RESEARCH ON THE DYNAMIC MODELING OF THE LONGITUDINAL VIBRATION OF A SHIP SHAFT-HULL SYSTEM

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A dynamic model to study the longitudinal vibration of a ship shaft-hull system is established using FRF-based substructuring method. The modes and nature frequencies of the shaft, the hull and the shaft-hull system are firstly analyzed separately. The coupled vibration characteristics of the shaft, the hull and the shaft hull system are then simulated. Finally, an experiment studying the vibration characteristics on a large scale shaft-hull system is carried out and compared with the numerical results, which shows good consistency.
Keywords: ship shaft-hull system, FRF-based substructuring method, coupled vibration

1. Introduction

The research of the dynamical characteristics of the ship shaft-hull coupled structure is one challenging but worthwhile work. A lot of researches have been done targeting the vibration mechanisms of the hull or the shaft itself.

The studying of the vibration of hull started from infinite cylindrical shell by Rayleigh[1], after that Forsberg[2] discussed the free vibration of finite cylindrical shell with different boundary conditions. Bulkheads, stiffeners and distributed concentrated mass caused a lot of trouble to model and study the vibration characteristics of hull. Bernblit[3] compared the dynamic response of hull with different type spacing stiffeners using finite element method. Wilken[4] used Admittance method to analyze the modal characteristics of a stiffened cylindrical hull and the stiffeners are considered as sub-structures. Laulagnet[5] described the rib force resultant moment with 4×4 impedance matrix, then the ribs and the shell is connected through the boundary conditions, and the solution is derived by complex equations. Caresta[6] utilized an analytical method, a wave approach and a power series solution, to calculate the cylindrical shell and the conical shell of the hull respectively.

Analytical method, transfer matrix method and finite element method are the common tools adopted to studying the dynamic characteristics of shaft. The coupled torsional, lateral and transversal vibration and the mutual influences between them had been extensively studied, such as Parsons[7], Schwibinger[8] and Kane[9]. Some simplification to simulate the propeller, water lubricated and thrust bearing were also implemented by Pan[10] and Schwanecke[11].

Owing to the complexity of the shaft-hull coupled system, the numerical and experimental methods are often implemented instead of analytical research. Cao[12] took the finite element method to investigate the longitudinal and transversal vibrations of a shaft-hull structure excited by propeller force. Missaoui[13] was the first simulating the coupling between the hull and shaft by introducing artificial springs via variation equation. Guo[14] used displacement admittance theory to deal with the force and moment acting on the plate and the shell, and the influence of the internal structure on the vibration of the structure is studied. Merz[15] used admittance matrix method studying the vibration of single layer and double layer cabin in the cylindrical shell, and the similar conclusion was obtained with Guo.

In this paper, a dynamic model to study the transverse vibration of a ship shaft-hull system is established using frequency response function based substructure synthesis method. The modes and nature frequencies of the shaft, the hull and the shaft-hull system are firstly analyzed separately. The coupling transverse vibration characteristics of the shaft, the hull and the shaft hull system are then simulated. Finally, an experiment studying the vibration characteristics on a large scale submarine is carried out and compared with the numerical results.

2. Ship shaft-hull system structure

The ship shaft-hull system structure studied in this paper is shown in Fig. 1. It consists of a hull shell and a propeller/shafting system. The hull shell includes six cylinder segments and a conical segment. The main structure is a pressure hull and bulkhead. The propeller/shafting system is assembled by a 17 kW motor and its base, three shafts and a propeller. A flexible coupling connects the motor to the thrust shaft and two rigid couplings connect the shafts to each other. Two water-lubricated bearings and an oil-lubricated bearing are fixed on the hull shell to support the shafts. The hull shell is supported by 14 air springs, shown by red arrows. Under each cabin section two springs are distributed symmetrically. The structure is in the air. In order to make the method appropriate for the structure in the water, the stiffness of the springs in the simulation model should be updated to keep the first frequency of the structure in the air is approximate to that in the water.

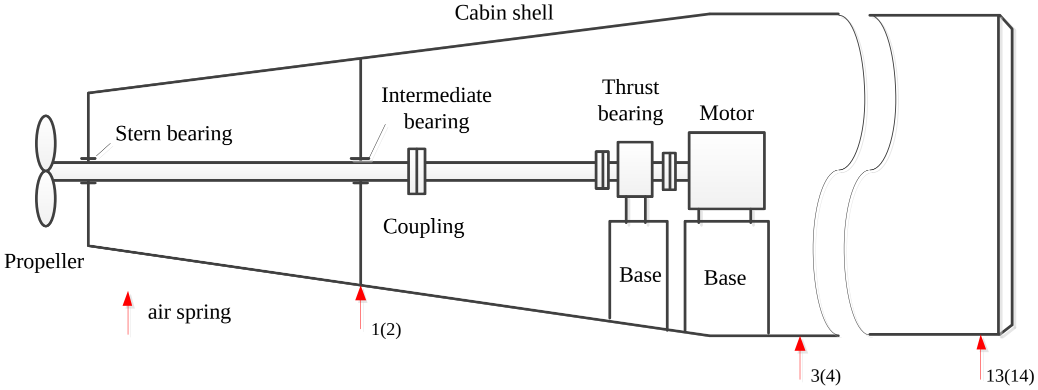


Figure 1: The ship shaft-hull system

The photographs of the main experimental set-up are shown in Fig. 2. The boundary condition of the ship hull-shell system is elastic because of the air springs.

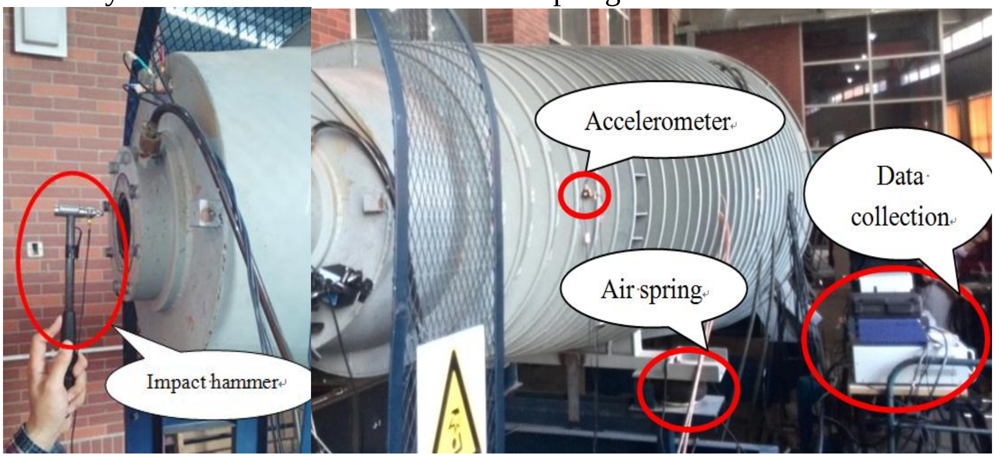


Figure 2: The main experimental set-up

A dynamic model to study the longitudinal vibration using FRF-based substructuring method will established according to this experiment setup, and the experiment studying the longitudinal vibration characteristics will also carried out on this testrig.

3. Numerical model

The ship shaft-hull system is established using FRF-based substructuring method, the modelling procedure is as follows.

3.1 Frequency response functions of subsystems

Consider the shaft and the hull are two separated subsystems. They can be simply implementation through classical finite element method. The dynamic equation for these two subsystem can both be presented as

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{Q}(t)\,, \tag{1}$$

where, **M**, **K** and **C** are mass, stiffness and damping matrices for each subsystem, u(t) represents the displacement of the subsystem dofs, and $\mathbf{Q}(t)$ is the external load.

The first n nature frequencies and their regularized corresponding modes are related to the following equation

$$\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_i = 1, (i = 1, 2, ..., n) \tag{2}$$

Assume the longitudinal nature frequencies their regularized corresponding modes are $\omega_{z,i}$ and 和 $W_i(\overline{z})$, the vibration frequency response function (FRF) of the subsystems can be obtained by former P order modal superposition, namely：

$$H_z(z_a, z_b) = \sum_{n=0}^{P} \frac{W_n(\overline{z_a})W_n(\overline{z_b})}{\omega_{z,n}^2 - \omega^2 + 2j\omega\omega_{z,n}\xi_r} \tag{3}$$

where, ω_n is the nature frequency of the subsystem， ω is the excitation frequency， ξ_r is the system modal damping ratio and $j = \sqrt{-1}$.

If P nodes of the subsystem is studied, the FRF of the subsystem can be represented as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}(u_1, u_1) & \cdots & \mathbf{H}(u_1, u_P) \\ \vdots & \cdots & \vdots \\ \mathbf{H}(u_P, u_1) & \cdots & \mathbf{H}(u_P, u_P) \end{bmatrix} \tag{4}$$

3.2 Connection of the two subsystems

Define the two subsystems (shaft and hull) are connected through elastic springs which can be defined using stiffness or using impedance, as shown in Fig. 3.

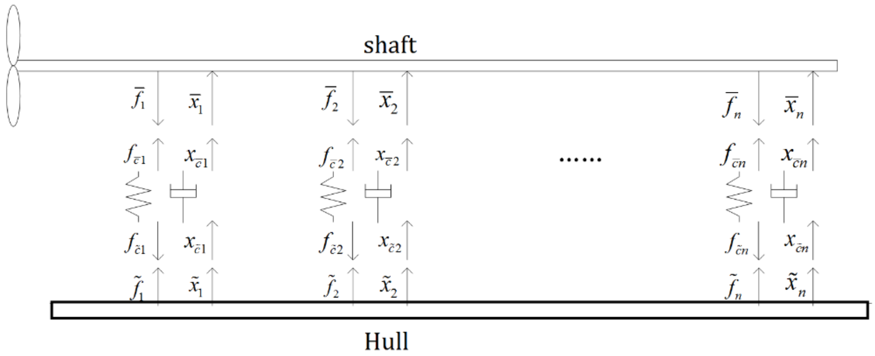


Figure 3: The coupling between two subsystems

The force and displacement relationship between at the connection point is

$$f = Zx \tag{5}$$

where：

$f = \left\{ \bar{f} \quad \tilde{f} \right\}$, $\bar{f} = \left\{ \bar{f}_1, \bar{f}_2, ..., \bar{f}_n \right\}$, $\tilde{f} = \left\{ \tilde{f}_1, \tilde{f}_2, ..., \tilde{f}_n \right\}$, $x_{\bar{c}} = \left\{ x_{\bar{c}1}, x_{\bar{c}2}, ..., x_{\bar{c}n} \right\}^{\mathrm{T}}$, $x_{\tilde{c}} = \left\{ x_{\tilde{c}1}, x_{\tilde{c}2}, ..., x_{\tilde{c}n} \right\}^{\mathrm{T}}$

Z is the impedance matrix of the elastic connections， where $Z = \begin{bmatrix} Z_{\bar{c}\bar{c}} & Z_{\bar{c}\tilde{c}} \\ Z_{\tilde{c}\bar{c}} & Z_{\tilde{c}\tilde{c}} \end{bmatrix}$， $Z_{\bar{c}\bar{c}} = Z_{\bar{c}\bar{c}}{}^{T}$, $Z_{\bar{c}\tilde{c}} = Z_{\tilde{c}\bar{c}}{}^{T}$ and $Z_{\tilde{c}\tilde{c}} = Z_{\tilde{c}\tilde{c}}{}^{T}$.

3.3 Frequency response function of the ship shaft-hull system

To resolve the force and displacement relationship at the connection point, a simple implementation can using inner force balancing before and after the connecting of two subsystems following reference [16]. Thus the FRF equation the shaft-hull system combined by shaft (subsystem A) and hull (subsystem B) is

$$\begin{Bmatrix} X_I^A \\ X_I^B \\ X_C^A \\ X_C^B \end{Bmatrix} = \begin{bmatrix} H_{II}^A & H_{II}^{AB} & H_{IC}^{AA} & H_{IC}^{AB} \\ & H_{II}^B & H_{IC}^{BA} & H_{IC}^{BB} \\ & & H_{CC}^A & H_{CC}^{AB} \\ sym & & & H_{CC}^B \end{bmatrix} \begin{Bmatrix} F_I^A \\ F_I^B \\ F_C^A \\ F_C^B \end{Bmatrix} \tag{6}$$

where

$H_{II} = H_{ii} + H_{i\bar{c}} D H_a + H_{i\bar{c}} D^T H_b$， $H_{I\bar{C}} = H_{i\bar{c}} D H_c - H_{i\tilde{c}} D^T H_{\bar{c}\bar{c}}$， $H_{\bar{C}\bar{C}} = H_{\bar{c}\bar{c}} D H_c$

4. Dynamic characteristics of the ship shaft-hull system

The modes and nature frequencies of the shaft, the hull and the shaft-hull system are firstly analyzed separately, then the longitudinal vibration characteristics of the shaft, hull and shaft-hull system are also simulated.

4.1 Nature frequencies and modes of the propeller/shaft system

The shaft is 3.44m long and with a diameter of 0.07m. The density is 7,800kg/m3, the elastic modulus E is 210GPa and the Poisson's ratio is 0.3. The finite element model of the shaft system is modeled using Timoshenko beam theory and meshed with 20 beam elements. The propeller, the middle coupling and other components which are attached to the shaft are simplified with associated concentrated mass and centralized inertia as given in Tab. 1. The gyroscopic effect of the propeller is considered. The model is established in the Cartesian coordinate system. X, Y and Z represent the axial, the horizontal and the vertical directions, respectively.

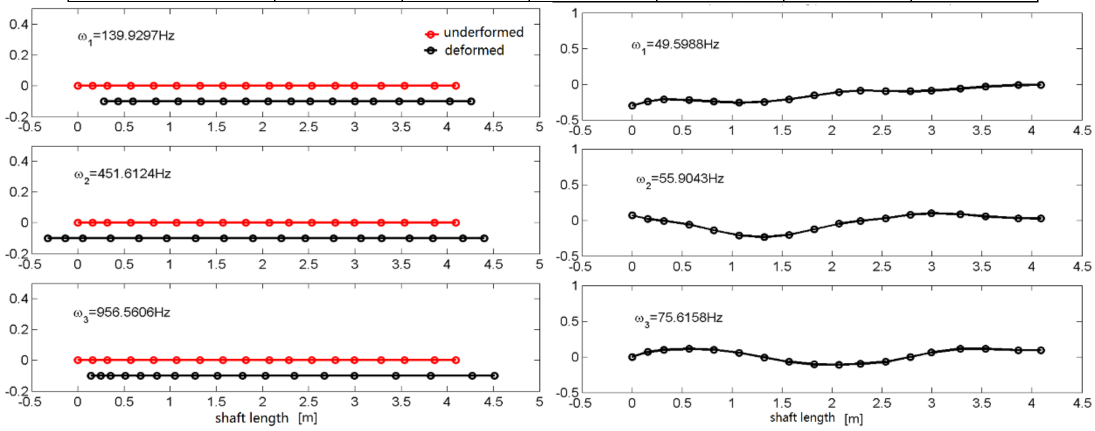
Table 1: Parameters of concentrated mass and centralized inertia

Component	Concentrated mass (kg)	Polar moment of inertia (kg·m2)	Moment of inertia (kg·m2)
Propeller	49.2	0.7433	0.3716
Middle coupling	5.51	0.0306	0.0153
Thrust plate	7.2	0.0339	0.0169

Getting rid of the zero frequency, the first three longitudinal and first three lateral natural frequencies of the shaft are shown in Tab. 2 and the first two corresponding mode shapes are given in Fig. 5.

Table 2: Natural frequency of the shaft system (Hz)

Order	Longitudinal			Lateral		
	1	2	3	1	2	3
Frequency	139.9	451.6	956.6	49.6	55.9	75.6



(a) Longitudinal nature frequency and modes (b) Lateral nature frequency and modes

Figure 5: Mode shapes of the shaft system

4.2 Nature frequencies and modes of the hull system

The hull is mainly composed by four parts: the bearing seat (including bearing shell), motor, base and the cabin shell. The cabin shell is the combination of six cylindrical and one conical segments. The length of the hull and the conical segment are 18.58m and 5.08m separately. The structure and main components of the hull are shown in Fig. 6.

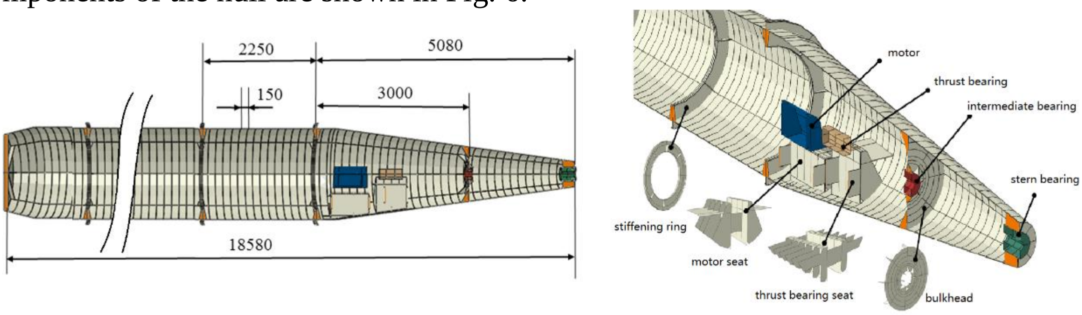
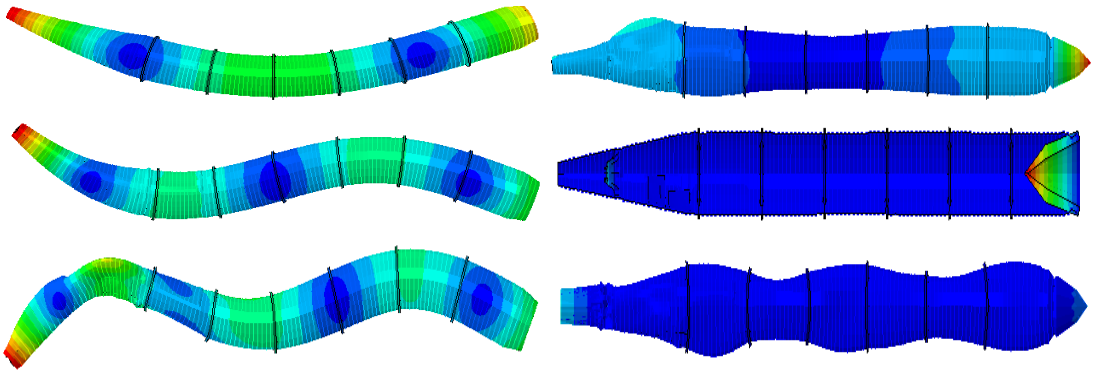


Figure 6: FE model of the hull

Also getting rid of the zero frequency, the first three lateral and first two longitudinal natural frequencies of the hull are shown in Tab. 3 and the first three corresponding mode shapes are given in Fig. 7.

Table 3: Natural frequency of the hull system (Hz)

Order	Longitudinal			Lateral		
	1	2	3	1	2	3
Frequency	105.1	134.1	153.4	24.8	56.4	78.2



(a) Lateral nature frequency and modes (b) Longitudinal nature frequency and modes

Figure 7: Mode shapes of the hull system

4.3 Frequency response function of the shaft-hull system

Using FRF-based substructuring method introduced in section 2, the lateral and longitudinal FRF responses of the shaft-hull system at thrust bearing is shown in Fig. 8. The external force is a sinusoidal signal from 1Hz to 200Hz with an amplitude of 1N and frequency resolution 0.1Hz added at the longitudinal direction at the middle of propeller hub of the shaft. The nature frequencies and modes of the shaft-hull system are also simulated using ABAQUS which are compared to the FRF-based substructuring method and listed in Tab. 4.

Table 4: Natural frequency of the shaft-hull system (Hz)

	FEM method		substructuring method	Error(%)
	modes	Nature Frequency	Nature frequency	
a	1St B Hull	24.6	25.2	2.4
b	1stB hull + 1st B Shaft	54.9	58.4	2.6
c	2nd B Shaft	59.7	64.5	2.8
d	3rd B Shaft	74.4	76.3	2.3
g	1st L Hull	104.6	108.6	3.8
h	2nd L Hull	128.6	131.1	1.9
i	1st L Shaft	140.4	139.9	0.1
j	3rd L Hull	152.6	159.8	4.7

* B for bending, L for longitudinal

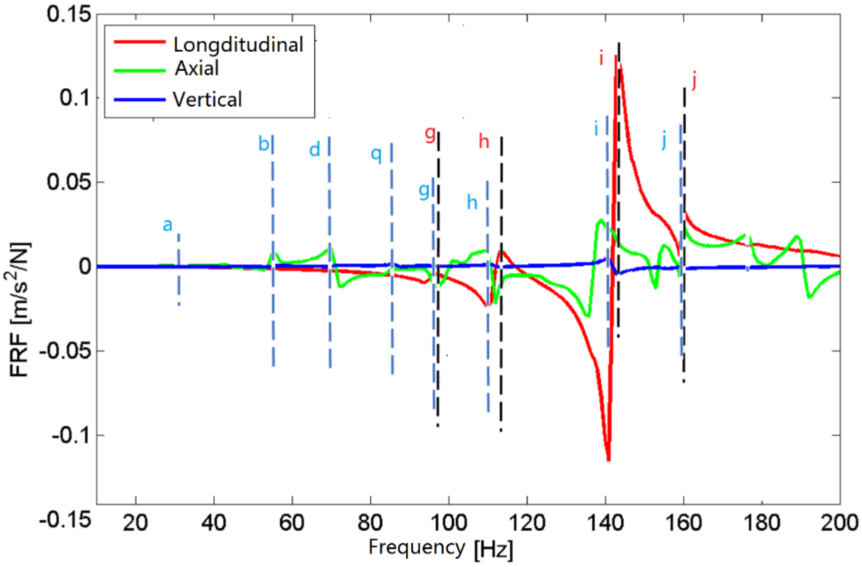


Figure 7: FRF responses of the shaft-hull system

5. Experiment results and conclusion

The Experiment is carried out using an impact hammer excited on the longitudinal direction of the propeller hub, the FRF response (Fig. 8) at the thrust bearing is acquisited using a three directional accelerometer amounted on the thrust bearing seat. The nature frequencies of the shaft-hull system are compared with experiment results and FRF-based substructuring method listed in Tab. 5.

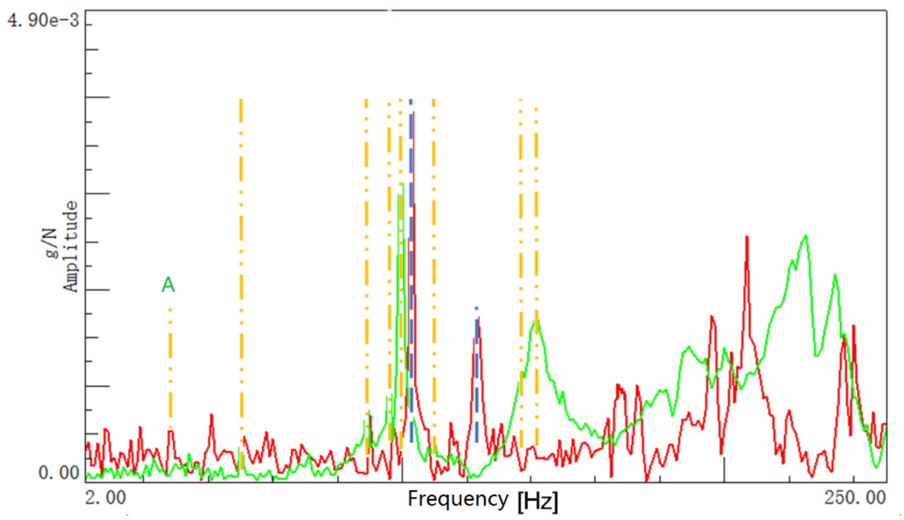


Figure 8: FRF responses of the shaft-hull system

Table 5: Compare of natural frequency of the shaft-hull system (Hz)

substructuring method		Experiment		Error（%）
Modes	Frequency	Modes	Frequency	
1St B Hull	25.2	1St B Hull	23.5	7.2
1stB hull + Shaft	58.4	1stB hull + Shaft	52.7	10.8
2nd B Shaft	64.5	2nd B Shaft	60.8	6.1
3rd B Shaft	76.3	3rd B Shaft	71.8	6.3
1st L Hull	108.6	1st L Hull	100.9	7.6
1st L Shaft	139.9	1st L Shaft	138.3	1.2

* B for bending, L for longitudinal

Compared the nature frequency and mode results to the shaft and hull subsystems with FRF based substructuring method, FE model and experiment results, the conclusions can be derived as:

(1) Under longitudinal force excitation, the frequencies of the amplitude of the shaft-hull system FRF response are very close to that using FE modal analysis, and the modes are corresponding, which verifies the correctness of the modelling procedure.

(2) The longitudinal nature frequencies and modes of the shaft or hull subsystem or the coupling of them are all been excited, e.g. amplitude of point g, h, j of FRF response represent the 1st to the 3rd longitudinal modes of the hull and amplitude of point i represents the 1st longitudinal mode of the shaft.

(3) Due to the asymmetry of the system, the lateral nature frequencies and modes are also been excited, such as the amplitude of point a, b, c of FRF response represent the 1st Bending of hull, the coupling of 1st bending of hull and 1st bending of shaft and the 2nd bending of shaft, respectively.

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