\mathbf{A}_n$ could also takes the form of $\mathbf{T}_n$ in Eq. (12) as:

$$\mathbf{A}_n = \begin{bmatrix} 1/|t_n|^2 & -r_n/|t_n|^2 & -r_n^*/|t_n|^2 & |r_n|^2/|t_n|^2 \\ -r_n^*\delta_n/t_n^{*2} & \delta_n/t_n^{*2} & r_n^{*2}\delta_n/t_n^{*2} & -r_n^*\delta_n/t_n^{*2} \\ -r_n/\delta_n t_n^2 & r_n^2/\delta_n t_n^2 & 1/\delta_n t_n^2 & -r_n/\delta_n t_n^2 \\ |r_n|^2/|t_n|^2 & -r_n/|t_n|^2 & -r_n^*/|t_n|^2 & 1/|t_n|^2 \end{bmatrix} \tag{13}$$

It should be noted that the value of $\mathbf{A}_n(4,4)$ is $1/|t_n|^2$. It means that the transmission loss between the two nearby unit can be presented as: $10\log_{10}(\mathbf{A}_n(4,4))$.

The HR system could also be represented in the vector form as: $\mathbf{e}_{n+1} = (\prod_{i=0}^{n} \mathbf{A}_n)\mathbf{e}_0 = \Lambda \mathbf{e}_0$. The Λ is the matrix of the whole ducted HR system and the $\overline{TL}$ of HR in the whole system could be expressed as: $10\log_{10}\left(\Lambda(4,4)\right)/N_{total}$.

5. Results and discussion

The geometries of the HR used in this paper are: neck area $S_n = 4\pi cm^2$, neck length $l_n = 2.5cm$,and cavity volume $V_c = 101.25\pi cm^3$. The cross-section area of the main duct is $S_d = 36cm^2$. The anechoic termination is applied at the end of the duct in both systems to avoid reflected waves from the termination. An oscillating sound pressure at a magnitude of $P_0 = 1$ is applied at the beginning of the duct.

5.1 Validation of the theoretical predictions of a periodic HR system

The periodic distance $d = \lambda_0/2$ is chosen here in order to obtain an broader noise attenuation band. The configurations of three periodic ducted HR system cases are illustrated in Fig. 4. The predicted $\overline{TL}$ of a periodic ducted HR system with different equivalent "one HR" (*N*=1,2,4) are exhibited in Fig. 5. It can be seen in Fig. 5 that with the increase number of HRs in the equivalent "one HR", a wider noise attenuation band could be achieved. Besides, it should be noted that the more HRs added on the system, the better noise attenuation performance of the whole system. A good agreement between the theoretical predicted $\overline{TL}$ and the FEM simulation results can also be seen in Fig. 5.

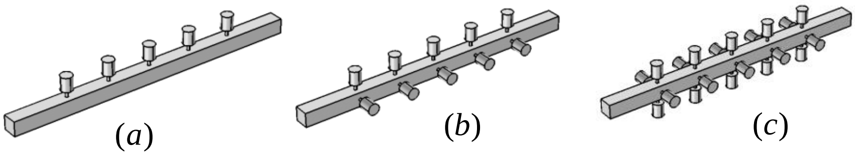

Figure 4: Configuration of three periodic ducted HR system cases: (a) *N=1*, (b) *N=2*, (b) *N=4*.

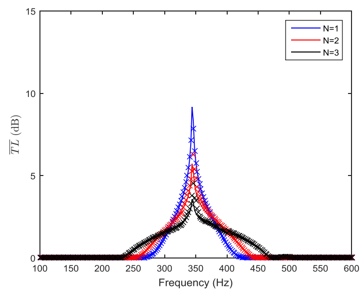

Figure 5: The average transmission loss of different ducted HR system cases (solid lines represent the theoretical predictions, and dotted crosses represent the FEM simulation results).

5.2 Validation of the theoretical predictions of a multiple HR arrays system

For a periodic ducted HR system, a broader noise attenuation band could be achieved due to the coupling of the Bragg reflection and HR's resonance. However, such kinds of noise attenuation system will occupy a large space and it is impractical to be used in an actual ventilation ductwork system. Two multiple HR array systems are investigated, as illustrated in Fig. 6. The periodic distance is also chosen as $d = \lambda_0/2$. The number of HRs mounted on the duct are both equal ten. Fig. 7 compares the average transmission loss of a periodic one (*N*=2), model 1 and model 2. The $\overline{TL}$ of these three system are nearly the same. It means that different arrangement of the HRs on the duct has no effect on the $\overline{TL}$ once the number of HRs and the distance between two nearby unit are the same The comparison of the thoretical predictions and the FEM simulation results are illustrated in Fig. 8, and the prediction results are in good agreement with the FEM simulation results.

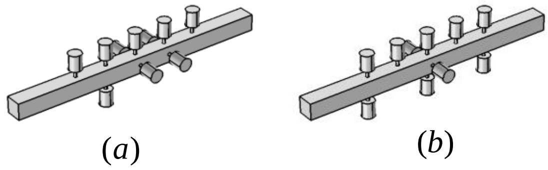

Figure 6: Configuration of two multiple HR arrays system cases: (a) model 1, (b) model 2.

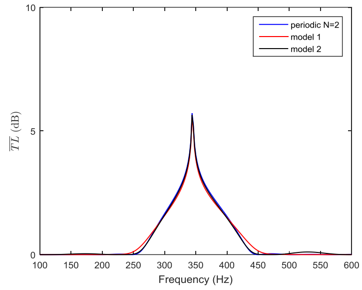

Figure 7: Comparison of different ducted HR system with the same number of HRs.

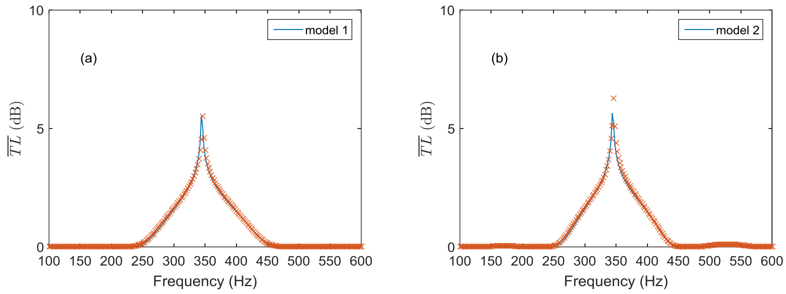

Figure 8: The average transmission loss of different modified HR systems (solid lines represent the theoretical predictions, and dotted crosses represent the FEM simulation results).

6. Conclusion

This paper focuses on improving the noise attenuation performance of a ducted HR arrays system. Owing to the coupling of Bragg reflection and HR's resonance, it is found that a periodic ducted HR system can provide much broader noise attenuation band. However, the noise attenuation band achieved by the HR system is fairly depended on the number of HRs, which is limited by the available space for HR's installation. In order to improve the noise attenuation performance of ducted HR system, HRs can be increased in number by using multiple HR arrays. Two cases are investigated: the periodic ducted HR system and the multiple HR arrays system. The multiple HR arrays system is based on the periodic system by adding identical resonators on the cross-section. Several identical resonators mounted on the same cross-section is considered as one unit in the system, the resonance frequency of the unit is the same as a single resonator. Only planar wave is assumed to propagate in the duct due to low frequency considered in this paper. The transfer matrix method and Bragg theory are used to investigate the wave propagation in the multiple HR array system and the periodic ducted HR system theoretically. The theoretical predictions are validated by Finite Element Method simulation and show a good agreement with the FEM simulation results. The multiple HR array system has a better noise attenuation performance, which has potential application in noise control to fully utilize the available space.

7. Acknowledgements

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. PolyU 152116/14E).

REFERENCES

1 Cory, W.T.W. *Fans & Ventilation: A Practical Guide,* Elsevier Science, (2005).

2 Ai, Z.T., Mak, C.M., Cui, D.J., Xue, P. Ventilation of air-conditioned residential buildings: A case study in Hong Kong. *Energy and Building*, **127**, 116-127, (2016).

3 Mak, C.M., Wu, J., Ye, C., Yang, J. Flow noise from spoilers in ducts. *Journal of Acoustical Society of America*, **125(6)**, 3756-3765, (2009).

4 Wong, T. J., Paurobally R., and Pan J. Hybrid active passive control of fan noise. *Applied Acoustics*, **64**, 885-901, (2003).

5 Mak, C.M., Wang, Z. Recent advances in building acoustics: An overview of prediction methods and their applications. *Building and Environment*, **91**, 118-126, (2015).

6 Ou, D.Y. Low frequency sound insulation analysis and evaluation of stiffened building structures. *Building and Environment*, **94(2)**, 802-809, (2015).

7 John, C.B. Active adaptive sound control in a duct: a computer simulation. *Journal of Acoustical Society of America*, **70**, 715-726, (1981).

8 Sen, M.K., Morgan D., *Active noise control systems: algorithms and DSP implementations*, John Wiley & Sons, Hoboken, NJ (1995).

9 Chu, Y.J. A new regularized subband ANC algorithm with online secondary-path modelling: Perforamnce analysis and application to buildings. *Building and Environment*, **94(2)**, 873-882, (2015).

10 Crocker, M.J., *Handbook of Noise and Vibration Control,* John Wiley & Sons, Hoboken, NJ (2007).

11 Ingard, U. On the theory and design of acoustic resonators. *Journal of Acoustical Society of America*, **25**, 1037-1061, (1953).

12 Cai, C., Mak, C.M., Shi, X. An extended neck versus a spiral neck of the Helmholz resonator. *Applied Acoustics*, **115**, 74-80, (2017).

13 Sugimoto, A., Horioka, T. Dispersion characteristics of sound waves in a tunnel with an array of Helmholtz resonators. *Journal of Acoustical Society of America*, **97(3)**, 1446-1459, (1995).

14 Bradley, C.E. Time harmonic acoustic Bloch wave propagation in periodic waveguides. Part I. Theory. *Journal of Acoustical Society of America*, **96**, 1844-1953, (1994).

15 Wang, X., Mak, C.M. Acoustic performance of a duct loaded with identical resonators. *Journal of Acoustical Society of America*, **131(4)**, EL316-EL322, (2012).

16 Wang, X., Mak, C.M. Disorder in a periodic Helmholtz resonator array. *Applied Acoustics*, **82,** 1-5, (2014).

17 Cai, C., Mak, C.M. Noise control zone for a periodic ducted Helmholtz resonator system. *Journal of Acoustical Society of America*, **140(6)**, EL471-EL477, (2016).

18 Coulon, J.M., Atalla, N., Desrochers, A. Optimization of concentric array resonators for wide band noise reduction. *Applied Acoustics*, **113,** 109-115, (2016).

19 Cai, C., Mak, C.M., Wang, X. Noise attenuation performance improvement by adding Helmholtz resonators on the periodic ducted Helmholtz resonator system. *Applied Acoustics*, **122**, 8-15, (2017).

20 Chiu, M.C. Numerical assessment for a broadband and tuned noise using hybrid mufflers and a simulated annealing method. *Journal of Sound and Vibration*, **332**, 2913-2940, (2013).

21 Langley, R.S. Wave transmission through one-dimensional near periodic structures: optimum and random disorder. *Journal of Sound and Vibration*, **188(5)**, 717-743, (1995).